Complex numbers - exam questions [49 marks]

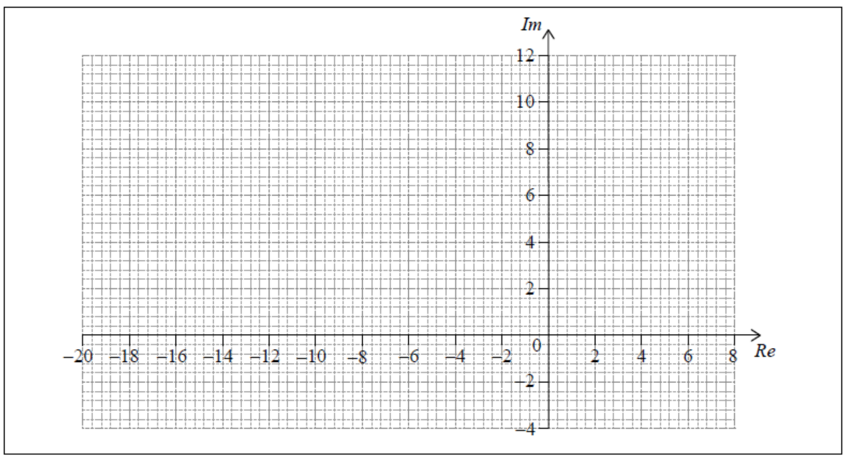
1. [Maximum mark: 7] SPM.1.AHL.TZ0.15

Let $w = a\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$, where $a \in \mathbb{R}^{+}$.

for $a = 2$,

(a.i) find the values of w^2, w^3, and w^4. [2]

(a.ii) draw w, w^2, w^3, and w^4 on the following Argand diagram.



[3]

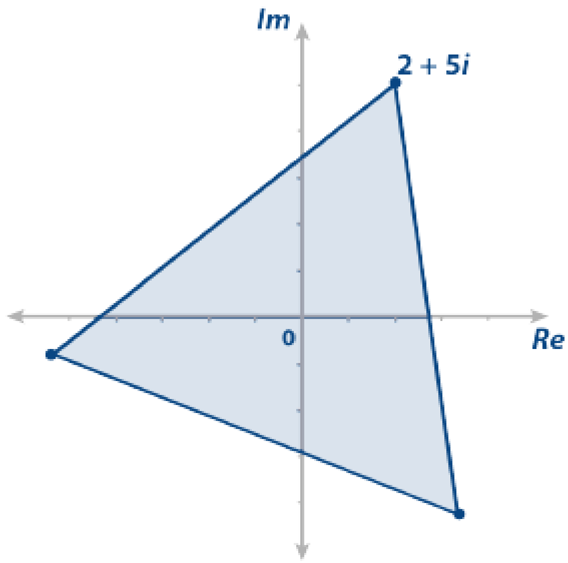
(b) Let $z = \frac{w}{2-\mathrm{i}}$.

Find the value of a for which successive powers of z lie on a circle. [2]

2. [Maximum mark: 5] EXN.1.AHL.TZ0.14

(a) Write down $2+5\mathrm{i}$ in exponential form. [2]

(b)

