AI HL 07.11 (vectors - revision) [126 marks]

1. [Maximum mark: 14] SPM.2.AHL.TZ0.4

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \text{km h}^{-1}$.

At 13:00 it is detected at a position 30 km east and 10 km north of the airport, and at a height of 5 km. Let t be the length of time in hours from 13:00.

(a) Write down a vector equation for the displacement, ***r*** of the aircraft in terms of t. [2]

Markscheme

r$= \begin{pmatrix} 30 \\ 10 \\ 5 \end{pmatrix} + t \begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix}$ ***A1A1***

[2 marks]

If the aircraft continued to fly with the velocity given

(b.i) verify that it would pass directly over the airport. [2]

Markscheme

when $x = 0, t = \frac{30}{150} = 0.2$ ***M1***

EITHER

when $y = 0, t = \frac{10}{150} = 0.2$ ***A1***

since the two values of t are equal the aircraft passes directly over the airport

OR

$t = 0.2, y = 0$ ***A1***

[2 marks]

(b.ii) state the height of the aircraft at this point. [1]

Markscheme

height = 5 – 0.2 × 20 = 1 km ***A1***

[1 mark]

(b.iii) find the time at which it would fly directly over the airport. [1]

Markscheme

time 13:12 ***A1***

[1 mark]

When the aircraft is 4 km above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point (0, 0, 0).

(c.i) Find the time at which the aircraft is 4 km above the ground. [2]

Markscheme

$5 - 20t = 4 \Rightarrow t = \frac{1}{20}$ (3 minutes) ***(M1)***

time 13:03 ***A1***

[2 marks]

(c.ii) Find the direct distance of the aircraft from the airport at this point. [3]

Markscheme

displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***A1*** distance is $\sqrt{22.5^2 + 7.5^2 + 4^2}$ ***(M1)***

= 24.1 km ***A1***

[3 marks]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \mathrm{km\,h^{-1}}$, find the value of a. [3]

Markscheme

METHOD 1

time until landing is $12 - 3 = 9$ minutes ***M1***

height to descend = $4\,\mathrm{km}$

$a = \frac{-4}{\frac{9}{60}}$ ***M1***

$= -26.7$ ***A1***

METHOD 2

$\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} = s\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***M1***

$-150 = 22.5\,s \Rightarrow s = -\frac{20}{3}$ ***M1***

$a = -\frac{20}{3} \times 4$

$= -26.7$ ***A1***

[3 marks]

2. [Maximum mark: 6] EXN.1.AHL.TZ0.7

The position of a helicopter relative to a communications tower at the top of a mountain at time t (hours) can be described by the vector equation below.

$$\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4.2 \\ 5.8 \\ -0.5 \end{pmatrix}$$

The entries in the column vector give the displacements east and north from the communications tower and above/below the top of the mountain respectively, all measured in kilometres.

(a) Find the speed of the helicopter. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$|v| = \sqrt{4.2^2 + 5.8^2 + 0.5^2}$ **(M1)**

$7.18\ (7.1784\ldots)\ (\text{kmh}^{-1})$ **A1**

[2 marks]

(b) Find the distance of the helicopter from the communications tower at $t = 0$. [2]

Markscheme

$$\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix}$$

$|\boldsymbol{r}| = \sqrt{20^2 + 25^2}$ **(M1)**

$= \sqrt{1025} = 32.0\ (32.0156\ldots)\ (\text{km})$ **A1**

[2 marks]

(c) Find the bearing on which the helicopter is travelling. [2]

Markscheme

Bearing is $\arctan\left(\frac{4.2}{5.8}\right)$ or 90°-$\arctan\left(\frac{5.8}{4.2}\right)$ **(M1)**

