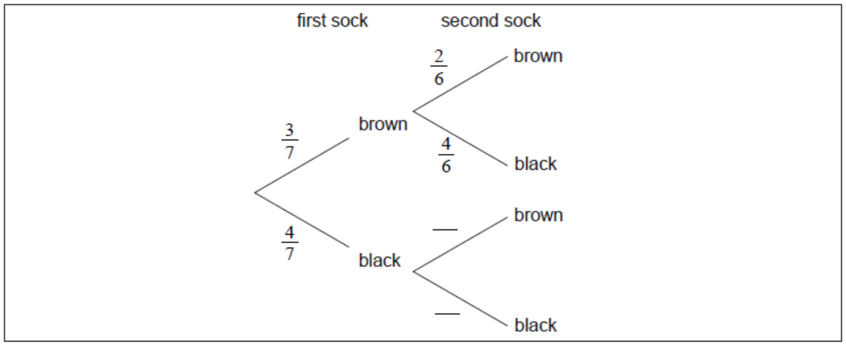
Basic probability [72 marks]

1. [Maximum mark: 6] 22M.1.SL.TZ1.10

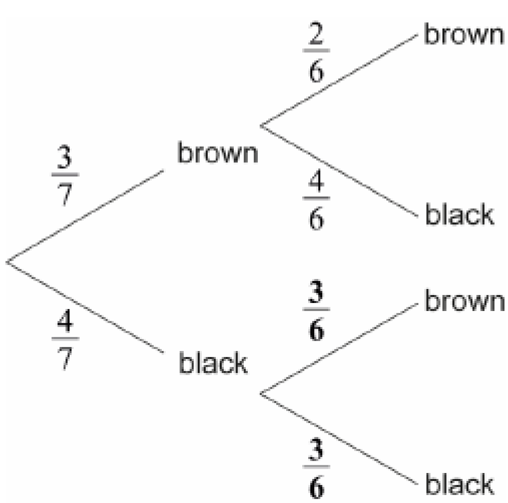
Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram.



[1]

Markscheme



A1

Note: Award ***A1*** for both missing probabilities correct.

[1 mark]

(b) Find the probability that Karl takes two socks of the same colour.

[2]

Markscheme

multiplying along branches and then adding outcomes ***(M1)***

$\frac{3}{7} \times \frac{2}{6} + \frac{4}{7} \times \frac{3}{6}$

$= \frac{18}{42} \ \left(= \frac{3}{7} \approx 0.429 \ (42.9\%)\right)$ ***A1***

[2 marks]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

Markscheme

use of conditional probability formula ***M1***

$\frac{\left(\frac{3}{7} \times \frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$ ***A1***

$= \frac{6}{18} \ \left(= \frac{1}{3}\right) \ \left(\frac{252}{756}, \ 0.333, \ 33.3\%\right)$ ***A1***

[3 marks]

2. [Maximum mark: 6] 22M.1.SL.TZ2.2

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	Rejected
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

Markscheme

$\left(\frac{17+25}{130}=\right)\ \frac{42}{130}\ \left(\frac{21}{65},\ 0.323076\ldots\right)$ ***A1***

[1 mark]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Markscheme

$\left(\frac{17}{17+25}=\right)\ \frac{17}{42}\ (0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

(c) Two different applicants are chosen at random from the original group.

Find the probability that both applicants applied to the Arts programme. [3]

Markscheme

$\frac{41}{130} \times \frac{40}{129}$ ***A1M1***

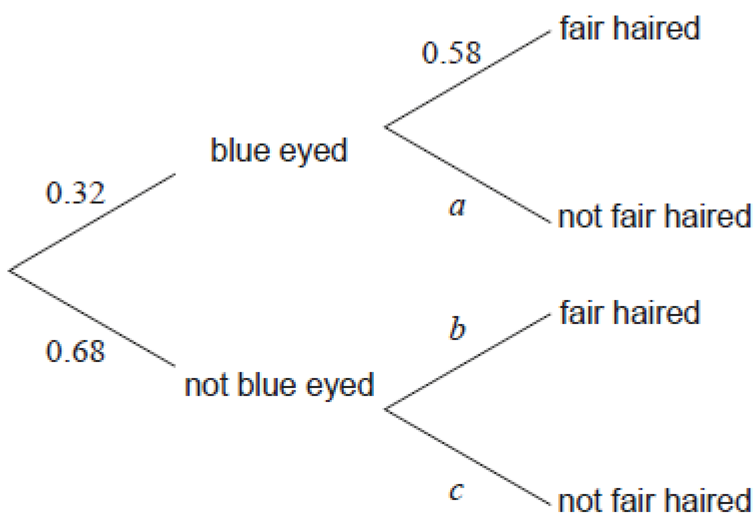
Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$= \frac{1640}{16770} \approx 0.0978 \ \left(0.0977936\ldots, \ \frac{164}{1677}\right)$ ***A1***

