Exponential function - modelling [37 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.5

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a.i) Find the population of the Bulbul birds at the start of the migration season. [1]

(a.ii) Find the population of the Bulbul birds after 5 days. [2]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

2. [Maximum mark: 15] 23M.2.SL.TZ2.3

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P, can be modelled by the function

$P(t) = 1200 \times k^t, t \geq 0$,

where t is the number of hours since the experiment began, and k is a positive constant.

(a.i) Write down the value of $P(0)$. [1]

(a.ii) Interpret what this value means in this context. [1]

3 hours after the experiment began, the population of the bacteria is $18\,750$.

(b) Find the value of k. [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria.

The population of this bacteria, S, can be modelled by the function

$S(t) = 5000 \times 1.65^t, t \geq 0$,

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach $19\,000$.

(e) Find the value of m, giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18} \mathrm{m}^3$, and the available volume inside the container is $2.1 \times 10^{-5} \mathrm{m}^3$.

(f) Determine how long it would take for the bacteria to fill the container. [3]

3. [Maximum mark: 6] 22N.1.SL.TZ0.5

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^\circ\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\mathrm{e}^{-0.0514t} + 23,\ t \geq 0,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

(b) Write down the equation of the horizontal asymptote. [1]

(c) Write down the room temperature. [1]

(d) Given that $T^{-1}(50) = k$, find the value of k. [2]

4. [Maximum mark: 4] 21N.1.SL.TZ0.3

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = A\mathrm{e}^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\mathrm{mm}^2$ at the start of the experiment and $360\,\mathrm{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

(b) Find the value of k. [3]

5. [Maximum mark: 6] 21M.1.SL.TZ1.7

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\mathrm{e}^{-pt}, t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]