035.9° $(35.909\ldots)$ **A1**

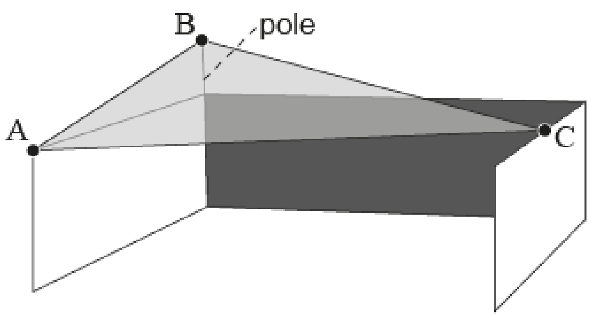
[2 marks]

3. [Maximum mark: 9] 23M.1.AHL.TZ1.7

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\text{m}$ wall, and at a point B, located at the top of a $1\,\text{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{\text{AB}} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{\text{AC}} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{\text{BC}} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}}$. [2]

Markscheme

attempt to find the vector product (e.g. one term correct) ***(M1)***

$$\begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \times \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -42 \end{pmatrix}$$ ***A1***

[2 marks]

(b) Hence find the area of the triangular cover. [2]

Markscheme

METHOD 1

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

area $= \frac{1}{2}\sqrt{3^2 + 7^2 + 42^2}$

$= 21.3\left(\text{m}^2\right)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

METHOD 2

find θ using $\overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}} = \left|\overrightarrow{\text{AB}}\right|\left|\overrightarrow{\text{AC}}\right| \sin\,\theta$ ***(M1)***

$\theta = 67.1$ $(67.1350^{\circ}\ldots,\ 1.171728\ldots$ radians)

then area $= \frac{1}{2}\left|\overrightarrow{\text{AB}}\right|\left|\overrightarrow{\text{AC}}\right| \sin\,\theta$

$= 21.3\left(\text{m}^2\right)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

[2 marks]

The point X on $[\text{AC}]$ is such that $[\text{BX}]$ is perpendicular to $[\text{AC}]$.

(c) Use your answer to part (b) to find the distance BX. [3]

Markscheme

$AC = 7.61577\ldots\ \left(\sqrt{58}\right)$ ***(A1)*** setting the area formula $\frac{1}{2}\times$base$\times$height equal to their part (b) *(M1)*

$\text{BX} = \frac{2\times 21.3424...}{\sqrt{58}}$

$= 5.60\ (5.60480\ldots)$ *A1*

Note: Award *A1* for 5.6.

Award *A1* for $5.59\ (5.5936\ldots)$ from the use of 21.3 to 3 sf.

[3 marks]

(d) Find the angle the cover makes with the horizontal plane. [2]

Markscheme

attempting to set up a trig ratio *(M1)*

angle is $\arcsin\left(\frac{1}{\text{BX}}\right)$

10.3° ($10.2776\ldots^\circ$, 0.179378 radians) *A1*

[2 marks]

4. [Maximum mark: 8] 23M.1.AHL.TZ2.14

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$

.

(a) Find the bearing on which ship A is sailing. [3]

Markscheme

$\boldsymbol{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right) \ (= 56.3099\ldots^\circ)$

$(90^\circ + 56.3099\ldots^\circ =)\ 146^\circ \ (146.3099\ldots^\circ)$ ***A1***

[3 marks]

(b) Find the value of t when ship B is directly south of ship A. [2]

Markscheme

setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

[2 marks]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

Markscheme

$\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67 \text{ (hrs.)} \left(1.66666\ldots,\ \frac{5}{3}\right)$ ***A1***

[3 marks]

5. [Maximum mark: 7] 22N.1.AHL.TZ0.8

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

(a) Find the coordinates of M. [3]

Markscheme

setting up at least two simultaneous equations ***(M1)***

$p = -0.8$ **OR** $q = 3.6$ ***(A1)***

M has coordinates $(1.6, \ -2.6, \ 8.2)$ ***A1***

[3 marks]

(b) Find the acute angle between the two lines. [4]

Markscheme

using vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ ***(M1)***

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 3$$ ***(A1)***

$$\cos\theta \frac{\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+(-1)^2+2^2}} \quad \left(\cos\theta = \frac{3}{\sqrt{14}\sqrt{6}}\right)$$ ***(M1)***

Note: Accept correct use of vector product.

