AI HL 24.10 (vectors) [49 marks]

1. [Maximum mark: 6] 19M.1.SL.TZ2.S_2

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

(a) parallel. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $\boldsymbol{b} = 2\boldsymbol{a}$, $\text{a} = k\boldsymbol{b}$, $\cos\theta = 1$, $\boldsymbol{a}\bullet\boldsymbol{b} = -|\boldsymbol{a}||\boldsymbol{b}|$, $2p = 18$

$p = 9$ ***A1 N2***

[2 marks]

(b) perpendicular. [4]

Markscheme

evidence of scalar product ***(M1)***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$, $(0)(0) + (3)(6) + p(18)$

recognizing $\boldsymbol{a}\bullet\boldsymbol{b} = 0$ (seen anywhere) ***(M1)***

correct working **(A1)**

eg $18 + 18p = 0$, $18p = -18$ ***(A1)***

$p = -1$ ***A1 N3***

[4 marks]

2. [Maximum mark: 6] 17N.2.SL.TZ0.S_3

Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

Markscheme

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correct substitution ***(A1)***

eg $\sqrt{4^2 + 1^2 + 2^2}$

4.58257

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{21}$ (exact), 4.58 ***A1 N2***

[2 marks]

(b) Let $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $\mathrm{B\hat{A}C}$. [4]

Markscheme

finding scalar product and $\left|\overrightarrow{\mathrm{AC}}\right|$ ***(A1)(A1)***

scalar product $= (4 \times 3) + (1 \times 0) + (2 \times 0)\ (= 12)$

$$\left|\overrightarrow{\mathrm{AC}}\right| = \sqrt{3^2 + 0 + 0}\ (= 3)$$

substituting **their** values into cosine formula *(M1)*

eg $\cos \mathrm{B}\hat{A}\mathrm{C} = \frac{4\times3+0+0}{\sqrt{3^2}\times\sqrt{21}},\ \frac{4}{\sqrt{21}},\ \cos\theta = 0.873$

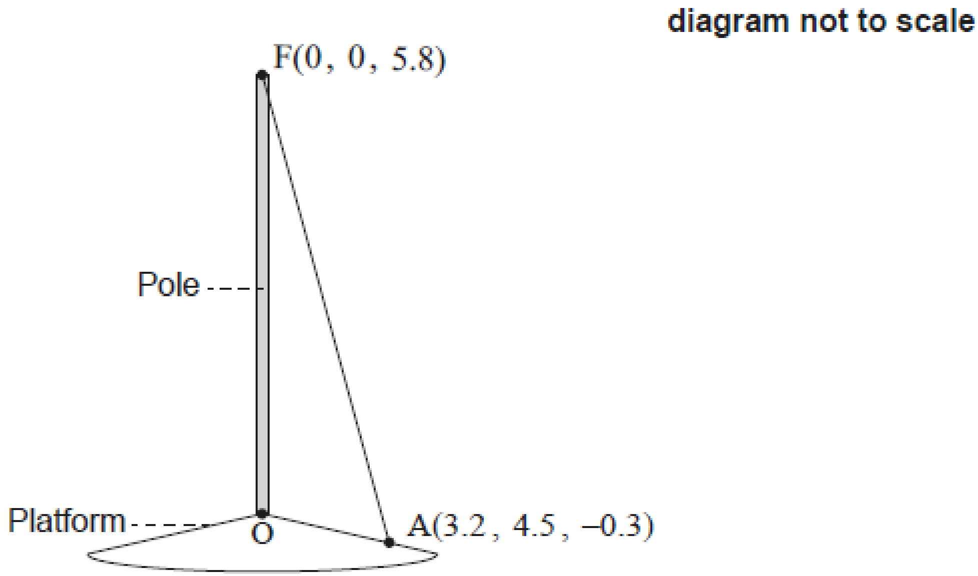
0.509739 (29.2059°)

$\mathrm{B\hat{A}C} = 0.510$ (29.2°) ***A1 N2***

[4 marks]

3. [Maximum mark: 8] 22M.1.AHL.TZ1.6

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0,\ 0,\ 5.8)$. All units are in metres.



The pole is held in place by ropes attached at F.

One of these ropes is attached to the platform at point $\mathrm{A}(3.2,\ 4.5,\ -0.3)$. The rope forms a straight line from A to F.

