Trig modelling - exam questions [60 marks]

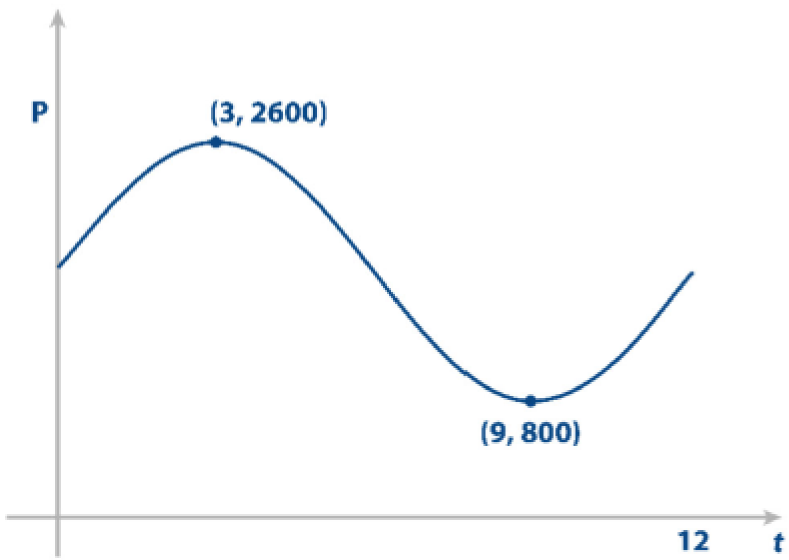
1. [Maximum mark: 7] EXN.1.SL.TZ0.6

The size of the population (P) of migrating birds in a particular town can be approximately modelled by the equation

$P = a\sin(bt) + c, \quad a, b, c \in \mathbb{R}^+$, where t is measured in months from the time of the initial measurements.

In a 12 month period the maximum population is 2600 and occurs when $t = 3$ and the minimum population is 800 and occurs when $t = 9$.

This information is shown on the graph below.



(a.i) Find the value of a. [2]

(a.ii) Find the value of b. [2]

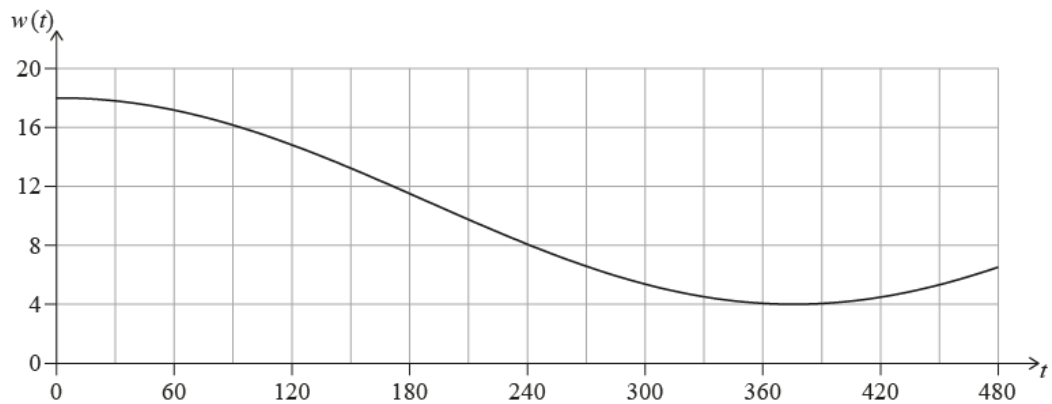
(a.iii) Find the value of c. [1]

(b) Find the value of t at which the population first reaches 2200. [2]

2. [Maximum mark: 15] 23M.2.SL.TZ1.3

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a\cos(bt^{\circ}) + d$ where t is the length of time, in minutes, after $06:00$.

On 20 January, the first high tide occurs at $06:00$, at which time the depth of water is $18\,\text{m}$. The following low tide occurs at $12:15$ when the depth of water is $4\,\text{m}$. This is shown in the diagram.



(a) Find the value of a. [2]

(b) Find the value of d. [2]

(c) Find the period of the function in minutes. [3]

(d) Find the value of b. [2]

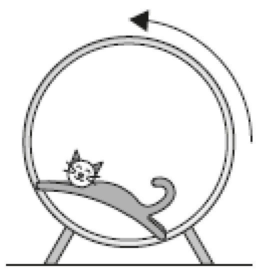
Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least $6\,\text{m}$.