[3 marks]

3. [Maximum mark: 5] 21N.1.SL.TZ0.11

In a city, 32% of people have blue eyes. If someone has blue eyes, the probability that they also have fair hair is 58%. This information is represented in the following tree diagram.



(a) Write down the value of a. [1]

Markscheme

$a = 0.42$ ***A1***

[1 mark]

(b) Find an expression, in terms of b, for the probability of a person not having blue eyes **and** having fair hair. [1]

Markscheme

$(\mathrm{P}(B' \cap F) =)\ b \times 0.68$ ***A1***

[1 mark]

It is known that 41% of people in this city have fair hair.

Calculate the value of

(c.i) b. [2]

Markscheme

$0.32 \times 0.58 + 0.68b = 0.41$ ***(M1)***

Note: Award ***(M1)*** for setting up equation for fair-haired or equivalent.

$b = 0.33$ ***A1***

[2 marks]

(c.ii) c. [1]

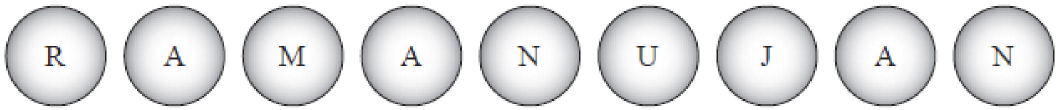
Markscheme

$c = 0.67$ ***A1***

[1 mark]

4. [Maximum mark: 6] 20N.1.SL.TZ0.T_6

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

(a.i) Find the probability that the first ball chosen is labelled A. [1]

Markscheme

$\frac{3}{9}$ $\left(\frac{1}{3},\ 0.333,\ 0.333333\ldots,\ 33.3\%\right)$ ***(A1) (C1)***

[1 mark]

(a.ii) Find the probability that the first ball chosen is labelled A or labelled N. [1]

Markscheme

$\frac{5}{9}$ $(0.556,\ 0.555555\ldots,\ 55.6\%)$ ***(A1) (C1)***

[1 mark]

(b) Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N. [2]

Markscheme

$\frac{3}{8}$ $(0.375,\ 37.5\%)$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

(c) Find the probability that both balls chosen are labelled N. [2]

Markscheme

$\frac{2}{9} \times \frac{1}{8}$ ***(M1)***

Note: Award ***(M1)*** for a correct compound probability calculation seen.

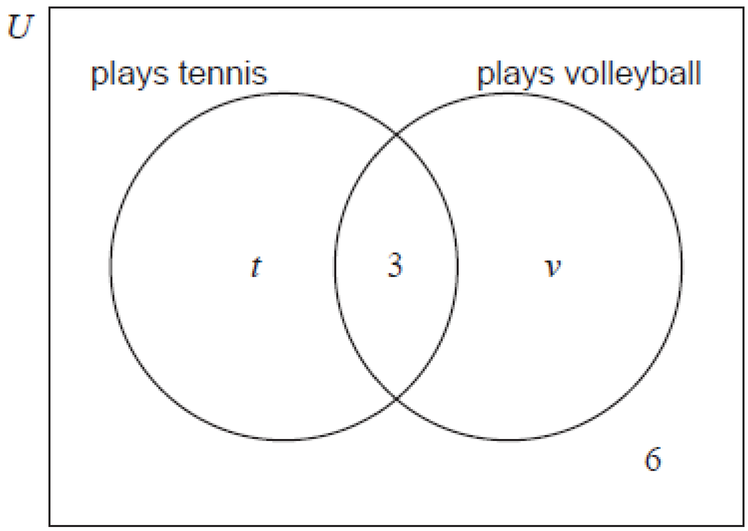
$\frac{2}{72}\ \left(\frac{1}{36},\ 0.0278,\ 0.0277777\ldots,\ 2.78\%\right)$ ***(A1) (C2)***

[2 marks]

5. [Maximum mark: 6] 20N.1.SL.TZ0.S_1

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events "plays tennis" and "plays volleyball". The values t and v represent numbers of students.



