Voronoi AI SL [58 marks]

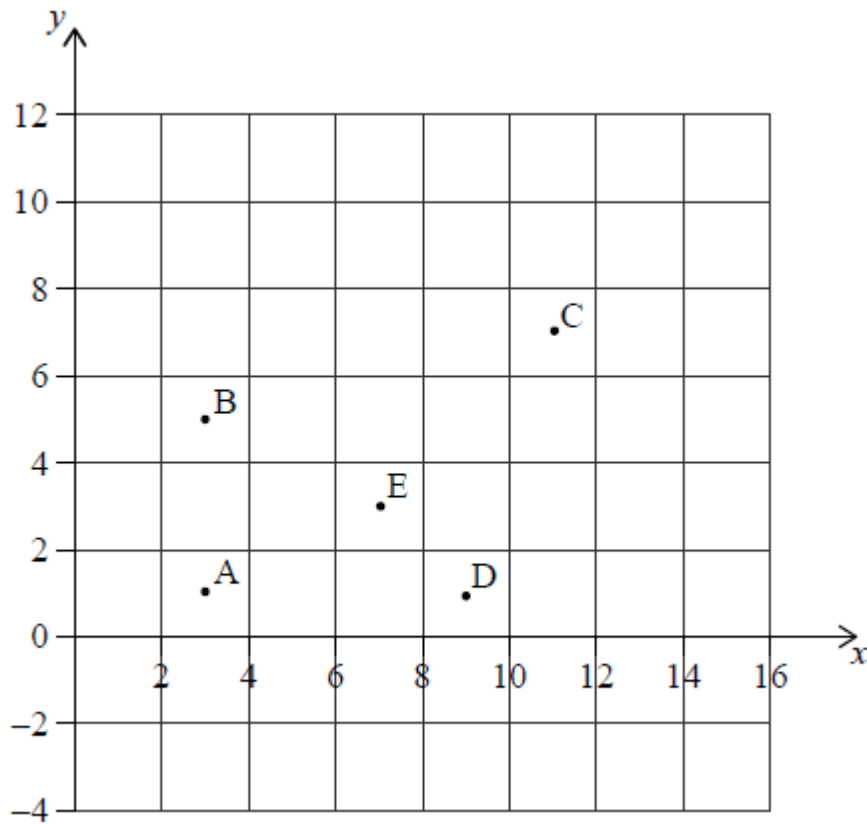**1.** [Maximum mark: 6]

Points A(3, 1), B(3, 5), C(11, 7), D(9, 1) and E(7, 3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.

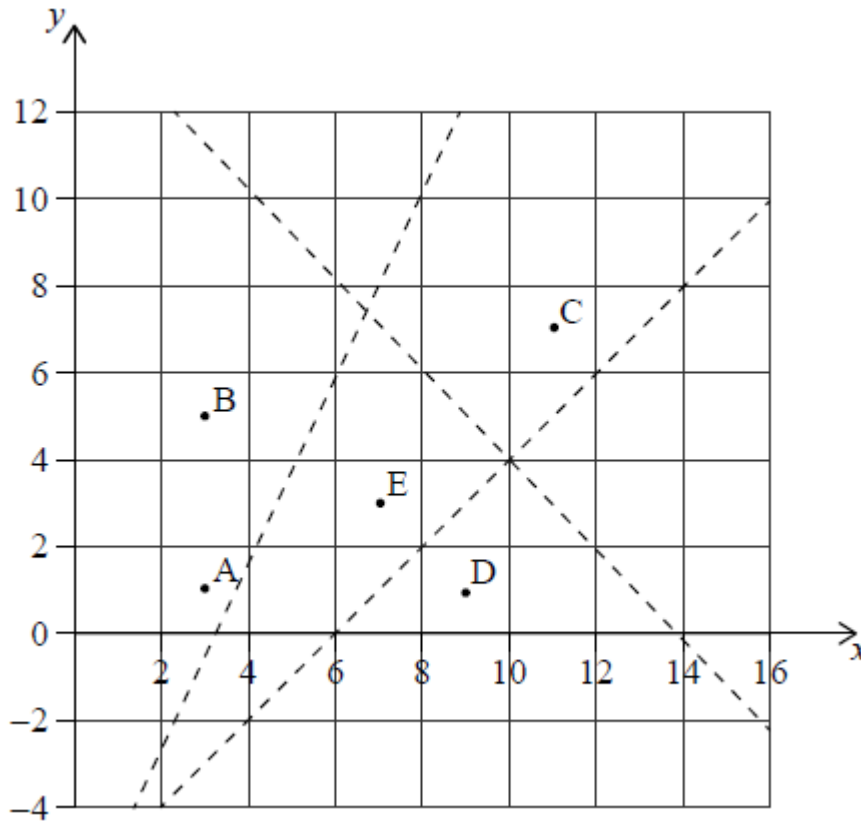(a) Calculate the gradient of the line segment AE. [2]

Markscheme

$\frac{3-1}{7-3}$ *(M1)*

$= 0.5$ *A1*

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.