(e) Find the latest time before $12:00$, to the nearest minute, that Naomi can enter the harbour. [4]

(f) Find the length of time (in minutes) between $06:00$ and $15:00$ on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

3. [Maximum mark: 6] 22N.1.SL.TZ0.12

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, $h\,\mathrm{cm}$, of a fixed point, P, on the wheel can be modelled by $h(t) = a\,\sin(bt) + c$ where t is the time in seconds and $a,\ b,\ c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of $78\,\mathrm{cm}$.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of $143\,\mathrm{cm}$.

(b.i) Find the value of a. [1]

(b.ii) Find the value of b. [2]

(c) Write down the minimum height of point P. [1]

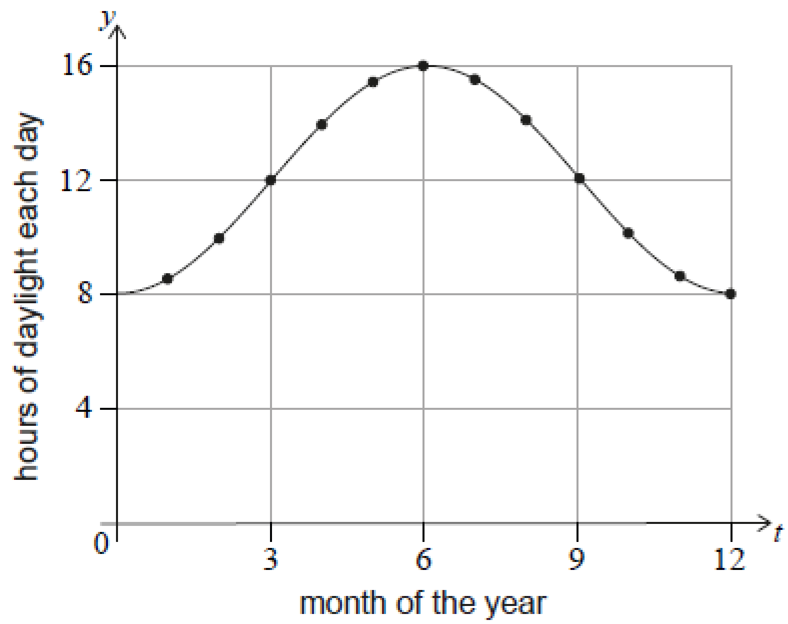
Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

4. [Maximum mark: 15] 22M.2.SL.TZ1.1

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0,\ 8)$ and maximum point $(6,\ 16)$ as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

(a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years. [1]

Paula thinks that a better model is $f(t) = a\cos(bt) + d, t \geq 0$, for specific values of $a,\ b$ and d.

For Paula's model, use the diagram to write down (b.i) the amplitude. [1]

(b.ii) the period. [1]

(b.iii) the equation of the principal axis. [2]

(c) Hence or otherwise find the equation of this model in the form:

$$f(t) = a\cos(bt) + d$$ [3]

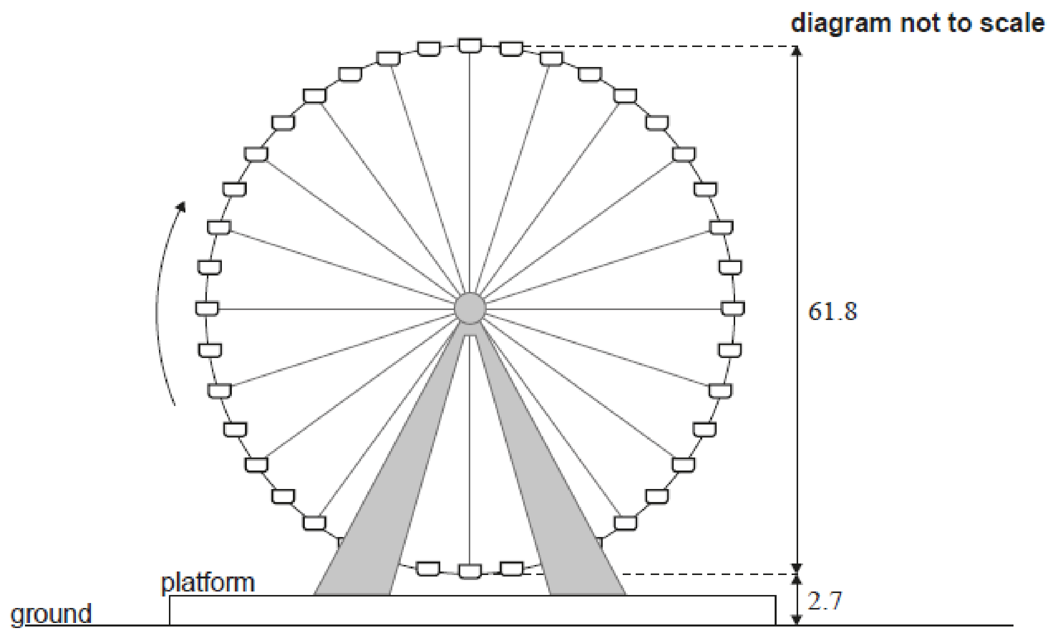
(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day. [4]

