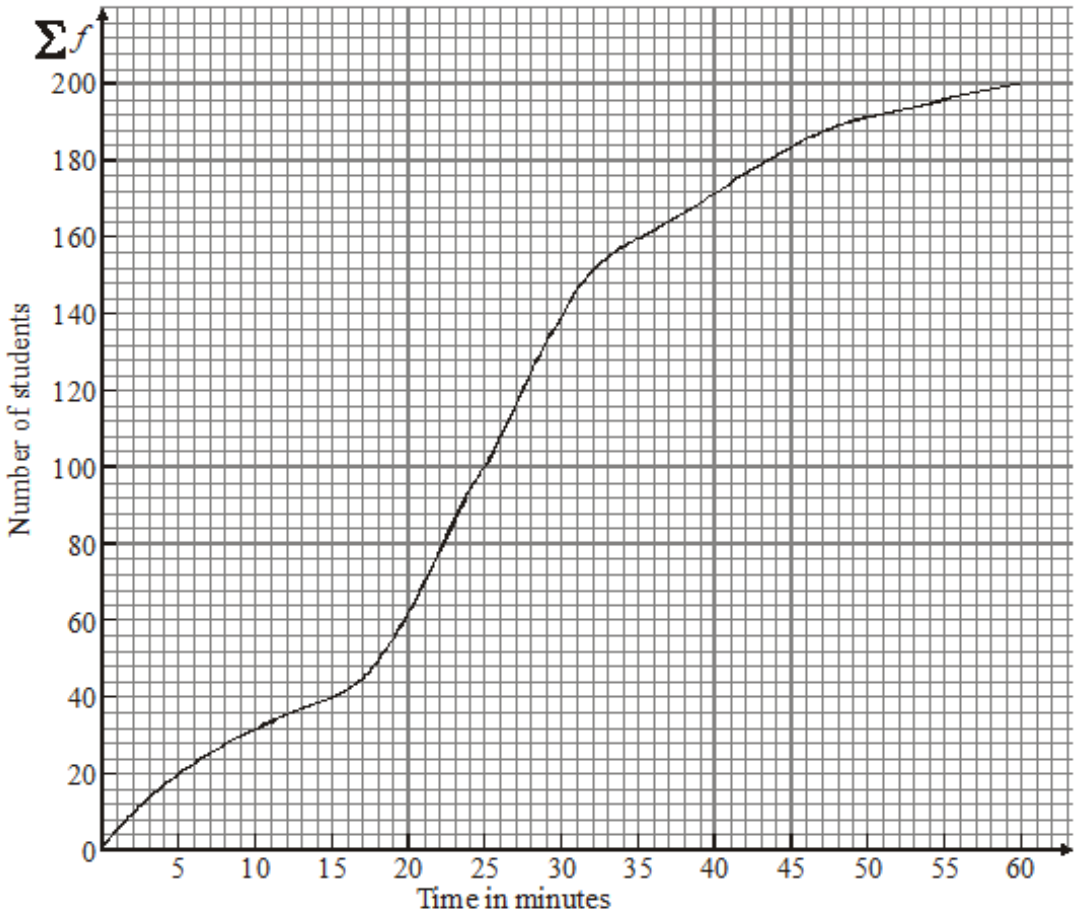
Name:
Result:

1. *(6 points)*

The cumulative frequency graph has been drawn from a frequency table showing the time it takes a number of students to complete a computer game.



(a) Find [3]

(i) the median time Med = 25 minutes

(ii) the interquartile range. IQR = 31.5 − 18 = 13.5 minutes

(b) Find the number of students who completed the game in more than 25 minutes. 100 [1]

(c) The quickest 10 % completed the game in no more than k minutes. Find k. $k = 5$ [2]

2. *(6 points)*

In this question the plane $Z = 0$ represents the ground level and distance are measured in metres.

Three vertical poles a, b, and c are placed at $A(-1, 1, 0)$, $B(2, 3, 0)$ and $C(1, 0, 0)$ respectively. The pole a is 3-metre high. The other two poles are 2-metre high.

(a) State the coordinates of the top of pole a. [1]

(b) Find the angle of elevation from the top of pole b to the top of pole a. [2]

(c) A triangular canvas is stretched and and its vertices are attached to the tops of the poles. Find the area of the canvas. [3]

(a) Top of pole A has coordinates $T_A = (-1, 1, 3)$

(b) The horizontal distance between the poles is $\sqrt{13}$ and the height difference is 1, so we have:

$$\tan\alpha = \frac{1}{\sqrt{13}} \qquad \text{so} \qquad \alpha = 15.5^\circ$$

(c) We have a triangle $T_AT_BT_C$ with $|T_AT_B| = \sqrt{14}, |T_AT_C| = \sqrt{6}$ and $|T_BT_C| = \sqrt{10}$. All distance were calculated using the 3d distance formula. Now we can proceed by calculating one of the angles using *cosine rule* (I will calculate the angle at vertex T_C):

$$\cos\gamma = \frac{6 + 10 - 14}{2\sqrt{6}\sqrt{10}} \qquad \text{so} \qquad \gamma = 82.5824...^\circ$$

Now we can calculate the area:

$$A = \frac{1}{2}\sqrt{6}\sqrt{10}\sin\gamma \approx 3.84\ m^2$$

3. *(6 points)*

A motion sensor is placed at $A(-1, 1, 2)$. A position of a particle is given by $(1, 2t, t-3)$, where $t \geq 0$.

