Quadratic modelling - exam questions [43 marks]

1. [Maximum mark: 8] 23M.1.SL.TZ2.10

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5,\ t \geq 0,$$

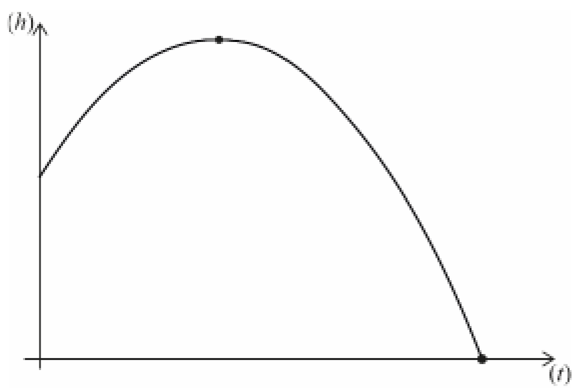
where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

Markscheme

METHOD 1

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052\ldots,\ \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =)\ -\frac{8.75}{2\times-4.75}$

0.921 (seconds) $\left(0.921052\ldots,\ \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

0.921 (seconds) $\left(0.921052\ldots,\ \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

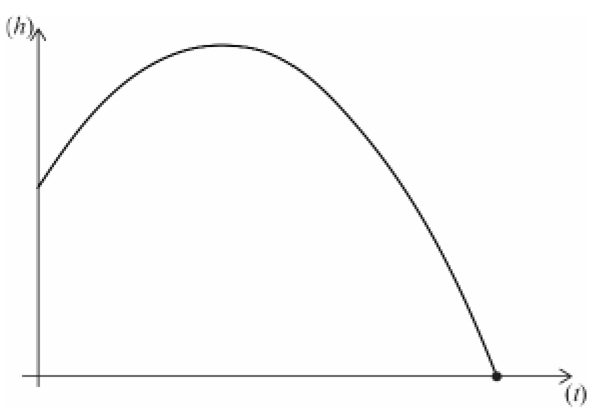
[2 marks]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Markscheme

METHOD 1

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

METHOD 2

setting the equation to zero ***(M1)***

$$-4.75t^2 + 8.75t + 1.5 = 0$$

2 (seconds) ***A1***

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

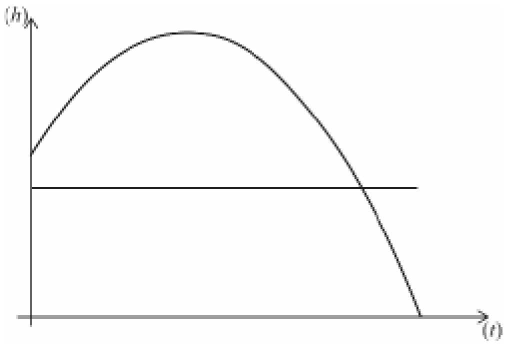
Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

Markscheme

METHOD 1

correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***



1.88 (seconds) ($1.87577\ldots$ (seconds)) ***A1***

METHOD 2

setting the equation equal to 1.2 ***(M1)***

$-4.75t^2 + 8.75t + 1.5 = 1.2$

1.88 (seconds) ($1.87577\ldots$ (seconds)) ***A1***

Note: Award ***(M1)A0*** if $-0.0336702\ldots$ seen as (part of) a final answer.

Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

(d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

Markscheme

*Award **R1** for a sensible reason in the context of the question:* ***R1R1***

e.g.

The model ignores air resistance (or wind)

The model treats the ball as a point

The model assumes gravity is constant

The model assumes that the ball continues to follow the trajectory even after hitting the ground

This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[2 marks]

2. [Maximum mark: 5] 22M.1.SL.TZ1.3

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

Markscheme

1.2 metres ***A1***

[1 mark]

(b) Find the value of t when the ball hits the ground. [2]

Markscheme

$-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)\ 4.43\text{s} \quad (4.431415\ldots\ \text{s})$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) State an appropriate domain for t in this model. [2]

Markscheme

$0 \leq t \leq 4.43$ **OR** $[0,\ 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]

3. [Maximum mark: 17] SPM.2.SL.TZ0.5

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, $d \text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, d **m**	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of 6 m s^{-1}, Model A can be represented by the equation $6p + q = 2$.

(a.i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1}. [2]

Markscheme

$p(10)^2 + q(10) = 60$ ***M1***

$10p + q = 6 \; (100p + 10q = 60)$ ***A1***

[2 marks]

(a.ii) Find the values of p and q. [2]

Markscheme

$p = 1, q = -4$ ***A1A1***

Note: If p and q are both incorrect then award ***M1A0*** for an attempt to solve simultaneous equations.