(a.i) Find the value of t. [2]

Markscheme

valid approach to find t ***(M1)***

eg $t+3=19,\ 19-3$

$t=16$ (may be seen on Venn diagram) ***A1 N2***

[2 marks]

(a.ii) Find the value of v. [2]

Markscheme

valid approach to find v ***(M1)***

eg $t + 3 + v + 6 = 30,\ 30 - 19 - 6$

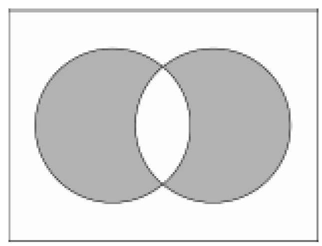
$v = 5$ (may be seen on Venn diagram) ***A1 N2***

[2 marks]

(b) Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both. [2]

Markscheme

valid approach ***(M1)***

eg $16 + 5, 21$ students, $1 - \frac{3+6}{30}$, 

$\frac{21}{30}\ \left(= \frac{7}{10}\right)$ ***A1 N2***

[2 marks]

6. [Maximum mark: 6] 20N.1.SL.TZ0.T_14

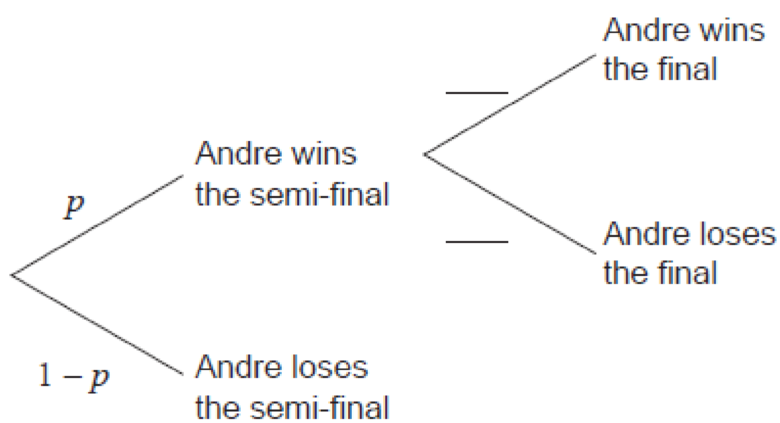
Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p. If Andre wins the semi-final, then the probability he will be the champion is 0.6.

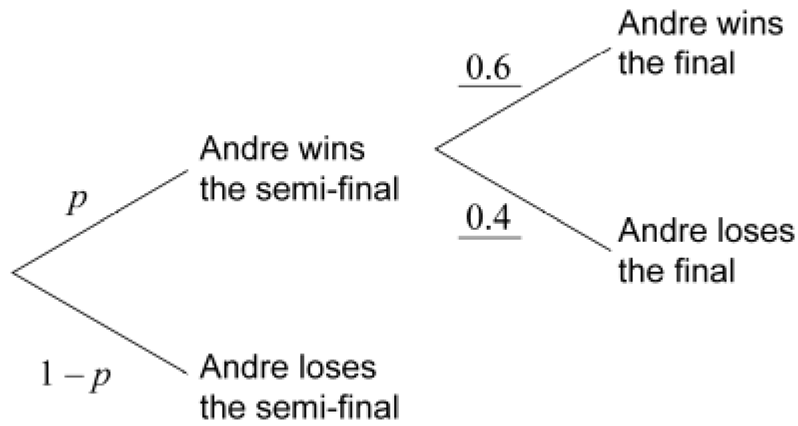
(a) Complete the values in the tree diagram.



[1]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1) (C1)

Note: Award *(A1)* for the correct pair of probabilities.

[1 mark]

The probability that Andre will not be the champion is 0.58.

(b) Find the value of p. [2]

Markscheme

$p \times 0.4 + (1 - p) = 0.58$ ***(M1)***

Note: Award *(M1)* for multiplying and adding correct probabilities for losing equated to 0.58.