$(\theta =)\ 1.24$ radians $(1.23732\ldots)$ $(70.9^\circ$ $(70.8933\ldots))$ ***A1***

[4 marks]

6. [Maximum mark: 5] 22M.1.AHL.TZ1.13

At $1:00\ \mathrm{pm}$ a ship is $1\,\mathrm{km}$ east and $4\,\mathrm{km}$ north of a harbour. A coordinate system is defined with the harbour at the origin. The position vector of the ship at $1:00\ \mathrm{pm}$ is given by $\begin{pmatrix}1\\4\end{pmatrix}$.

The ship has a constant velocity of $\begin{pmatrix}1.2\\-0.6\end{pmatrix}$ kilometres per hour ($\mathrm{km\,h^{-1}}$).

(a) Write down an expression for the position vector $\boldsymbol{r}$ of the ship, t hours after $1:00\ \mathrm{pm}$. [1]

Markscheme

$(\boldsymbol{r}=)\begin{pmatrix}1\\4\end{pmatrix}+t\begin{pmatrix}1.2\\-0.6\end{pmatrix}$ ***A1***

Note: Do not condone the use of λ or any other variable apart from t.

[1 mark]

(b) Find the time at which the bearing of the ship from the harbour is 045°. [4]

Markscheme

when the bearing from the port is 045°, the distance east from the port is equal to the distance north from the port ***(M1)***

$1+1.2t=4-0.6t$ ***(A1)***

$1.8t=3$

$t=\frac{5}{3}$ ($1.6666\ldots$, 1 hour 40 minutes) ***(A1)***

time is $2:40\ \mathrm{pm}\quad(14:40)$ ***A1***

[4 marks]

7. [Maximum mark: 9] 21N.1.AHL.TZ0.16

A ship S is travelling with a constant velocity, v, measured in kilometres per hour, where

$$v = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time $t = 0$ the ship is at a point $\mathrm{A}(300,\ 100)$ relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector $\overrightarrow{\mathrm{OS}}$ of the ship at time t hours. [1]

Markscheme

$\overrightarrow{\mathrm{OS}} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

[1 mark]

A lighthouse is located at a point $(129,\ 283)$.

(b) Find the value of t when the ship will be closest to the lighthouse. [6]

Markscheme

attempt to find the vector from L to S ***(M1)***

$\overrightarrow{\mathrm{LS}} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

EITHER

$\left|\overrightarrow{\mathrm{LS}}\right| = \sqrt{(171 - 12t)^2 + (15t - 183)^2}$ ***(M1)(A1)***

minimize to find t on GDC ***(M1)***

OR

S closest when $\overrightarrow{\mathrm{LS}} \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}\right) \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$$

$-2052 + 144t - 2745 + 225t = 0$ ***(M1)(A1)***

OR

S closest when $\overrightarrow{\mathrm{LS}} \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$$\overrightarrow{\mathrm{LS}} = \begin{pmatrix} 5k \\ 4k \end{pmatrix}$$

$\overrightarrow{\mathrm{OS}} = \begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix}$ ***(A1)***

$$\begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix} = \begin{pmatrix} 300 - 12t \\ 100 + 15t \end{pmatrix}$$

Solving simultaneously ***(M1)***

THEN

$t = 13$ ***A1***

[6 marks]

(c) An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

State whether the alarm will sound. Give a reason for your answer. [2]

Markscheme

the alarm will sound ***A1***

$$\left|\overrightarrow{\mathrm{LS}}\right| = 19.2\ldots < 20$$ ***R1***

Note: Do not award ***A1R0***.