(a) Find the distance between the sensor and the particle when $t = 0$. [1]

(b) Find t for which the distance is $\sqrt{22}$. [2]

(c) Find t for which the distance is minimal and calculate this minimal distance. [3]

(a) When $t = 0$ we have $A(-1, 1, 2)$ and $P(1, 0, -3)$, so

$$|AP| = \sqrt{2^2 + (-1)^2 + (-5)^2} = \sqrt{30} \approx 5.48$$

(b) We want to solve:

$$\sqrt{2^2 + (2t-1)^2 + (t-5)^2} = \sqrt{22}$$

After squaring both sides and rearranging we should get:

$$5t^2 - 14t + 8 = 0$$

Now use the Polynomial Root finder on GDC or factorize:

$$(5t - 4)(t - 2) = 0$$

So $t = 0.8$ or $t = 2$.

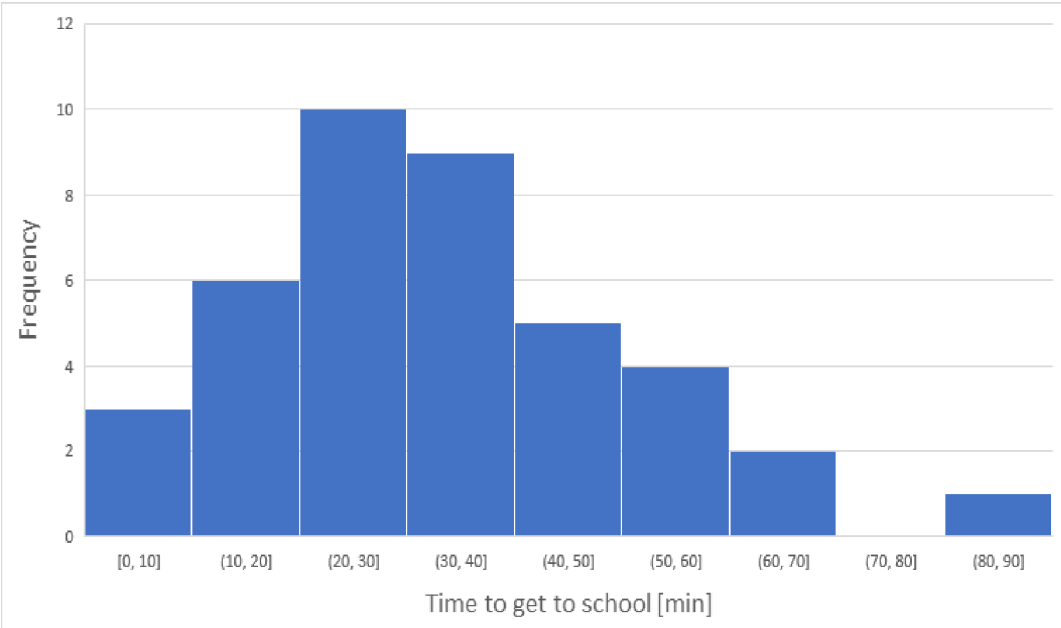
(c) We have:

$$|AP| = \sqrt{2^2 + (-1)^2 + (-5)^2} = \sqrt{5t^2 - 14t + 30}$$

Now we can use GDC to find the minimum. We can also use our knowledge of quadratic function. A quadratic expression $ax^2 + bx + c$ is minimal at $x = -\frac{b}{2a}$ (provided $a > 0$, otherwise its maximal there). So in our case minimum occurs at $t = \frac{14}{10} = 1.4$. The minimum value there is $|AP| = \sqrt{5 \cdot (1.4)^2 - 14 \cdot 1.4 + 30} \approx 4.49$.

4. *(5 points)*

Students of certain school were surveyed on how long it takes them to get to school. The following histogram shows the results:



(a) Is the data **discrete** or **continuous**? continuous [1]

(b) Comment on the distribution of data. skewed to the right [1]

(c) How many students took part in the survey. 40 [1]

(d) Estimate the standard deviation of the times it takes the surveyed students to get to school. $\sigma \approx 17.6$ [2]

5. *(11 points)*

Maria wants to buy a new car which costs 180 000 PLN. She considers taking a loan. Bank offers her two options:

Option 1
20% deposit required. Annual interest rate on the loan is 12% and is compounded quarterly. The loan is to be repaid in 10 years with quarterly instalments.

Option 2
No deposit required. Annual interest rate on the loan is r% and is compounded monthly. The loan is to be repaid in 12 years with monthly instalments of 2702 PLN.

