

# Quadratic equations

# Introduction

In this presentation we will review different methods for solving quadratic equations.

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- factorization,
- completing the square,
- quadratic formula.

We will now review these methods.



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Factorization should always be your first choice. If you can factorize a given expression quickly, then you will save lots of time. Only if you can't factorize the given quadratic, should you move on to a different method.

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Solve

$$x^2 - 3x - 18 = 0$$

We factorize the left hand side to get:

$$(x - 6)(x + 3) = 0$$

So  $x - 6 = 0$  or  $x + 3 = 0$ . Which gives  $x = 6$  or  $x = -3$ .

# Factorization - examples

a) Solve:

$$x^2 + 2x - 15 = 0$$

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# Factorization

Remember that we constantly use the fact that if a product of two numbers is 0, then one of the numbers must be 0.

## Important property

If  $a \times b = 0$ , then  $a = 0$  or  $b = 0$ .

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## Useless property

If  $a \times b = 7$  (or any other non-zero number), then we don't know much about  $a$  or  $b$ .

Factorization doesn't always work and if after a few seconds we cannot factorize the given expression, then we should try a different approach.

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We have  $x^2 + 4x = (x + 2)^2 - 4$

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We get:

$$(x + 2)^2 - 16 = 0$$

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$x + 2$  squared gives 16, so  $x + 2 = 4$  or  $x + 2 = -4$ , which gives  $x = 2$  or  $x = -6$ .

# Completing the square

The method is fairly simple:

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We want to change the left hand side to the form

$$(x \dots)^2 - \dots = 0$$

We just need to put appropriate numbers in place of dots.



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$(x + 2)^2 = x^2 + 4x + 4$ . the first two terms agree, we need to change the last one. We want  $-12$  and we have  $4$ , so we need to subtract  $16$ .

Finally we have  $x^2 + 4x - 12 = (x + 2)^2 - 16$ .

# Names

Let's look at the equation once more:

$$x^2 + 4x - 12 = 0$$

The left hand side of the equation is a quadratic in a **standard form**.

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$$(x - 2)(x + 6) = 0$$

This form is called a **factored form**.

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This is called a **vertex form**.

We will talk more about these forms when we will be covering quadratic functions.



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Turn  $x^2 + 6x - 2$  into vertex form. Hence solve  $x^2 + 6x - 2 = 0$ .

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So we have  $(x + 3)^2$ , which gives  $(x + 3)^2 = x^2 + 6x + 9$ , but instead of  $9$  we want  $-2$ , so we need to subtract  $11$ .

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So we have  $(x + 3)^2$ , which gives  $(x + 3)^2 = x^2 + 6x + 9$ , but instead of  $9$  we want  $-2$ , so we need to subtract  $11$ . Finally:

$$x^2 + 6x - 2 = (x + 3)^2 - 11$$

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Now we want to solve:

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We turn the left hand side into vertex form:

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so:

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so  $x + 3 = \sqrt{11}$  or  $x + 3 = -\sqrt{11}$ .



## Completing the square - example

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We turn the left hand side into vertex form:

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so  $x + 3 = \sqrt{11}$  or  $x + 3 = -\sqrt{11}$ .

This gives  $x = -3 + \sqrt{11}$  or  $x = -3 - \sqrt{11}$ .

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so  $x + 3 = \sqrt{11}$  or  $x + 3 = -\sqrt{11}$ .

This gives  $x = -3 + \sqrt{11}$  or  $x = -3 - \sqrt{11}$ .

Note that we wouldn't be able to solve the equation  $x^2 + 6x - 2 = 0$  by factorizing it, or at least it would be very hard.

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We will first divide both sides by 2, this gives:

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If we want to solve an equation like:

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We will first divide both sides by 2, this gives:

$$x^2 + 3x - \frac{3}{2} = 0$$

Now we complete the square:

$$\left(x + \frac{3}{2}\right)^2 - \frac{15}{4} = 0$$

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If we want to solve an equation like:

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We will first divide both sides by 2, this gives:

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$$\left(x + \frac{3}{2}\right)^2 - \frac{15}{4} = 0$$

So

$$\left(x + \frac{3}{2}\right)^2 = \frac{15}{4}$$

which gives  $x + \frac{3}{2} = \pm \frac{\sqrt{15}}{2}$ , so  $x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}$ .

## Completing the square - example

If we want to solve an equation like:

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We will first divide both sides by 2, this gives:

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Now we complete the square:

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So

$$\left(x + \frac{3}{2}\right)^2 = \frac{15}{4}$$

which gives  $x + \frac{3}{2} = \pm \frac{\sqrt{15}}{2}$ , so  $x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}$ . Note  $\pm$  means that there are two solutions, one when we add the given number, the other when we subtract it.

# Quadratic formula

The method of completing the square led us to a formula for solving quadratic equations:

$$ax^2 + bx + c = 0$$



# Quadratic formula

The method of completing the square led us to a formula for solving quadratic equations:

$$ax^2 + bx + c = 0$$

The formula we derived is  $x = \frac{-b \pm \sqrt{\Delta}}{2a}$ , where  $\Delta = b^2 - 4ac$ .

# Quadratic formula

If we want to solve:

$$2x^2 + 6x - 3 = 0$$

then we have  $a = 2$ ,  $b = 6$  and  $c = -3$ .

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We first calculate  $\Delta$ :

$$\Delta = 6^2 - 4(2)(-3) = 60$$

# Quadratic formula

If we want to solve:

$$2x^2 + 6x - 3 = 0$$

then we have  $a = 2$ ,  $b = 6$  and  $c = -3$ .