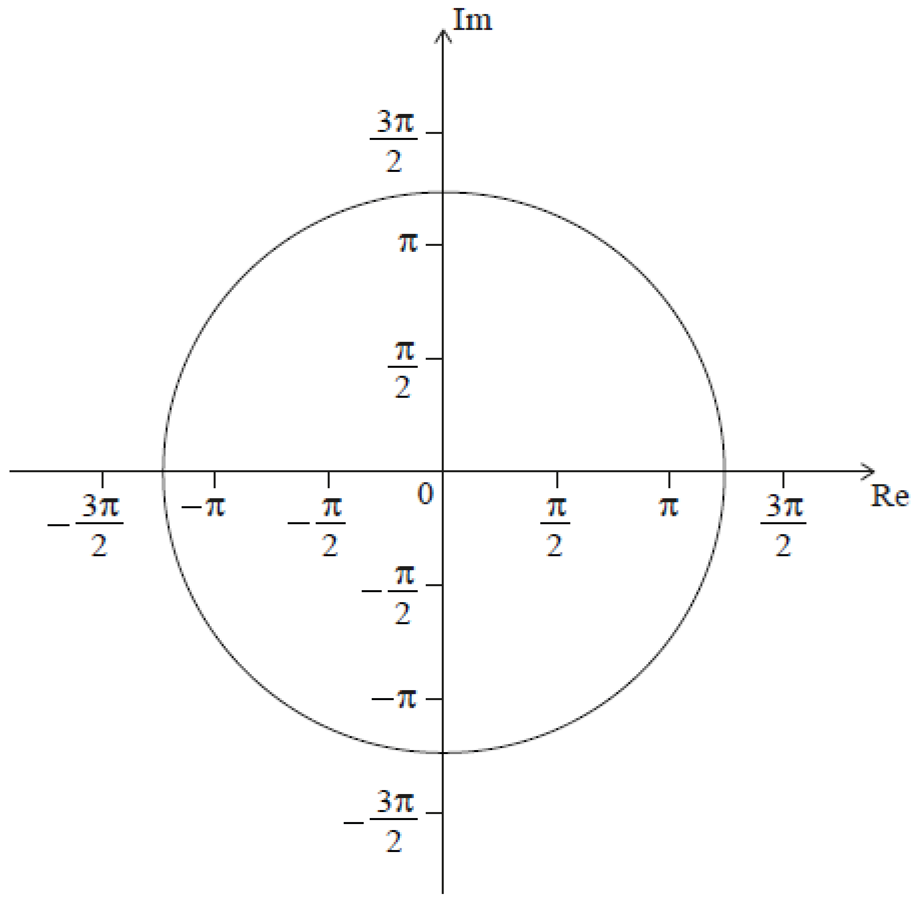


An equilateral triangle is to be drawn on the Argand plane with one of the vertices at the point corresponding to $2+5\mathrm{i}$ and all the vertices equidistant from 0.

Find the points that correspond to the other two vertices. Give your answers in Cartesian form. [3]

3. [Maximum mark: 7] 22M.1.AHL.TZ1.10

The following Argand diagram shows a circle centre 0 with a radius of 4 units.



A set of points, $\{z_\theta\}$, on the Argand plane are defined by the equation

$z_\theta = \frac{1}{2}\theta e^{\theta i}$, where $\theta \geq 0$.

Plot on the Argand diagram the points corresponding to

(a.i) $\theta = \frac{\pi}{2}$. [1]

(a.ii) $\theta = \pi$. [1]

(a.iii) $\theta = \frac{3\pi}{2}$. [1]

Consider the case where $|z_\theta| = 4$.

(b.i) Find this value of θ. [2]

(b.ii) For this value of θ, plot the approximate position of z_θ on the Argand diagram. [2]

4. [Maximum mark: 8] 21M.1.AHL.TZ1.9

Consider $w = \mathrm{i}z + 1$, where $w,\ z \in \mathbb{C}$.

Find w when

(a.i) $z = 2\mathrm{i}$. [2]

(a.ii) $z = 1 + \mathrm{i}$. [1]

Point z on the Argand diagram can be transformed to point w by two transformations.

(b) Describe these two transformations and give the order in which they are applied. [3]

(c) Hence, or otherwise, find the value of z when $w = 2 - \mathrm{i}$. [2]

5. [Maximum mark: 8] 21M.1.AHL.TZ2.12

It is given that $z_1 = 3\,\mathrm{cis}\left(\frac{3\pi}{4}\right)$ and $z_2 = 2\,\mathrm{cis}\left(\frac{n\pi}{16}\right),\ n \in \mathbb{Z}^+$.

In parts (a)(i) and (a)(ii), give your answers in the form $re^{i\theta},\ r \geq 0,\ \ -\pi < \theta \leq \pi$.

(a.i) Find the value of ${z_1}^3$. [2]

(a.ii) Find the value of $\left(\frac{z_1}{z_2}\right)^4$ for $n = 2$. [3]

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$. [3]

6. [Maximum mark: 14] 18M.1.AHL.TZ1.H_11

Consider $w = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$

(a.i) Express w^2 and w^3 in modulus-argument form. [3]

(a.ii) Sketch on an Argand diagram the points represented by w^0, w^1, w^2 and w^3. [2]

These four points form the vertices of a quadrilateral, Q.

(b) Show that the area of the quadrilateral Q is $\frac{21\sqrt{3}}{2}$. [3]

(c) Let $z = 2\left(\cos\frac{\pi}{n} + \mathrm{i}\sin\frac{\pi}{n}\right),\ n \in \mathbb{Z}^+$. The points represented on an Argand diagram by $z^0,\ z^1,\ z^2,\ \ldots,\ z^n$ form the vertices of a polygon P_n.

Show that the area of the polygon P_n can be expressed in the form $a\left(b^n - 1\right)\sin\frac{\pi}{n}$, where $a,\ b \in \mathbb{R}$. [6]