[2 marks]

8. [Maximum mark: 7] 21M.1.AHL.TZ1.13

A submarine is located in a sea at coordinates $(0.8,\ 1.3,\ -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

Markscheme

$$\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$$ ***A1A1***

Note: Award **A1** for each correct vector. Award ***A0A1*** if their "$\boldsymbol{r} =$" is omitted.

[2 marks]

The submarine reaches the surface of the sea at the point P.

(b.i) Find the coordinates of P. [3]

Markscheme

$-0.3 + \lambda = 0$ ***(M1)***

$\Rightarrow \lambda = 0.3$

$$\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$$ ***(M1)***

P has coordinates $(0.2,\ 0.4,\ 0)$ ***A1***

Note: Accept the coordinates of P in vector form.

[3 marks]

(b.ii) Find OP. [2]

Markscheme

$\sqrt{0.2^2 + 0.4^2}$ ***(M1)***

$= 0.447\ \mathrm{km}\ \ (= 447\,\mathrm{m})$ ***A1***

[2 marks]

9. [Maximum mark: 6] 21M.1.AHL.TZ1.5

A garden has a triangular sunshade suspended from three points $\mathrm{A}(2,\ 0,\ 2),\ \mathrm{B}(8,\ 0,\ 2)$ and $\mathrm{C}(5,\ 4,\ 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a.i) Find $\overrightarrow{\mathrm{CA}}$. [1]

Markscheme

$\overrightarrow{\mathrm{CA}} = \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[1 mark]

(a.ii) Find $\overrightarrow{\mathrm{CB}}$. [1]

Markscheme

$\overrightarrow{\mathrm{CB}} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[1 mark]

(b) Find $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [2]

Markscheme

$\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ ***(M1)A1***

Note: Do not award ***(M1)*** if less than 2 entries are correct.

[2 marks]

(c) Hence find the area of the triangle ABC. [2]

Markscheme

area is $\frac{1}{2}\sqrt{6^2+24^2}=12.4\ \mathrm{m}^2\ \left(12.3693\ldots,\ 3\sqrt{17}\right)$ ***(M1)A1***

[2 marks]

10. [Maximum mark: 7] 21M.1.AHL.TZ2.8

Two lines L_1 and L_2 are given by the following equations, where $p \in \mathbb{R}$.

$$L_1 : r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that L_1 and L_2 are perpendicular.

(a) Find the possible value(s) for p. [3]

Markscheme

setting a dot product of the direction vectors equal to zero ***(M1)***

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} \cdot \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$p(p+4) + 8p - 28 = 0$ ***(A1)***

$p^2 + 12p - 28 = 0$

$(p+14)(p-2) = 0$

$p = -14,\ p = 2$ ***A1***

[3 marks]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

Markscheme

$p = -14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$$2 - 14\lambda = 14 - 10\mu$$

$$-5 - 28\lambda = 7 + 4\mu$$ ***(M1)***

$$-3 + 4\lambda = -2 - 7\mu$$

METHOD 1

solving the first two equations simultaneously

$\lambda = -\frac{1}{2},\ \mu = \frac{1}{2}$ ***A1***

substitute into the third equation: ***M1***

$$-3 + 4\left(-\tfrac{1}{2}\right) \neq -2 + \tfrac{1}{2}(-7)$$

so lines do not intersect. ***R1***

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC ***M1***

GDC indicates no solution ***A1***

so lines do not intersect ***R1***

[4 marks]

11. [Maximum mark: 15] 20N.1.SL.TZ0.S_9

Points A and B have coordinates $(1,\ 1,\ 2)$ and $(9,\ m,\ -6)$ respectively.

(a) Express $\overrightarrow{\mathrm{AB}}$ in terms of m. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach to find $\overrightarrow{\mathrm{AB}}$ ***(M1)***

eg $\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}\ ,\ \mathrm{A} - \mathrm{B}$

$$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 8 \\ m-1 \\ -8 \end{pmatrix}$$ ***A1 N2***

[2 marks]

The line L, which passes through B, has equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$.