OR

$p \times 0.6 = 1 - 0.58$ ***(M1)***

Note: Award *(M1)* for multiplying correct probabilities for winning equated to $1 - 0.58$ or 0.42.

$(p =)\ 0.7$ ***(A1)*(ft)** *(C2)*

Note: Follow through from their part (a). Award the final ***(A1)*(ft)** only if their p is within the range $0 < p < 1$.

[2 marks]

(c) Given that Andre did not become the champion, find the probability that he lost in the semi-final. [3]

Markscheme

$\frac{0.3}{0.58}\ \left(\frac{1-0.7}{0.58}\right)$ ***(A1)*(ft)*(A1)***

Note: Award ***(A1)*(ft)** for their correct numerator. Follow through from part (b). Award ***(A1)*** for the correct denominator.

OR

$\frac{0.3}{0.3+0.7\times0.4}$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for their correct numerator. Follow through from part (b). Award ***(A1)*(ft)** for their correct calculation of Andre losing the semi-final or winning the semi-final and then losing in the final. Follow through from their parts (a) and (b).

$\frac{15}{29}\ (0.517,\ 0.517241\ldots,\ 51.7\%)$ ***(A1)*(ft)** *(C3)*

Note: Follow through from parts (a) and (b).

[3 marks]

7. [Maximum mark: 6] 19N.1.SL.TZ0.T_4

Let the universal set, U, be the set of all integers x such that $1 \leq x < 11$.
A, B and C are subsets of U.

$$A = \{1, 2, 3, 4, 6, 8\}$$
$$B = \{2, 3, 5, 7\}$$
$$C = \{1, 3, 5, 7, 9\}$$

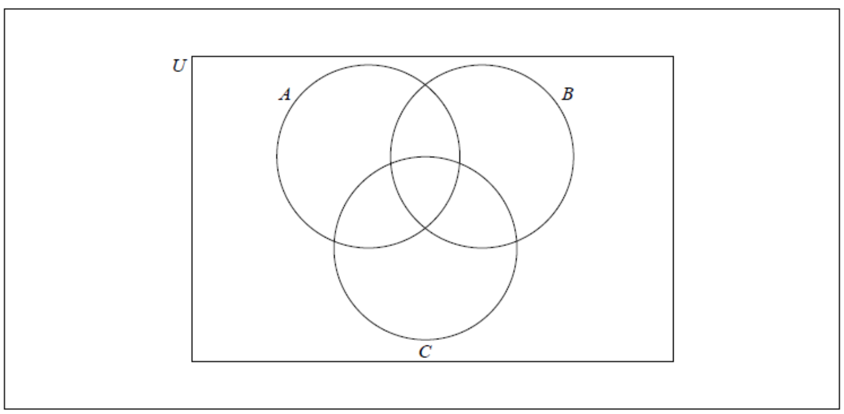
(a) Write down $n(B)$. [1]

Markscheme

4 ***(A1) (C1)***

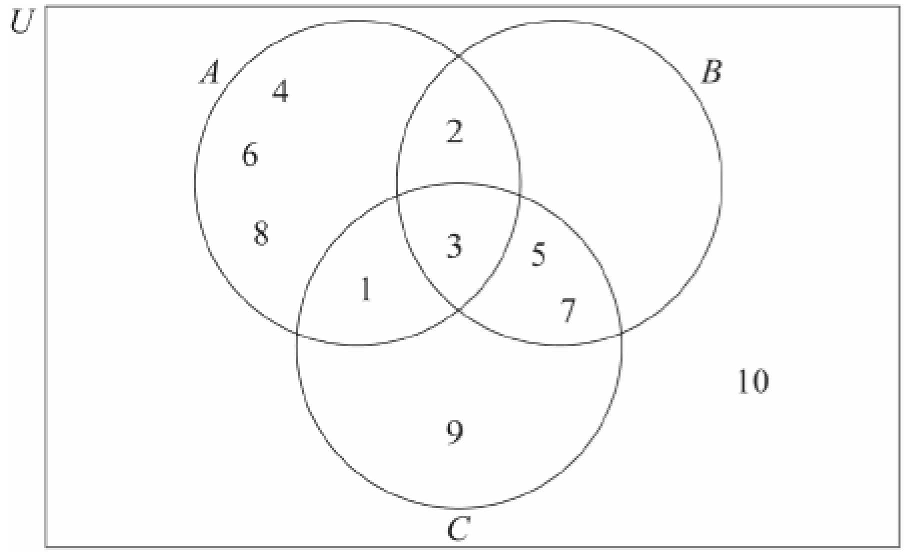
[1 mark]

(b) Complete the following Venn diagram using **all** elements of U.



