

Approximations and errors

Things you need to learn

- How to approximate calculations.
- How to specify lower and upper boundaries of approximated answers/calculations.
- How to calculate absolute and percentage errors in approximations.

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The exact average speed was $v_{ave} = \frac{325km}{4.1h} = 79.268... \frac{km}{h}$.

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width w has to be such that $85\text{cm} \leq w < 95\text{cm}$,
- ii. perimeter $P = 2(l + w)$, so the lower bound for P is $2(55 + 85) = 280\text{cm}$ and the upper bound is $2(65 + 95) = 320\text{cm}$.
We have $280\text{cm} \leq P < 320\text{cm}$.
- iii. area is given by $A = l \times w$, so the lower bound for A is $55 \times 85 = 4675\text{cm}^2$ and the upper bound is $65 \times 95 = 6175\text{cm}^2$.
We have $4675\text{cm}^2 \leq A < 6175\text{cm}^2$.

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The area of a triangle is $A = \frac{1}{2}bh$,
so the lower bound for the area is $0.5 \times 11.5\text{cm} \times 7.5\text{cm} = 43.125\text{cm}^2$.

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So we have $43.125\text{cm}^2 \leq A < 53.125\text{cm}^2$

Errors

Definition

Let v_e denote the exact value of a certain quantity and v_a denote its approximated value. Then the absolute error ϵ of the approximation is given by

$$\epsilon = |v_e - v_a|$$

And the percentage error $\epsilon_{\%}$ is given by:

$$\epsilon_{\%} = \left| \frac{v_e - v_a}{v_e} \right| \times 100\%$$

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and the percentage error:

$$\epsilon_{\%} = \frac{23337}{1223337} \times 100\% = 1.90765\dots\%$$

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The percentage error is:

$$\epsilon_{\%} = \frac{356}{5356} \times 100\% \approx 6.64675\ldots\%$$

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The approximated area is given by $v_a = 3 \times 3^2 = 27$

So we have the absolute error:

$$\epsilon = |v_e - v_a| = 0.33971\dots$$

$$\epsilon_{\%} = \frac{0.33971\dots}{8.7025\pi} \times 100\% \approx 1.24255\dots\%$$

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This gives the absolute and percentage errors:

$$\epsilon = |v_e - v_a| \approx 0.230769\dots\text{ h}$$

$$\epsilon\% = \frac{0.230769\dots}{3.76923\dots} \times 100\% \approx 6.122489\dots\%$$

If you have any questions or doubts email me at T.J.Lechowski@gmail.com