[2 marks]

(b) Find the coordinates of the vertex of the graph of $y = d\left(s\right)$. [2]

Markscheme

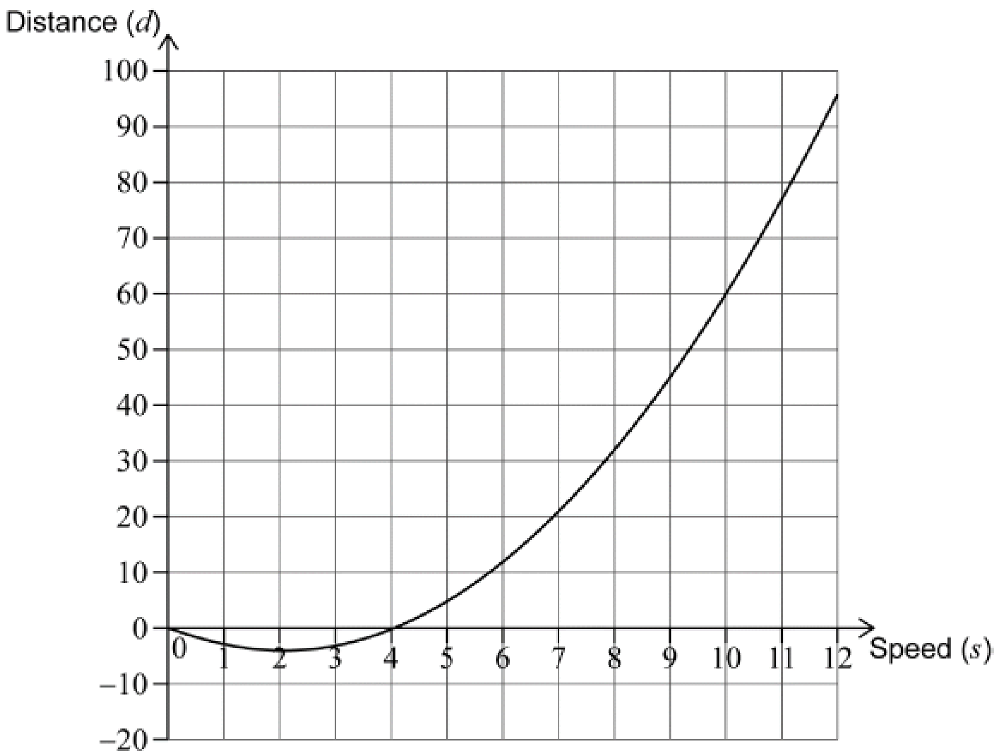
(2, −4) ***A1A1***

Note: Award ***A1*** for each correct coordinate.
Award ***A0A1*** if parentheses are missing.

[2 marks]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d\left(s\right)$ for 0 ≤ s ≤ 10 and −10 ≤ d ≤ 60, clearly showing the vertex. [3]

Markscheme



A3

Note: Award ***A1*** for smooth quadratic curve on labelled axes and within correct window.
Award ***A1*** for the curve passing through (0, 0) and (10, 60). Award ***A1*** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Markscheme

the graph indicates there are negative stopping distances (for low speeds) ***R1***

Note: Award ***R1*** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances…).

[1 mark]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d\left(s\right) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \mathrm{m\,s^{-1}}$. [2]

Markscheme

$0.95 \times 20^2 - 3.92 \times 20$ ***(M1)***

$= 302\ \left(\mathrm{m}\right)\ \left(301.6\ldots\right)$ ***A1***

[2 marks]

The actual braking distance at $20\ \mathrm{m\,s^{-1}}$ is $320\ \mathrm{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

Markscheme

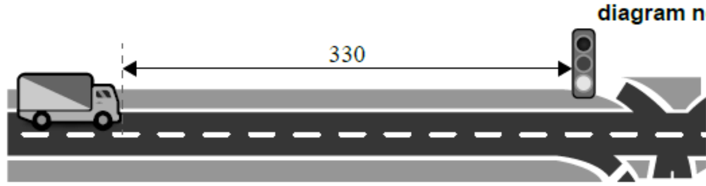
$\left|\frac{301.6 - 320}{320}\right| \times 100$ ***M1***

$= 5.75$ (%) ***A1***

[2 marks]

(g) It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ \mathrm{m\,s^{-1}}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\ \mathrm{m}$.



Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]

Markscheme

$330 = 1.6 \times s + 0.95 \times s^2 - 3.92 \times s$ ***M1A1***

Note: Award ***M1*** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330. Award ***A1*** for a completely correct equation.

$19.9\ (\mathrm{m\,s^{-1}})\ \ (19.8988\ldots)$ ***A1***

[3 marks]

4. [Maximum mark: 13] EXM.2.SL.TZ0.3

Urvashi wants to model the height of a moving object. She collects the following data showing the height, h metres, of the object at time t seconds.

t (seconds)	2	5	7
h (metres)	34	38	24

She believes the height can be modeled by a quadratic function,
$h\left(t\right)=at^2+bt+c$, where $a,\ b,\ c\in\mathbb{R}$.

(a) Show that $4a+2b+c=34$. [1]

Markscheme

$t=2,\ h=34\Rightarrow\ \ 34=a2^2+2b+c$ ***M1***

$\Rightarrow\ 34=4a+2b+c$ ***AG***

[1 mark]

(b) Write down two more equations for a, b and c. [3]

Markscheme

attempt to substitute either (5, 38) or (7, 24) ***M1***

$25a+5b+c=38$ ***A1***

$49a+7b+c=24$ ***A1***

[3 marks]

(c) Solve this system of three equations to find the value of a, b and c. [4]

Markscheme

$a = -\frac{5}{3},\ b = 13,\ c = \frac{44}{3}$ ***M1A1A1A1***

[3 marks]

Hence find

(d.i) when the height of the object is zero. [3]

Markscheme

$-\frac{5}{3}t^2 + 13t + \frac{44}{3} = 0$ ***M1***

$t = 8.8$ seconds ***M1A1***

[3 marks]

(d.ii) the maximum height of the object. [2]

Markscheme

attempt to find maximum height, e.g. sketch of graph ***M1***

$h = 40.0$ metres ***A1***

[2 marks]