AI HL 24.10 (vectors) [49 marks]

1. [Maximum mark: 6] 19M.1.SL.TZ2.S_2

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

(a) parallel. [2]

(b) perpendicular. [4]

2. [Maximum mark: 6] 17N.2.SL.TZ0.S_3

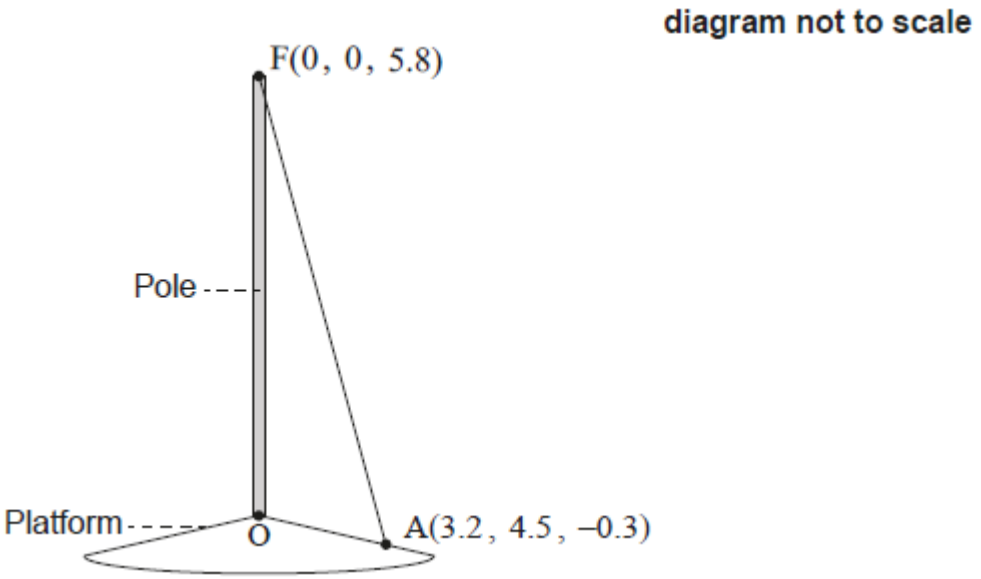
Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

(b) Let $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $\mathrm{B\hat{A}C}$. [4]

3. [Maximum mark: 8] 22M.1.AHL.TZ1.6

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0,\ 0,\ 5.8)$. All units are in metres.



The pole is held in place by ropes attached at F.

One of these ropes is attached to the platform at point $\mathrm{A}(3.2,\ 4.5,\ -0.3)$. The rope forms a straight line from A to F.

(a) Find $\overrightarrow{\mathrm{AF}}$. [1]

(b) Find the length of the rope. [2]

(c) Find $\mathrm{F\hat{A}O}$, the angle the rope makes with the platform. [5]

4. [Maximum mark: 7] 19M.1.SL.TZ1.S_6

The magnitudes of two vectors, ***u*** and ***v***, are 4 and $\sqrt{3}$ respectively. The angle between ***u*** and ***v*** is $\frac{\pi}{6}$.

Let ***w*** = ***u*** − ***v***. Find the magnitude of ***w***. [7]

5. [Maximum mark: 4] 19M.1.AHL.TZ1.H_1

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}, k \in \mathbb{R}$.

Given that ***a*** and ***b*** are perpendicular, find the possible values of k. [4]

6. [Maximum mark: 6] 19M.1.AHL.TZ2.H_2

Three points in three-dimensional space have coordinates A(0, 0, 2), B(0, 2, 0) and C(3, 1, 0).

(a.i) Find the vector $\overrightarrow{\mathrm{AB}}$. [1]

(a.ii) Find the vector $\overrightarrow{\mathrm{AC}}$. [1]

(b) Hence or otherwise, find the area of the triangle ABC. [4]

7. [Maximum mark: 6] 18N.1.AHL.TZ0.H_5

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \cdot \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse . [4]

8. [Maximum mark: 6] SPM.1.AHL.TZ0.11

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

(b) The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]