

Name:

Result:

1. (4 points)

Tomasz invests 150 000 PLN into savings account that pays 8% p.a. compounded quarterly.

(a) Find the value of the investment after 5 years. [2]

The average inflation rate over these 5 years has been equal to 6.5% per year.

(b) Estimate the real value of Tomasz's investment after 5 years. [2]

(a) Either $FV = 150000 \cdot (1.02)^{20} = 222892.11$ or:

$$n = 20 \quad I\% = 8 \quad PV = -150000 \quad PMT = 0 \quad FV = ? \quad P/Y = C/Y = 4$$

and solve for FV to get 222892.11

(b) We want $\frac{FV}{1.065^5} = 162684.68$.

NOTE: The IB also accepts a different method where you subtract the inflation rate from the interest rate. This gives a different answer - both would be accepted on the exam. However only the one we use gives an accurate real value.

2.*(5 points)*

Explorers Wanda and Tomasz are located at $(3, 4, 1)$ and $(-1, 2, 3)$ respectively. The units are kilometres, $Z = 0$ represents the sea level and all coordinates are given to the nearest kilometre.

(a) Use the above measurements to find the angle of elevation from Wanda to Tomasz. Give your answer to 4 significant figures. [2]

(b) Find the maximum percentage error of your answer to part (a). [3]

(a) Let us calculate the horizontal and vertical distances between W and T . The vertical distance is simply 2. The horizontal distance is $\sqrt{4^2 + 2^2} = \sqrt{20}$. The angle of elevation is then (using *SOHCAHTOA*):

$$\tan \theta = \frac{2}{\sqrt{20}}$$

so $\theta \approx 24.09^\circ$.

(b) θ is largest when the horizontal distance is smallest and the vertical distance is largest. The largest vertical distance is 3. The smallest horizontal distance is $\sqrt{3^2 + 1^2} = \sqrt{10}$, so we have:

$$\theta_{max} = \tan^{-1} \left(\frac{3}{\sqrt{10}} \right) = 43.491519\dots^\circ$$

Similarly θ is smallest when the horizontal distance is largest and the vertical distance is smallest. The smallest vertical distance is 1. The largest horizontal distance is $\sqrt{5^2 + 3^2} = \sqrt{34}$, so:

$$\theta_{min} = \tan^{-1} \left(\frac{1}{\sqrt{34}} \right) = 9.731475\dots^\circ$$

We then have two options. Underestimation:

$$\epsilon\% = \frac{|43.491519\dots - 24.09|}{43.491519\dots} \cdot 100\% \approx 44.6\%$$

Overestimation:

$$\epsilon\% = \frac{|9.731475\dots - 24.09|}{9.731475\dots} \cdot 100\% \approx 148\%$$

So the largest possible percentage error is approximately 148%.

3. (6 points)
 The base of a square-based pyramid has coordinates $A(3, 0, 1)$, $B(-1, 1, 1)$, $C(0, 5, 1)$ and D .
 The apex E lies directly above the centre of the base. 1 unit represents 10 metres.

(a) Find the coordinates of D . [2]

The angles that the edges AE , BE , CE and DE make with the base plane are all equal to 70° .

(b) Find the height of the pyramid. [2]

(c) Find the volume of the pyramid in square metres. Express your answer in the standard form. [2]

(a) The midpoint of AC is also the midpoint of BD . $M_{AC} = (1.5, 2.5, 1)$, so $D = (4, 4, 1)$.

(b) Note that we have $h = |M_{AC}E|$ and $\angle M_{AC}AE = 70^\circ$. Using *SOHCAHTOA*:

$$\tan 70^\circ = \frac{h}{|M_{AC}A|}$$

But $|M_{AC}A| = \sqrt{(1.5)^2 + (2.5)^2 + 0^2} \approx 2.92$, so:

$$h = 2.9154759... \cdot \tan(70^\circ) \approx 8.01 = 80.1 \text{ m}$$

(c) The base is a square with side-length $|AB| = \sqrt{4^2 + 1^2 + 0^2} = \sqrt{17} \approx 4.12 = 41.2 \text{ m}$, so the volume of the pyramid is:

$$V = \frac{1}{3} \cdot (41.231056...)^2 \cdot 80.10204.. \approx 45400 \text{ m}^3 = 4.54 \cdot 10^4 \text{ m}^3$$

4.

(8 points)

Maria wants to buy a new Tesla model 3 which costs 220 000 PLN. She saved 80 000 PLN and wants to loan the remaining funds. The terms of the loan are as follows. The interest rate is 11.5% p.a. compounded monthly and the loan is to be repaid in equal monthly instalments in 10 years.

(a) How much does Maria need to borrow? [1]

(b) Calculate the monthly repayments. [2]

(c) Calculate the total interest that Maria will pay. [2]

After 5 years Maria is allowed to repay the loan in one final payments.

(d) Calculate how much she would save, if she would do so. [3]

(a) 140 000 PLN

(b) We have:

$$n = 120 \quad I\% = 11.5 \quad PV = 140000 \quad PMT = ? \quad FV = 0 \quad P/Y = C/Y = 12$$

Solve for PMT and we get 1968.34 PLN.

(c) The total interest is:

$$120 \cdot 1968.34 - 140000 = 96200.8 \text{ PLN}$$

(d) The outstanding balance after 5 years is:

$$n = 60 \quad I\% = 11.5 \quad PV = 140000 \quad PMT = -1968.34 \quad FV = ? \quad P/Y = C/Y = 12$$

89499.60 PLN. So this will be her final payment. If she didn't make the payment, she would need to pay 60 instalments of 1968.34 PLN, so she can save:

$$60 \cdot 1968.34 - 89499.60 = 28600.8 \text{ PLN}$$

5.*(7 points)*

Consider a triangle ABC , with $A(1, -3, 2)$, $B(k, 0, 1)$ and C

(a) The distance from point A and B is $\sqrt{14}$. Find the value of k , given that $k > 0$. [2]

(b) The midpoint between A and C is $(-1, 1, 0)$. Find the coordinates of C . [2]

(c) Calculate the angle BAC . [3]

(a) $|AB| = \sqrt{(k-1)^2 + 3^2 + 1^2}$, so we have:

$$\sqrt{(k-1)^2 + 10} = \sqrt{14}$$

This gives $(k-1)^2 = 4$, so $k-1 = \pm 2$, so $k = 3$ as it needs to be positive.

(b) $C = (-3, 5, -2)$.

(c) We want to apply the cosine rule, but we need to calculate all the sides first. We already have $|AB| = \sqrt{14}$.

$$|AC| = \sqrt{4^2 + 8^2 + 4^2} = \sqrt{96} \quad |BC| = \sqrt{6^2 + 5^2 + 3^2} = \sqrt{70}$$

the side opposite to the angle BAC is the side BC , so we have:

$$\cos \alpha = \frac{14 + 96 - 70}{2 \cdot \sqrt{14} \cdot \sqrt{96}} = 0.5455447\dots$$

so $\alpha \approx 56.9^\circ$.