(a) Find $\overrightarrow{\mathrm{AF}}$. [1]

Markscheme

$$\begin{pmatrix} -3.2 \\ -4.5 \\ 6.1 \end{pmatrix}$$ ***A1***

[1 mark]

(b) Find the length of the rope. [2]

Markscheme

$\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2}$ ***(M1)***

$8.22800\ldots \approx 8.23\,\text{m}$ ***A1***

[2 marks]

(c) Find $\text{F}\hat{\text{A}}\text{O}$, the angle the rope makes with the platform. [5]

Markscheme

EITHER

$\overrightarrow{\text{AO}} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix}$ ***A1***

$\cos\theta = \frac{\overrightarrow{\text{AO}} \cdot \overrightarrow{\text{AF}}}{\left|\overrightarrow{\text{AO}}\right|\left|\overrightarrow{\text{AF}}\right|}$

$\overrightarrow{\text{AO}} \cdot \overrightarrow{\text{AF}} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) \ (= 32.32)$ ***(A1)***

$\cos\theta = \frac{32.32}{\sqrt{3.2^2+4.5^2+0.3^2} \times 8.22800\ldots}$ ***(M1)***

$= 0.710326\ldots$ ***(A1)***

Note: If $\overrightarrow{\mathrm{OA}}$ is used in place of $\overrightarrow{\mathrm{AO}}$ then $\cos\,\theta$ will be negative.
Award ***A1(A1)(M1)(A1)*** as above. In order to award the final ***A1***, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$\mathrm{AO} = \sqrt{3.\,2^2 + 4.\,5^2 + 0.\,3^2}\ \ (= 5.\,52991\ldots)$ ***(A1)***

$\cos\,\theta = \frac{8.22800\ldots^2 + 5.52991\ldots^2 - 5.8^2}{2 \times 8.22800\ldots \times 5.52991\ldots}$ ***(M1)(A1)***

$= 0.\,710326\ldots$ ***(A1)***

THEN

$\theta = 0.\,780833\ldots \approx 0.\,781$ OR $44.\,7384\ldots^\circ \approx 44.\,7^\circ$ ***A1***

[5 marks]

4. [Maximum mark: 7] 19M.1.SL.TZ1.S_6

The magnitudes of two vectors, ***u*** and ***v***, are 4 and $\sqrt{3}$ respectively. The angle between ***u*** and ***v*** is $\frac{\pi}{6}$.

Let ***w*** = ***u*** − ***v***. Find the magnitude of ***w***. [7]

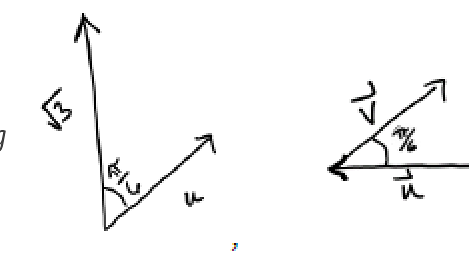
Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1 (cosine rule)

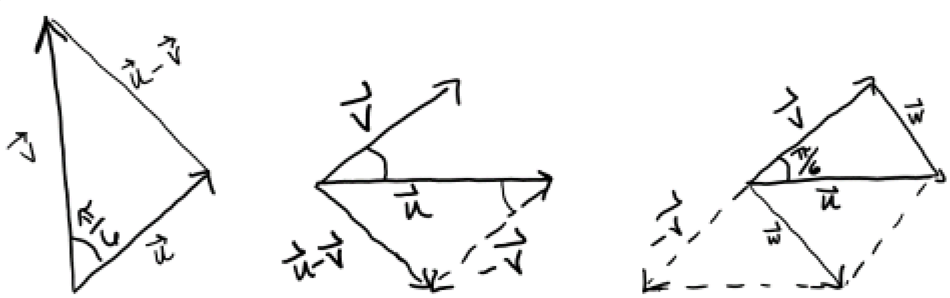
diagram including ***u***, ***v*** and included angle of $\frac{\pi}{6}$ ***(M1)***

eg



sketch of triangle with ***w*** (does not need to be to scale) ***(A1)***

eg



choosing cosine rule ***(M1)***

eg $a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

eg $4^2 + \left(\sqrt{3}\right)^2 - 2\left(4\right)\left(\sqrt{3}\right)\cos\frac{\pi}{6}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ (seen anywhere) ***(A1)***

correct working ***(A1)***

eg 16 + 3 – 12

$|\boldsymbol{w}| = \sqrt{7}$ ***A1 N2***

METHOD 2 (scalar product)

valid approach, in terms of u and v (seen anywhere) ***(M1)***

eg $|\boldsymbol{w}|^2 = (\boldsymbol{u} - \boldsymbol{v})\bullet(\boldsymbol{u} - \boldsymbol{v})$, $|\boldsymbol{w}|^2 = \boldsymbol{u}\bullet\boldsymbol{u} - 2\boldsymbol{u}\bullet\boldsymbol{v} + \boldsymbol{v}\bullet\boldsymbol{v}$, $|\boldsymbol{w}|^2 = (u_1 - v_1)^2 + (u_2 - v_2)^2$,

$|\boldsymbol{w}| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2}$

correct value for $\boldsymbol{u}\bullet\boldsymbol{u}$ (seen anywhere) ***(A1)***

eg $|\boldsymbol{u}|^2 = 16$, $\boldsymbol{u}\bullet\boldsymbol{u} = 16$, ${u_1}^2 + {u_2}^2 = 16$

correct value for $\boldsymbol{v}\bullet\boldsymbol{v}$ (seen anywhere) ***(A1)***

eg $|\boldsymbol{v}|^2 = 16$, $\boldsymbol{v}\bullet\boldsymbol{v} = 3$, ${v_1}^2 + {v_2}^2 + {v_3}^2 = 3$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (seen anywhere) ***(A1)***