(b) Find the value of m. [5]

Markscheme

valid approach ***(M1)***

eg $L = \begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix}, \begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$

one correct equation ***(A1)***

eg $-3 + 2s = 9,\ -6 = 24 - 5s$

correct value for s ***A1***

eg $s = 6$

substituting **their** s value into their expression/equation to find m ***(M1)***

eg $-19 + 6 \times 4$

$m = 5$ ***A1 N3***

[5 marks]

(c) Consider a unit vector $\boldsymbol{u}$, such that $\boldsymbol{u} = p\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} + \frac{1}{3}\boldsymbol{k}$, where $p > 0$.

Point C is such that $\overrightarrow{\mathrm{BC}} = 9\boldsymbol{u}$.

Find the coordinates of C. [8]

Markscheme

valid approach ***(M1)***

eg $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 9p \\ -6 \\ 3 \end{pmatrix}$, $C = 9\boldsymbol{u} + B$, $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} x-9 \\ y-5 \\ z+6 \end{pmatrix}$

correct working to find C ***(A1)***

eg $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 9p+9 \\ -1 \\ -3 \end{pmatrix}$, $\mathrm{C} = 9\begin{pmatrix} p \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}$, $y = -1$ and $z = -3$

correct approach to find $|\boldsymbol{u}|$ (seen anywhere) ***A1***

eg $p^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2$, $\sqrt{p^2 + \frac{4}{9} + \frac{1}{9}}$

recognizing unit vector has magnitude of 1 ***(M1)***

eg $|\boldsymbol{u}| = 1$, $\sqrt{p^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1$, $p^2 + \frac{5}{9} = 1$

correct working ***(A1)***

eg $p^2 = \frac{4}{9}$, $p = \pm\frac{2}{3}$

$p = \frac{2}{3}$ ***A1***

substituting **their** value of p ***(M1)***

eg

$$\begin{pmatrix} x-9 \\ y-5 \\ z+6 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix}, \ \mathrm{C} = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}, \ \mathrm{C} = 9\begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}, \ x-9=6$$

$\mathrm{C}(15, \ -1, \ -3)$ (accept $\begin{pmatrix} 15 \\ -1 \\ -3 \end{pmatrix}$) ***A1 N4***

Note: The marks for finding p are independent of the first two marks.
For example, it is possible to award marks such as ***(M0)(A0)A1(M1)(A1)A1 (M0)A0*** or ***(M0)(A0)A1(M1)(A0)A0 (M1)A0***.

[8 marks]

12. [Maximum mark: 7] 20N.1.SL.TZ0.S_7

In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $s(t) = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

(a) Find an expression for the velocity of P_1 at time t. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing velocity is derivative of displacement ***(M1)***

eg $v = \frac{\mathrm{d}s}{\mathrm{d}t}$, $\frac{\mathrm{d}}{\mathrm{d}t}\left(10 - \frac{7}{4}t^2\right)$

velocity$= -\frac{14}{4}t \quad \left(= -\frac{7}{2}t\right)$ ***A1 N2***

[2 marks]

(b) Particle P_2 also moves in a straight line. The position of P_2 is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$.

Find the value of q. [5]

Markscheme

valid approach to find speed of P_2 ***(M1)***

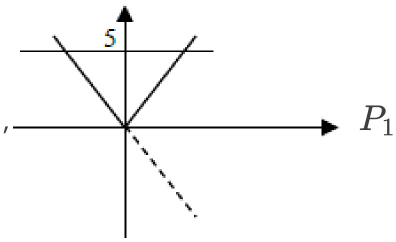
eg $\left|\begin{pmatrix} 4 \\ -3 \end{pmatrix}\right|$, $\sqrt{4^2 + (-3)^2}$, velocity$= \sqrt{4^2 + (-3)^2}$