The true maximum number of daylight hours was 16 hours and 14 minutes.

(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram. [3]

5. [Maximum mark: 17] 22M.2.SL.TZ2.4

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of 61.8 m. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is 2.7 m above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.



The height of a chair above the ground, h, measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a\cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

Calculate the value of

(a.i) a. [2]

(a.ii) b. [2]

(a.iii) d. [2]

A ride on the Ferris wheel lasts for 12 minutes in total.

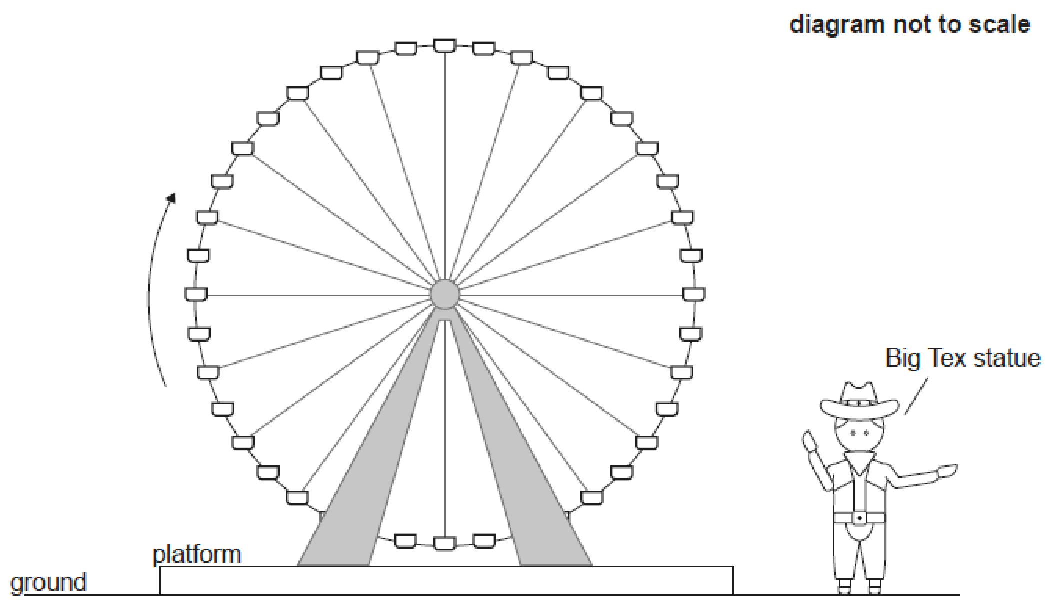
(b) Calculate the number of revolutions of the Ferris wheel per ride. [2]

For exactly one ride on the Ferris wheel, suggest

(c.i) an appropriate domain for $h(t)$. [1]

(c.ii) an appropriate range for $h(t)$. [2]

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.



[Source: Aline Escobar., n.d. Cowboy. [image online] Available at: https://thenounproject.com/search/?q=cowboy&i=1080130

 [Accessed 13/05/2021]. Source adapted.]

(d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to $65.2\,\mathrm{m}$. This will change the value of one parameter, a, b or d, found in part (a).

(e.i) Identify which parameter will change. [1]

(e.ii) Find the new value of the parameter identified in part (e)(i). [2]