We first calculate  $\Delta$ :

$$\Delta = 6^2 - 4(2)(-3) = 60$$

$$\text{So } x = \frac{-6 \pm \sqrt{60}}{4} = \frac{-6 \pm 2\sqrt{15}}{4} = \frac{-3 \pm \sqrt{15}}{2}$$

# Practice

When you solve a quadratic equation, you should start by trying factorization, then if it doesn't work use the quadratic formula. The completing the square method is still important and we will use it when we will be dealing with quadratic functions.

# Practice

Solve:

$$x^2 - 6x - 7 = 0$$

# Practice

Solve:

$$x^2 - 6x - 7 = 0$$

Method:

# Practice

Solve:

$$x^2 - 6x - 7 = 0$$

Method: factorization!



# Practice

Solve:

$$x^2 - 6x - 7 = 0$$

Method: factorization!

$$(x - 7)(x + 1) = 0$$

so  $x = 7$  oraz  $x = -1$ .

# Practice

Solve:

$$2x^2 - x - 15 = 0$$

# Practice

Solve:

$$2x^2 - x - 15 = 0$$

Method:

# Practice

Solve:

$$2x^2 - x - 15 = 0$$

Method: factorization!

# Practice

Solve:

$$2x^2 - x - 15 = 0$$

Method: factorization!

$$(2x + 5)(x - 3) = 0$$

so  $x = -2.5$  oraz  $x = 3$ .

# Practice

Solve:

$$x^2 + 5x + 1 = 0$$

# Practice

Solve:

$$x^2 + 5x + 1 = 0$$

Method:

# Practice

Solve:

$$x^2 + 5x + 1 = 0$$

Method: quadratic formula (factorization doesn't work nicely)



# Practice

Solve:

$$x^2 + 5x + 1 = 0$$

Method: quadratic formula (factorization doesn't work nicely)

$a = 1$ ,  $b = 5$ ,  $c = 1$ , so

$$\Delta = 25 - 4(1)(1) = 21$$

# Practice

Solve:

$$x^2 + 5x + 1 = 0$$

Method: quadratic formula (factorization doesn't work nicely)

$a = 1$ ,  $b = 5$ ,  $c = 1$ , so

$$\Delta = 25 - 4(1)(1) = 21$$

So we have:

$$x = \frac{-5 \pm \sqrt{21}}{2}$$

# Practice

Solve:

$$3x^2 + 5x = 0$$

# Practice

Solve:

$$3x^2 + 5x = 0$$

Method:

# Practice

Solve:

$$3x^2 + 5x = 0$$

Method: factorization!

# Practice

Solve:

$$3x^2 + 5x = 0$$

Method: factorization!

$$x(3x + 5) = 0$$

so  $x = 0$  oraz  $x = -\frac{5}{3}$ .

# Practice

Solve:

$$2x^2 + 3x - 1 = 0$$

# Practice

Solve:

$$2x^2 + 3x - 1 = 0$$

Method:



# Practice

Solve:

$$2x^2 + 3x - 1 = 0$$

Method: quadratic formula (factorization doesn't work)

# Practice

Solve:

$$2x^2 + 3x - 1 = 0$$

Method: quadratic formula (factorization doesn't work)

$a = 2$ ,  $b = 3$ ,  $c = -1$ , so

$$\Delta = 9 - 4(2)(-1) = 17$$

# Practice

Solve:

$$2x^2 + 3x - 1 = 0$$

Method: quadratic formula (factorization doesn't work)

$a = 2$ ,  $b = 3$ ,  $c = -1$ , so

$$\Delta = 9 - 4(2)(-1) = 17$$

So we have:

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

# Practice

Solve:

$$9x^2 - 4 = 0$$

# Practice

Solve:

$$9x^2 - 4 = 0$$

Method:

# Practice

Solve:

$$9x^2 - 4 = 0$$

Method: factorization!

# Practice

Solve:

$$9x^2 - 4 = 0$$

Method: factorization!

$$(3x - 2)(3x + 2) = 0$$

so  $x = \frac{2}{3}$  oraz  $x = -\frac{2}{3}$ .

# Practice

Solve:

$$3x^2 + 14x + 8 = 0$$



# Practice

Solve:

$$3x^2 + 14x + 8 = 0$$

Method:

# Practice

Solve:

$$3x^2 + 14x + 8 = 0$$

Method: factorization!

# Practice

Solve:

$$3x^2 + 14x + 8 = 0$$

Method: factorization!

$$(3x + 2)(x + 4) = 0$$

so  $x = -\frac{2}{3}$  oraz  $x = -4$ .

# Practice

Solve:

$$2x^2 - 6x + 3 = 0$$

# Practice

Solve:

$$2x^2 - 6x + 3 = 0$$

Method:

# Practice

Solve:

$$2x^2 - 6x + 3 = 0$$

Method: quadratic formula (factorization doesn't work)

# Practice

Solve:

$$2x^2 - 6x + 3 = 0$$

Method: quadratic formula (factorization doesn't work)

$a = 2$ ,  $b = -6$ ,  $c = 3$ , so

$$\Delta = 36 - 4(2)(3) = 12$$

# Practice

Solve:

$$2x^2 - 6x + 3 = 0$$

Method: quadratic formula (factorization doesn't work)

$a = 2$ ,  $b = -6$ ,  $c = 3$ , so

$$\Delta = 36 - 4(2)(3) = 12$$

So we have:

$$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$



Make sure you practice all three methods and are confident using them all.