correct speed ***(A1)***

eg $5\,\mathrm{m\,s^{-1}}$

recognizing relationship between speed and velocity (may be seen in inequality/equation) ***R1***

eg $\left|-\frac{7}{2}t\right|$, speed = | velocity | , graph of P_1 speed ,



speed $= \frac{7}{2}t$, P_2 velocity $= -5$

correct inequality or equation that compares speed or velocity (accept any variable for q) ***A1***

eg $\left|-\frac{7}{2}t\right| > 5$, $-\frac{7}{2}q < -5$, $\frac{7}{2}q > 5$, $\frac{7}{2}q = 5$

$q = \frac{10}{7}$ (seconds) (accept $t > \frac{10}{7}$, do not accept $t = \frac{10}{7}$) ***A1 N2***

Note: Do not award the last two ***A1*** marks without the ***R1***.

[5 marks]

13. [Maximum mark: 6] 19M.1.SL.TZ1.S_2

A line, L_1, has equation $r = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}\ (15,\ 9,\ c)$ lies on L_1.

(a) Find c. [4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct equation ***(A1)***

eg $-3 + 6s = 15$, $6s = 18$

$s = 3$ ***(A1)***

substitute their s value into z component ***(M1)***

eg $10 + 3\,(2)$, $10 + 6$

$c = 16$ ***A1 N3***

[4 marks]

(b) A second line, L_2, is parallel to L_1 and passes through (1, 2, 3).

Write down a vector equation for L_2. [2]

Markscheme

$r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$ $(= (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + t(6\boldsymbol{i} + 2\boldsymbol{k}))$ ***A2 N2***

Note: Accept any scalar multiple of $\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$ for the direction vector.

Award **A1** for $\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A1*** for $L_2=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A0*** for $r=\begin{pmatrix}6\\0\\2\end{pmatrix}+t\begin{pmatrix}1\\2\\3\end{pmatrix}$.

[2 marks]

14. [Maximum mark: 6] 19M.2.SL.TZ2.S_7

The vector equation of line L is given by **r** $= \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + t \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$.

Point P is the point on L that is closest to the origin. Find the coordinates of P. [6]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1 (Distance between the origin and P)

correct position vector for OP ***(A1)***

eg $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\mathrm{P} = (-1+4t,\ 3+5t,\ 8-t)$

correct expression for OP or OP^2 (seen anywhere) ***A1***

eg $\sqrt{(-1+4t)^2 + (3+5t)^2 + (8-t)^2}$,
$(-1+4x)^2 + (3+5x)^2 + (8-x)^2$

valid attempt to find the minimum of OP ***(M1)***

eg $d' = 0$, root on sketch of d', min indicated on sketch of d

$t = -\frac{1}{14},\ -0.0714285$ ***(A1)***

substitute their value of t into L (only award if there is working to find t) ***(M1)***

eg one correct coordinate, $-1 + 4\left(-\frac{1}{14}\right)$

$(-1.28571,\ 2.64285,\ 8.07142)$

$\left(-\frac{9}{7},\ \frac{37}{14},\ \frac{113}{14}\right) = (-1.29,\ 2.64,\ 8.07)$ ***A1 N2***

METHOD 2 (Perpendicular vectors)

recognizing that closest implies perpendicular ***(M1)*** *eg* $\overrightarrow{\mathrm{OP}} \perp L$ (may be seen on sketch), $a \bullet b = 0$

valid approach involving $\overrightarrow{\mathrm{OP}}$ ***(M1)***

eg $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \bullet \overrightarrow{\mathrm{OP}}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \perp \overrightarrow{\mathrm{OP}}$

correct scalar product ***A1***

eg $4(-1+4t) + 5(3+5t) - 1(8-t)$,
$-4 + 16t + 15 + 25t - 8 + t = 0$, $42t + 3$

$t = -\frac{1}{14},\ -0.0714285$ ***(A1)***

substitute their value of t into L or $\overrightarrow{\mathrm{OP}}$ (only award if scalar product used to find t) ***(M1)***

eg one correct coordinate, $-1 + 4\left(-\frac{1}{14}\right)$

$(-1.28571,\ 2.64285,\ 8.07142)$

$\left(-\frac{9}{7},\ \frac{37}{14},\ \frac{113}{14}\right) = (-1.29,\ 2.64,\ 8.07)$ ***A1 N2***