[4]

Markscheme



(A1)

(A1)(A1)(A1) (C4)

Note: Award *(A1)* for 3 in the correct place. Award *(A1)* for $1, 2, 5$ and 7 in the correct places. Award *(A1)* for $4, 6, 8, 9$ in the correct places. Award *(A1)* for 10 outside of the three circles **and** 11 not shown in the diagram.

If any entry is duplicated within its region, award at most *(A3)*.

[4 marks]

(c) Write down an element that belongs to $(A \cup B)' \cap C$. [1]

Markscheme

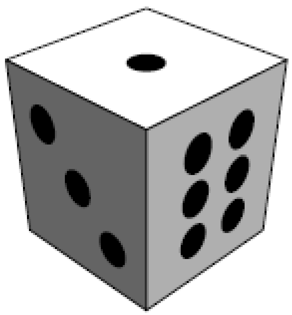
9 *(A1)***(ft)** *(C1)*

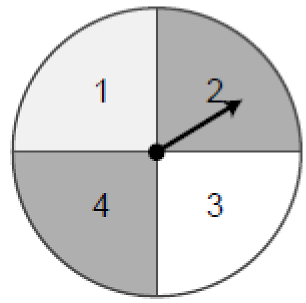
Note: Award *(A1)* for the correct element. Follow through from *their* Venn diagram in part (b). Award *(A0)* if additional incorrect elements are included in their answer.

[1 mark]

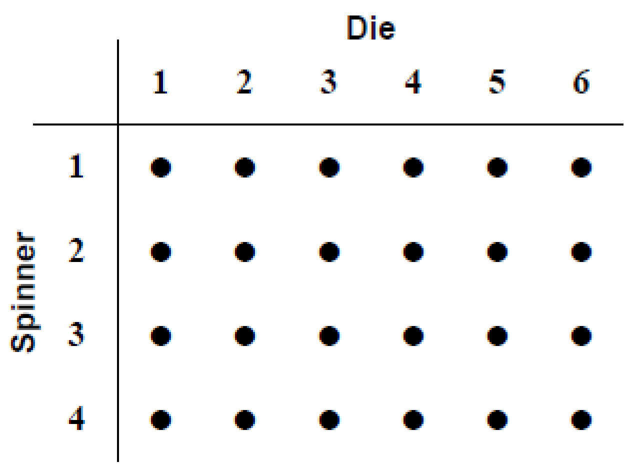
8. [Maximum mark: 6] 19N.1.SL.TZ0.T_9

Sungwon plays a game where she rolls a fair 6-sided die and spins a fair spinner with 4 equal sectors. During each turn in the game, the die is rolled once and the spinner is spun once. The **score** for each turn is the sum of the two results. For example, 1 on the die and 2 on the spinner would receive a score of 3.





The following diagram represents the sample space.



(a) Find the probability that Sungwon's score on her first turn is greater than 4. [2]

Markscheme

$\frac{18}{24}\left(\frac{3}{4},\ 0.75,\ 75\%\right)$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

Sungwon takes a second turn.

(b) Find the probability that Sungwon scores greater than 4 on both of her first two turns. [2]

Markscheme

$\frac{18}{24} \times \frac{18}{24}$ ***(M1)***

Note: Award ***(M1)*** for the square of *their* probability in part (a).

$= 0.563 \left(\frac{9}{16}, \frac{324}{576}, 0.5625, 56.3\%\right)$ ***(A1)*(ft) *(C2)***

Note: Follow through from part (a), provided *their* answer is less than or equal to 1.

[2 marks]

(c) Sungwon will play the game for 11 turns.

Find the expected number of times the score on a turn is greater than 4. [2]

Markscheme

$11 \times \frac{18}{24}$ ***(M1)***

Note: Award ***(M1)*** for multiplying *their* part (a) by 11.