(a) For **option 1** calculate:

(i) The amount borrowed. 80% of 180 000 PLN which is 144 000 PLN [1]

(ii) The quarterly repayments. 6229.79 [2]

(iii) The total amount paid for the car. $6229.79 \cdot 40 + 36000 = 285191.6$ [1]

(b) For **option 2** calculate:

(i) The annual interest rate, r. 15% [2]

(ii) The interest paid on the loan. $2702 \cdot 144 - 180000 = 209088$ [2]

(c) State which option Maria should choose. Justify your answer. You can choose either and justify it. Option 1 has lower interest, option 2 does not require any deposit. [1]

(d) The value of the car depreciates by on average 17%. After n years the value of the car will fall below 40 000 PLN for the first time. Find n. [2]

$$180000 \cdot 0.83^n < 40000$$

Rearrange to get:

$$n > \frac{\ln \frac{2}{9}}{\ln 0.83}$$

So $n > 8.072...$, which means $n = 9$.

6. *(8 points)*

A sample of 6 measurements was taken, each rounded to the nearest metre. The mean of these rounded measurements is exactly 16 metres.

(a) State the lower and upper bound for the mean of the exact values of these measurements. [1]

(b) The seventh measurement increased the mean to 18 metres (rounded to the nearest metre). Find the possible values of the seventh measurement. [3]

A new sample was taken. The results are as follows:

12, 18, 19, 11, 19, 21, 22, 13, 15, 16, 14, 15, 17, 14, 14, 11, 16, 20

(c) Decide if this sample contains any outliers. [2]

(d) After the last measurement, the researcher noticed that each following measurements gives the same result (20). How many more measurements need to be taken so that the mean of is greater than 18. Find the least such number. [2]

(a) Lower bound = 15.5, upper bound 16.5.

(b) The sum of the six measurements is between $6 \cdot 15.5 = 93$ and $6 \cdot 16.5 = 99$.

The sum of the seven measurements is between $7 \cdot 17.5 = 122.5$ and $7 \cdot 18.5 = 129.5$.

So the least the seventh measurement could be is $122.5 - 99 = 23.5$ and the most it could be is $129.5 - 93 = 36.5$.

(c) $Q_1 = 14, Q_3 = 19$ so an outlier is any number $< 14 - 1.5 \cdot 5 = 6.5$ or $> 19 + 1.5 \cdot 5 = 26.5$. There are no such numbers.

(d) We want to solve:

$$\frac{287 + 20x}{18 + x} > 18$$

We get that $x > 18.5$, so $x = 19$.

7. *(18 points)*

Tomasz is a Mathematics teacher. He wants to investigate the relationship between the grades of his students and their attitude towards the subject. He teaches a total of 110 students: 40 in year IV, 20 in year III, 30 in year II and 20 in year I. He decides to select a sample of 11 students. He wants the proportion of students in each year in the sample to be the same as in the population. He chooses the students at random within each year.

(a) State the sampling method described. stratified sampling [1]

(b) State the number of students from year IV that Tomasz should select. 4 [1]

The students were asked to rate their attitude towards Mathematics on a scale from 1 to 10, where 1 means that they despise Maths and 10 that they love the subject. The results of the survey are presented below:

Attitude towards Mathematics (x)	8	7	7	10	7	9	5	6	10	5	5
Grade average (y)	4.3	4.1	4.3	4.9	4.0	4.8	3.7	2.0	4.8	3.2	3.7

(c) Calculate the unbiased estimates of the mean and standard deviation of the attitude towards Mathematics amongst all of Tomasz's students. [3]
$\mu = 7.18$ $s_{n-1} = 1.89$

(c) Calculate the Pearson's correlation coefficient r. [1]
$r = 0.744$

(d) Find the regression line in the form $y = ax + b$. [2]
$y = 0.332x + 1.60$

(e) Interpret the coefficient a in the context of the question. [1]
Each additional point in attitude toward Mathematics category corresponds to an increase in grade average by on average 0.332.

(f) Use the regression to predict the grade average of a students whose attitude is 2. Give a reason why your prediction may no be valid. [2]
$y(2) \approx 2.26$, outside of range of analysed attitudes (extrapolation).

(g) Give one reason, why it may be more appropriate to calculate the Spearman's correlation coefficient instead of Pearson's correlation coefficient. [1]
Data contains a clear outlier, and Spearman's correlation coefficient is less sensitive to outliers.

(h) Complete the following table: [2]

Rank of attitude	4	6	6	1.5	6	3	10	8	1.5	10	10
Rank of grade average	4.5	6	4.5	1	7	2.5	8.5	11	2.5	10	8.5

(i) Calculate r_s and interpret its value. [2]

$r_s = 0.914$, strong positive correlation between rank of attitude and rank of grade average.

(j) The student whose grade average was 2.0 retook one of the tests and improved his grade average to 2.3. Explain how this change affects r_s, justify your answer. [2]

r_s does not change as the grade average of 2.3 is still the lowest grade average, so its rank does not change.