$\boldsymbol{u}\bullet\boldsymbol{v} = 4 \times \sqrt{3} \times \frac{\sqrt{3}}{2}$ (= 6) (seen anywhere) ***A1***

correct substitution into $\boldsymbol{u}\bullet\boldsymbol{u} - 2\boldsymbol{u}\bullet\boldsymbol{v} + \boldsymbol{v}\bullet\boldsymbol{v}$ or ${u_1}^2 + {u_2}^2 + {v_1}^2 + {v_2}^2 - 2\left(u_1v_1 + u_2v_2\right)$ (2 or 3 dimensions) ***(A1)***

eg 16 – 2(6) + 3 (= 7)

$|\boldsymbol{w}| = \sqrt{7}$ ***A1 N2***

5. [Maximum mark: 4] 19M.1.AHL.TZ1.H_1

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$, $k \in \mathbb{R}$.

Given that ***a*** and ***b*** are perpendicular, find the possible values of k. [4]

Markscheme

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$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$

$= -6 + k(k+2) - k$ ***A1***

$\boldsymbol{a} \cdot \boldsymbol{b} = 0$ ***(M1)***

$k^2 + k - 6 = 0$

attempt at solving their quadratic equation ***(M1)***

$(k+3)(k-2) = 0$

$k = -3,\ 2$ ***A1***

Note: Attempt at solving using $|\boldsymbol{a}||\boldsymbol{b}| = |\boldsymbol{a} \times \boldsymbol{b}|$ will be ***M1A0A0A0*** if neither answer found ***M1(A1)A1A0***
for one correct answer and ***M1(A1)A1A1*** for two correct answers.

[4 marks]

6. [Maximum mark: 6] 19M.1.AHL.TZ2.H_2

Three points in three-dimensional space have coordinates A(0, 0, 2), B(0, 2, 0) and C(3, 1, 0).

(a.i) Find the vector $\overrightarrow{AB}$. [1]

Markscheme

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[1 mark]

(a.ii) Find the vector $\overrightarrow{AC}$. [1]

Markscheme

$\overrightarrow{AC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[1 mark]

(b) Hence or otherwise, find the area of the triangle ABC. [4]

Markscheme

METHOD 1

attempt at vector product using $\overrightarrow{AB}$ and $\overrightarrow{AC}$. ***(M1)***

$\pm(2\boldsymbol{i}+6\boldsymbol{j}+6\boldsymbol{k})$ ***A1***

attempt to use area $=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$ ***M1***

$=\frac{\sqrt{76}}{2}\ \left(=\sqrt{19}\right)$ ***A1***

METHOD 2

attempt to use $\overrightarrow{AB}\bullet\overrightarrow{AC}=\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\cos\theta$ ***M1***

$$\begin{pmatrix}0\\2\\-2\end{pmatrix}\cdot\begin{pmatrix}3\\1\\-2\end{pmatrix}=\sqrt{0^2+2^2+(-2)^2}\sqrt{3^2+1^2+(-2)^2}\cos\theta$$

$6=\sqrt{8}\sqrt{14}\cos\theta$ ***A1***

$\cos\theta=\frac{6}{\sqrt{8}\sqrt{14}}=\frac{6}{\sqrt{112}}$

attempt to use area $=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|\sin\theta$ ***M1***

$=\frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{1-\frac{36}{112}}\ \left(=\frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{\frac{76}{112}}\right)$

$=\frac{\sqrt{76}}{2}\ \left(=\sqrt{19}\right)$ ***A1***

[4 marks]

7. [Maximum mark: 6] 18N.1.AHL.TZ0.H_5

The vectors ***a*** and ***b*** are defined by ***a*** = $\begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, ***b*** = $\begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for ***a*** • ***b*** in terms of t. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

a • ***b*** = $(1 \times 0) + (1 \times -t) + (t \times 4t)$ ***(M1)***

= $-t + 4t^2$ ***A1***

[2 marks]

(b) Hence or otherwise, find the values of t for which the angle between ***a*** and ***b*** is obtuse . [4]

Markscheme

recognition that ***a*** • ***b*** = |***a***||***b***|cos θ ***(M1)***

a • ***b*** < 0 or $-t + 4t^2$ < 0 or cos θ < 0 ***R1***

Note: Allow ≤ for ***R1***.

attempt to solve using sketch or sign diagram ***(M1)***

$0 < t < \frac{1}{4}$ ***A1***

[4 marks]

8. [Maximum mark: 6] SPM.1.AHL.TZ0.11

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

Markscheme

$15 \times 0 + 2d + 4 = 0$ ***(M1)***

$d = -2$ ***A1***

[2 marks]

(b) The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

Markscheme

$a \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ ***(M1)***

$= a \begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$ ***A1***

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ ***M1***

$a = \frac{14}{35} \ (= 0.4)$ ***A1***

[4 marks]