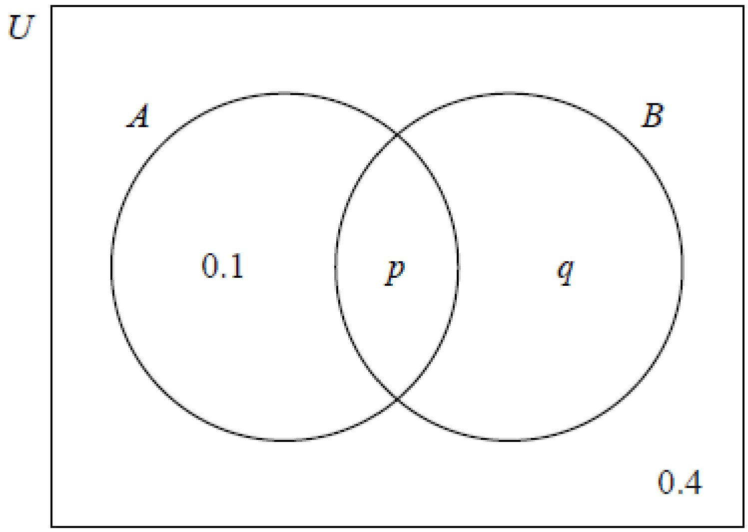
$8.25 \left(\frac{33}{4}\right)$ ***(A1)*(ft) *(C2)***

Note: Follow through from part (a), provided *their* answer is less than or equal to 1.

[2 marks]

9. [Maximum mark: 6] 19M.1.SL.TZ1.S_1

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$.
The values shown are probabilities.



(a) Find the value of p. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $0.30 - 0.1, p + 0.1 = 0.3$

$p = 0.2$ ***A1 N2***

[2 marks]

(b) Find the value of q. [2]

Markscheme

valid approach ***(M1)***

eg $1 - (0.3 + 0.4), 1 - 0.4 - 0.1 - p$

$q = 0.3$ ***A1 N2***

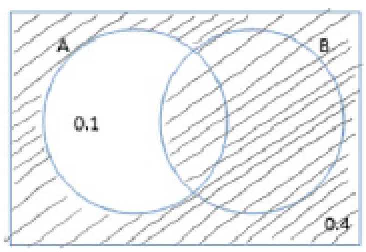
[2 marks]

(c) Find $\mathrm{P}\left(A' \cup B\right)$. [2]

Markscheme

valid approach ***(M1)***

eg $0.7 + 0.5 - 0.3$, $p + q + 0.4$, $1 - 0.1$,
$\mathrm{P}\left(A' \cup B\right) = \mathrm{P}\left(A'\right) + \mathrm{P}\left(B\right) - \mathrm{P}\left(A' \cap B\right)$



$\mathrm{P}\left(A' \cup B\right) = 0.9$ ***A1 N2***

[2 marks]

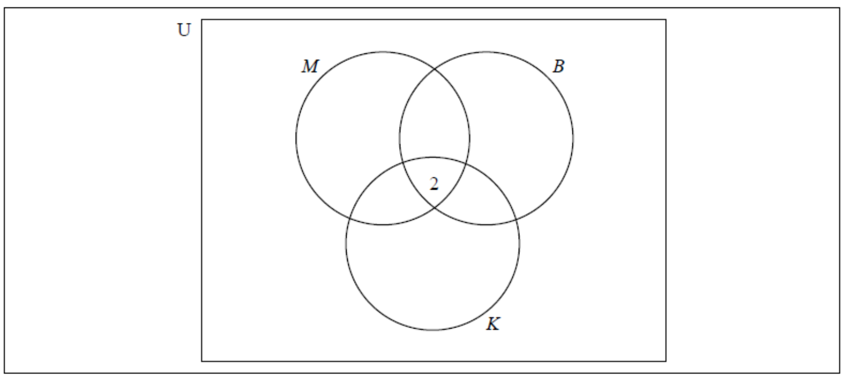
10. [Maximum mark: 6] 19M.1.SL.TZ2.T_5

A school café sells three flavours of smoothies: mango (M), kiwi fruit (K) and banana (B).

