AI HL 07.11 (vectors - revision) [126 marks]

1. [Maximum mark: 14] SPM.2.AHL.TZ0.4

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \mathrm{km\,h^{-1}}$.

At 13:00 it is detected at a position 30 km east and 10 km north of the airport, and at a height of 5 km. Let t be the length of time in hours from 13:00.

(a) Write down a vector equation for the displacement, ***r*** of the aircraft in terms of t. [2]

If the aircraft continued to fly with the velocity given

(b.i) verify that it would pass directly over the airport. [2]

(b.ii) state the height of the aircraft at this point. [1]

(b.iii) find the time at which it would fly directly over the airport. [1]

When the aircraft is 4 km above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point (0, 0, 0).

(c.i) Find the time at which the aircraft is 4 km above the ground. [2]

(c.ii) Find the direct distance of the aircraft from the airport at this point. [3]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \mathrm{km\,h^{-1}}$, find the value of a.

[3]

2. [Maximum mark: 6] EXN.1.AHL.TZ0.7

The position of a helicopter relative to a communications tower at the top of a mountain at time t (hours) can be described by the vector equation below.

$$\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4.2 \\ 5.8 \\ -0.5 \end{pmatrix}$$

The entries in the column vector give the displacements east and north from the communications tower and above/below the top of the mountain respectively, all measured in kilometres.

(a) Find the speed of the helicopter. [2]

(b) Find the distance of the helicopter from the communications tower at $t = 0$. [2]

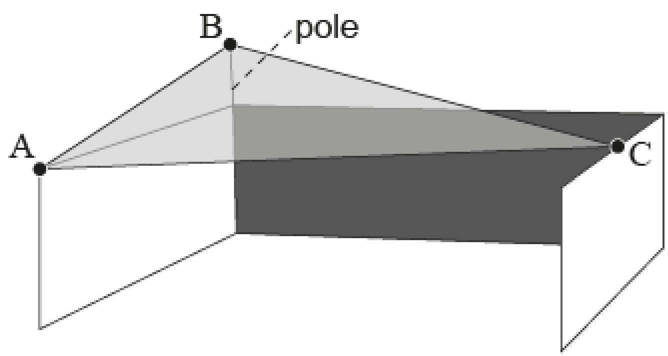
(c) Find the bearing on which the helicopter is travelling. [2]

3. [Maximum mark: 9] 23M.1.AHL.TZ1.7

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\mathrm{m}$ wall, and at a point B, located at the top of a $1\,\mathrm{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on $[\mathrm{AC}]$ is such that $[\mathrm{BX}]$ is perpendicular to $[\mathrm{AC}]$.

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

4. [Maximum mark: 8] 23M.1.AHL.TZ2.14

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as
$\boldsymbol{r}_A = (1+2t)\boldsymbol{i} + (3-3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as
$\boldsymbol{r}_B = (-2+4t)\boldsymbol{i} + (-4+t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

5. [Maximum mark: 7] 22N.1.AHL.TZ0.8

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

(a) Find the coordinates of M. [3]

(b) Find the acute angle between the two lines. [4]

6. [Maximum mark: 5] 22M.1.AHL.TZ1.13

At $1:00\text{ pm}$ a ship is 1 km east and 4 km north of a harbour. A coordinate system is defined with the harbour at the origin. The position vector of the ship at $1:00\text{ pm}$ is given by $\begin{pmatrix}1\\4\end{pmatrix}$.

The ship has a constant velocity of $\begin{pmatrix}1.2\\-0.6\end{pmatrix}$ kilometres per hour (km h^{-1}).

(a) Write down an expression for the position vector $\boldsymbol{r}$ of the ship, t hours after $1:00\text{ pm}$. [1]

(b) Find the time at which the bearing of the ship from the harbour is $045°$. [4]

7. [Maximum mark: 9] 21N.1.AHL.TZ0.16

A ship S is travelling with a constant velocity, v, measured in kilometres per hour, where

$$v = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time $t = 0$ the ship is at a point $\mathrm{A}(300,\ 100)$ relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector $\overrightarrow{\mathrm{OS}}$ of the ship at time t hours. [1]

A lighthouse is located at a point $(129,\ 283)$.

(b) Find the value of t when the ship will be closest to the lighthouse. [6]

(c) An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

State whether the alarm will sound. Give a reason for your answer. [2]

8. [Maximum mark: 7] 21M.1.AHL.TZ1.13

A submarine is located in a sea at coordinates $(0.8,\ 1.3,\ \ -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

The submarine reaches the surface of the sea at the point P.

(b.i) Find the coordinates of P. [3]

(b.ii) Find OP. [2]

9. [Maximum mark: 6] 21M.1.AHL.TZ1.5

A garden has a triangular sunshade suspended from three points $\mathrm{A}(2,\ 0,\ 2),\ \mathrm{B}(8,\ 0,\ 2)$ and $\mathrm{C}(5,\ 4,\ 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a.i) Find $\overrightarrow{\mathrm{CA}}$. [1]

(a.ii) Find $\overrightarrow{\mathrm{CB}}$. [1]

(b) Find $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [2]

(c) Hence find the area of the triangle ABC. [2]

10. [Maximum mark: 7] 21M.1.AHL.TZ2.8

Two lines L_1 and L_2 are given by the following equations, where $p \in \mathbb{R}$.

$$L_1 : r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that L_1 and L_2 are perpendicular.

(a) Find the possible value(s) for p. [3]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

11. [Maximum mark: 15] 20N.1.SL.TZ0.S_9

Points A and B have coordinates $(1,\ 1,\ 2)$ and $(9,\ m,\ -6)$ respectively.

(a) Express $\overrightarrow{\mathrm{AB}}$ in terms of m. [2]

The line L, which passes through B, has equation

$$\boldsymbol{r} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}.$$

(b) Find the value of m. [5]

(c) Consider a unit vector $\boldsymbol{u}$, such that $\boldsymbol{u} = p\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} + \frac{1}{3}\boldsymbol{k}$, where $p > 0$.

Point C is such that $\overrightarrow{\mathrm{BC}} = 9\boldsymbol{u}$.

Find the coordinates of C. [8]

12. [Maximum mark: 7] 20N.1.SL.TZ0.S_7

In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $s(t) = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

(a) Find an expression for the velocity of P_1 at time t. [2]

(b) Particle P_2 also moves in a straight line. The position of P_2 is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$.

Find the value of q. [5]

13. [Maximum mark: 6] 19M.1.SL.TZ1.S_2

A line, L_1, has equation $r = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}\ (15,\ 9,\ c)$ lies on L_1.

(a) Find c. [4]

(b) A second line, L_2, is parallel to L_1 and passes through (1, 2, 3).

Write down a vector equation for L_2. [2]

14. [Maximum mark: 6] 19M.2.SL.TZ2.S_7

The vector equation of line L is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$.

Point P is the point on L that is closest to the origin. Find the coordinates of P. [6]

15. [Maximum mark: 14] 18N.2.SL.TZ0.S_8

Consider the points A(−3, 4, 2) and B(8, −1, 5).

(a.ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

A line *L* has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point C (5, y, 1) lies on line *L*.

(b.i) Find the value of y. [3]

(b.ii) Show that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 8 \\ -10 \\ -1 \end{pmatrix}$. [2]

(c) Find the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [5]

(d) Find the area of triangle ABC. [2]