[6 marks]

15. [Maximum mark: 14] 18N.2.SL.TZ0.S_8

Consider the points A(−3, 4, 2) and B(8, −1, 5).

(a.ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

Markscheme

Note: There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

correct substitution into formula ***(A1)***

eg $\sqrt{11^2+(-5)^2+3^2}$

12.4498

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{155}$ (exact), 12.4 ***A1 N2***

[2 marks]

A line *L* has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point C (5, y, 1) lies on line *L*.

(b.i) Find the value of y. [3]

Markscheme

Note: There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

valid approach to find t ***(M1)***

eg $\begin{pmatrix}5\\y\\1\end{pmatrix} = \begin{pmatrix}2\\0\\-5\end{pmatrix} + t\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $5 = 2 + t,\ 1 = -5 + 2t$

$t = 3$ (seen anywhere) ***(A1)***

attempt to substitute **their** parameter into the vector equation ***(M1)***

eg $\begin{pmatrix}5\\y\\1\end{pmatrix} = \begin{pmatrix}2\\0\\-5\end{pmatrix} + 3\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $3 \cdot (-2)$

$y = -6$ ***A1 N2***

[3 marks]

(b.ii)

Show that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix}8\\-10\\-1\end{pmatrix}$. [2]

Markscheme

Note: There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

correct approach ***A1***

eg $\begin{pmatrix}5\\-6\\1\end{pmatrix} - \begin{pmatrix}-3\\4\\2\end{pmatrix}$, AO + OC, $c - a$

$\overrightarrow{\mathrm{AC}} = \begin{pmatrix}8\\-10\\-1\end{pmatrix}$ ***AG N0***

Note: Do not award ***A1*** in part (ii) unless answer in part (i) is correct and does not result from working backwards.

[2 marks]

(c) Find the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [5]

Markscheme

Note: There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

finding scalar product and magnitude ***(A1)(A1)***

scalar product = $11 \times 8 + -5 \times -10 + 3 \times -1$ (=135)

$$|\overrightarrow{\mathrm{AC}}| = \sqrt{8^2 + (-10)^2 + (-1)^2}\ \left(= \sqrt{165},\ 12.8452\right)$$

evidence of substitution into formula ***(M1)***

eg $\cos\theta = \frac{11\times8+-5\times-10+3\times-1}{\left|\overrightarrow{\mathrm{AB}}\right|\times\sqrt{8^2+(-10)^2+(-1)^2}}$, $\cos\theta = \frac{\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AC}}}{\sqrt{155}\times\sqrt{8^2+(-10)^2+(-1)^2}}$

correct substitution ***(A1)***

eg $\cos\theta = \frac{11\times8+-5\times-10+3\times-1}{\sqrt{155}\times\sqrt{8^2+(-10)^2+(-1)^2}}$, $\cos\theta = \frac{135}{159.921\ldots}$,

$$\cos\theta = 0.844162\ldots$$

0.565795, 32.4177°

$\hat{A}$ = 0.566, 32.4° ***A1 N3***

[5 marks]

(d) Find the area of triangle ABC. [2]

Markscheme

Note: There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

correct substitution into area formula ***(A1)***

eg $\frac{1}{2} \times \sqrt{155} \times \sqrt{165} \times \sin(0.566\ldots)$, $\frac{1}{2} \times \sqrt{155 \times 165} \times \sin(32.4)$

42.8660

area = 42.9 ***A1 N2***

[2 marks]