85 students were surveyed about which of these three flavours they like.

35 students liked mango, 37 liked banana, and 26 liked kiwi fruit
2 liked all three flavours
20 liked both mango and banana
14 liked mango and kiwi fruit
3 liked banana and kiwi fruit

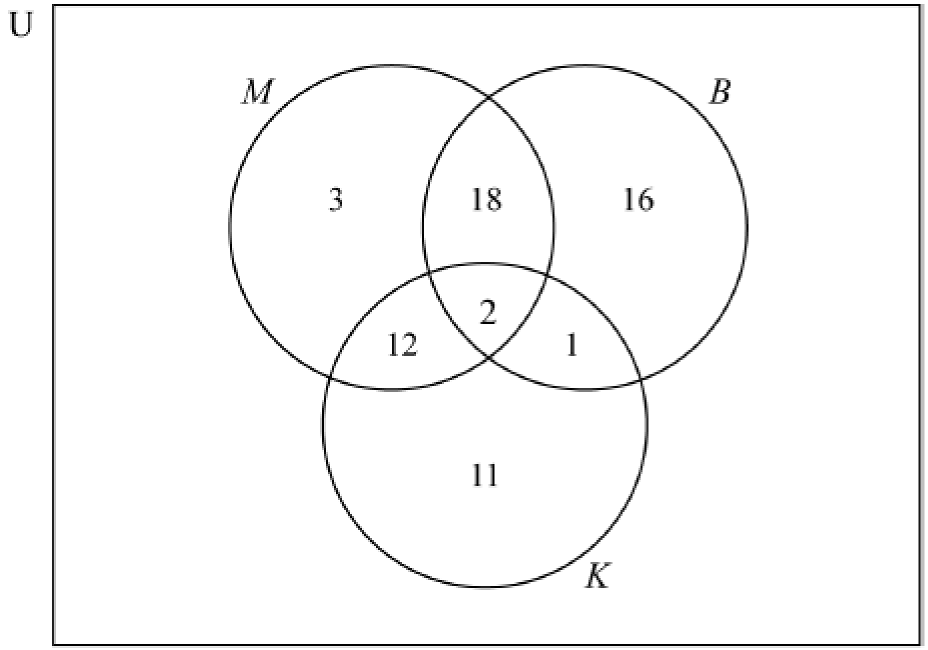
(a) Using the given information, complete the following Venn diagram.



[2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1) (C2)

Note: Award ***(A1)*** for 18, 12 and 1 in correct place on Venn diagram, ***(A1)*** for 3, 16 and 11 in correct place on Venn diagram.

[2 marks]

(b) Find the number of surveyed students who did not like any of the three flavours. [2]

Markscheme

85 – (3 + 16 + 11 + 18 + 12 + 1 + 2) ***(M1)***

Note: Award ***(M1)*** for subtracting the sum of their values from 85.

22 ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their Venn diagram in part (a).
If any numbers that are being subtracted are negative award ***(M1)(A0)***.

[2 marks]

(c) A student is chosen at random from the surveyed students.

Find the probability that this student likes kiwi fruit smoothies given that they like mango smoothies. [2]

Markscheme

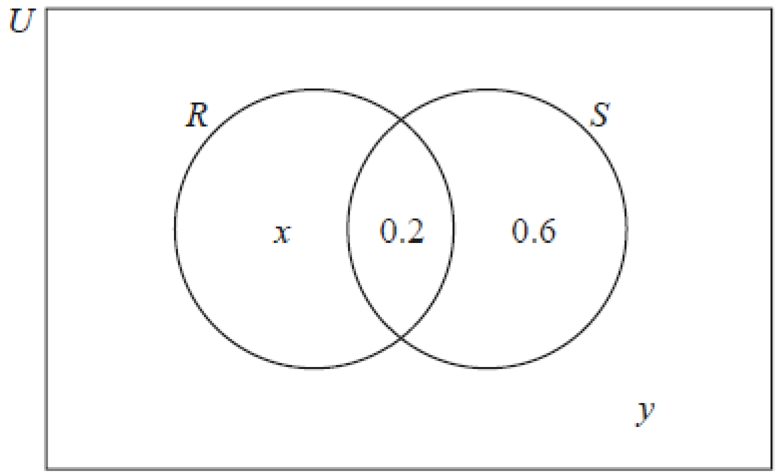
$\frac{14}{35}$ $\left(\frac{2}{5},\ 0.4,\ 40\%\right)$ ***(A1)*****(ft)*****(A1)*****(ft)** ***(C2)***

Note: Award ***(A1)*** for correct numerator; ***(A1)*** for correct denominator. Follow through from their Venn diagram.

[2 marks]

11. [Maximum mark: 7] 23M.1.AHL.TZ2.3

The following Venn diagram shows two independent events, R and S. The values in the diagram represent probabilities.



(a) Find the value of x. [3]

Markscheme

attempting to use $\mathrm{P}(R \cap S) = \mathrm{P}(R)\mathrm{P}(S)$ ***(M1)***

$0.2 = 0.8(0.2 + x)$ ***(A1)***

$x = 0.05$ ***A1***

[3 marks]

(b) Find the value of y. [2]

Markscheme

$x + 0.2 + 0.6 + y = 1$ ***(M1)***

$y = 0.15$ ***A1***

[2 marks]

(c) Find $\mathrm{P}(R\prime|S\prime)$. [2]

Markscheme

METHOD 1

attempting to apply $\mathrm{P}\left(R\prime\middle|S\prime\right) = \frac{\mathrm{P}(R\prime \cap S\prime)}{\mathrm{P}(S\prime)}$ ***(M1)***

$\frac{0.15}{0.2}$

$= \frac{3}{4}$ ***A1***

METHOD 2

$\mathrm{P}(R\prime|S\prime) = \mathrm{P}(R\prime)$ (because R, S are independent) ***(M1)***

$= 1 - 0.25 = 0.75$ ***A1***

Note: ***FT*** from their values of x or y.

[2 marks]

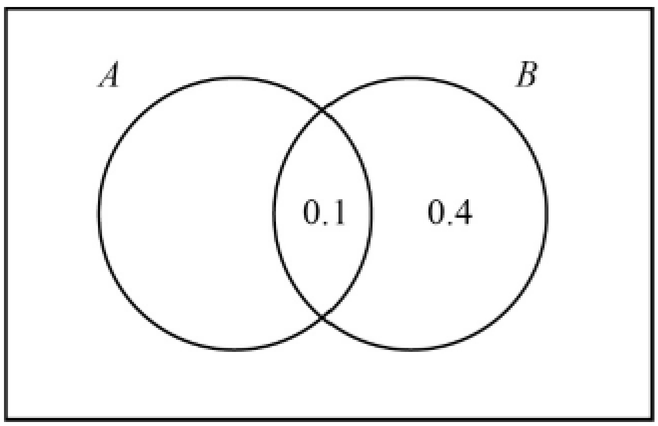
12. [Maximum mark: 6] 18N.1.AHL.TZ0.H_1

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(M1)

Note: Award ***M1*** for a Venn diagram with at least one probability in the correct region.

EITHER

$\mathrm{P}(A \cap B') = 0.3$ ***(A1)***

$\mathrm{P}(A \cup B) = 0.3 + 0.4 + 0.1 = 0.8$ ***A1***

OR

$\mathrm{P}(B) = 0.5$ ***(A1)***

$\mathrm{P}(A \cup B) = 0.5 + 0.4 - 0.1 = 0.8$ ***A1***

[3 marks]

(b) Show that the events A and B are not independent. [3]

Markscheme

METHOD 1

$\mathrm{P}\,(A)\mathrm{P}\,(B) = 0.4 \times 0.5$ ***(M1)***

= 0.2 ***A1***

statement that their $\mathrm{P}\,(A)\mathrm{P}\,(B) \neq \mathrm{P}\,(A \cap B)$ ***R1***

Note: Award ***R1*** for correct reasoning from their value.

⇒ A, B not independent ***AG***

METHOD 2

$\mathrm{P}\,(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{0.1}{0.5}$ ***(M1)***

= 0.2 ***A1***

statement that their $\mathrm{P}\,(A|B) \neq \mathrm{P}\,(A)$ ***R1***

Note: Award ***R1*** for correct reasoning from their value.

⇒ A, B not independent ***AG***

Note: Accept equivalent argument using $\mathrm{P}\,(B|A) = 0.25$.

[3 marks]