(b)   Find the equation of the line which would complete the Voronoi cell containing site E.

Give your answer in the form $ax + by + d = 0$ where $a, b,$ $d \in \mathbb{Z}$.

[3]

Markscheme

$y - 2 = -2\left(x - 5\right)$ *(A1) (M1)*

Note: Award *(A1)* for their -2 seen, award *(M1)* for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ **A1**

[3 marks]

(c)   In the context of the question, explain the significance of the
        Voronoi cell containing site E.   [1]

Markscheme

every point in the cell is closer to E than any other snow shelter   **A1**

[1 mark]

2. [Maximum mark: 9]
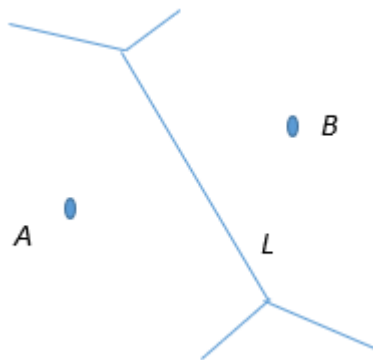
The diagram below is part of a Voronoi diagram.

Diagram not to scale

A and *B* are sites with *B* having the co-ordinates of (4, 6). *L* is an edge; the equation of this perpendicular bisector of the line segment from *A* to *B* is $y = -2x + 9$

Find the co-ordinates of the point *A*. [9]

Markscheme

Line from *A* to *B* will have the form $y = \frac{1}{2}x + c$ **M1A1**

Through $(4, \, 6) \Rightarrow c = 4$ so line is $y = \frac{1}{2}x + 4$ **M1A1**

Intersection of $y = -2x + 9$ and $y = \frac{1}{2}x + 4$ is (2, 5) **M1A1**

Let $A = (p, \, q)$ then $(2, \, 5) = \left(\frac{p+4}{2}, \, \frac{q+6}{2} \right) \Rightarrow p = 0, \, q = 4$

M1A1A1
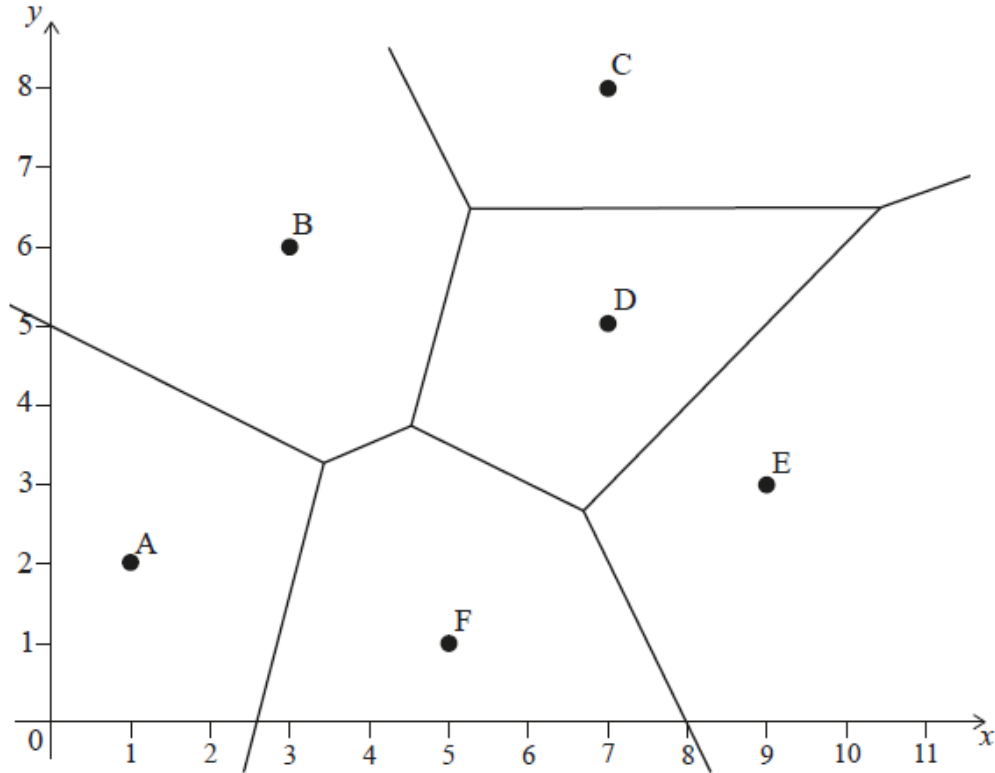
$A = (0, 4)$

[9 marks]

3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.

Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(a.i) $(2, 7)$.   [1]

> **Markscheme**
>
> B   *A1*
>
> *[1 mark]*

(a.ii) $(0, 1)$, when restaurant A is closed.

Restaurant C is at $(7,\ 8)$ and restaurant D is at $(7,\ 5)$.

(b)   Find the equation of the perpendicular bisector of CD.   [2]

Restaurant B is at $(3,\ 6)$.

(c)   Find the equation of the perpendicular bisector of BC.   [5]

correct use of gradient formula   *(M1)*

$\frac{8-6}{7-3}$

gradient of $BC = 0.5$ *(A1)*

negative reciprocal of gradient   *(M1)*

perpendicular gradient $= -2$

$y - 7 = -2(x - 5)$ (or $y = -2x + 17$)   *A1*

[5 marks]

Hence find

(d.i)   the coordinates of the point which is of equal distance from B, C and D.   [2]

Markscheme

attempt to find the intersection of two perpendicular bisectors (BC & CD)   *(M1)*

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x - 5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ OR $y = 4x - 14.5$

$x = 5.25$, $y = 6.5$ **OR** $(5.25, 6.5)$ *A1*

Note: The x-coordinate must be exact or expressed to at least 3 sf.

[2 marks]

(d.ii)   the distance of this point from D.   [2]

4. [Maximum mark: 18]   22M.2.SL.TZ2.3

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, \ 0), B(6, \ 0), C(0, \ 6)$ and $D(2, \ 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.

diagram not to scale
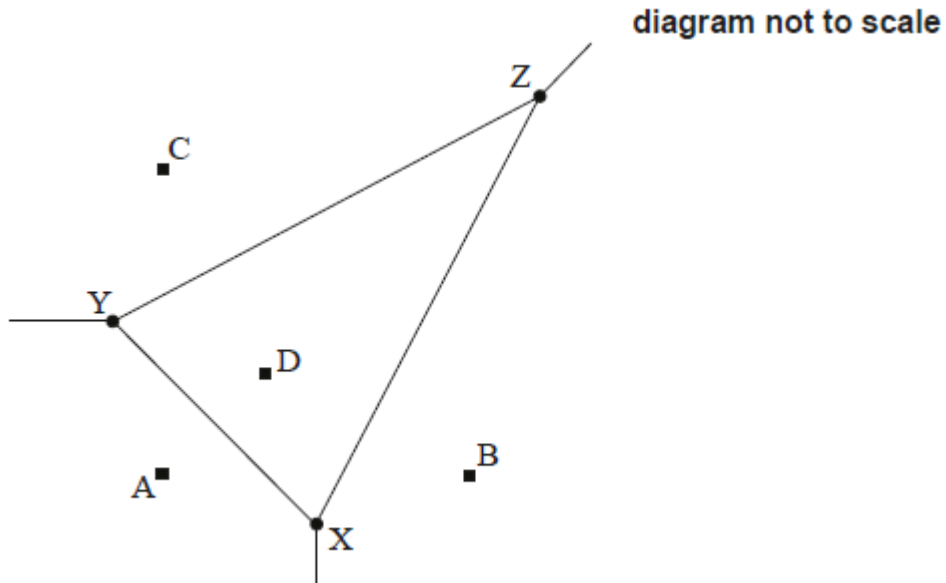
(a)   Find the midpoint of $[BD]$.   [2]

Markscheme

$\left(\frac{2+6}{2}, \ \frac{2+0}{2}\right)$ *(M1)*

$(4, \ 1)$ *A1*

Note: Award *A0* if parentheses are omitted in the final answer.

[2 marks]

(b)   Find the equation of (XZ).   [4]

attempt to substitute values into gradient formula   *(M1)*

$\left(\frac{0-2}{6-2} =\right) - \frac{1}{2}$ *(A1)*

therefore the gradient of perpendicular bisector is 2 *(M1)*

so $y - 1 = 2(x - 4)$ $(y = 2x - 7)$ *A1*

[4 marks]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c)   Find the coordinates of X.   [3]

identifying the correct equations to use: *(M1)*

$y = 2 - x$ and $y = 2x - 7$

evidence of solving their correct equations or of finding intersection point graphically   *(M1)*

$(3, \ -1)$ *A1*

Note: Accept an answer expressed as "$x = 3, \ y = -1$".

[3 marks]

The coordinates of Y are $(-1,\ 3)$ and the coordinates of Z are $(7,\ 7)$.

(d)   Determine the exact length of $[YZ]$.   [2]

Markscheme

attempt to use distance formula   *(M1)*

$$YZ = \sqrt{(7-(-1))^2 + (7-3)^2}$$

$$= \sqrt{80}\ \left(4\sqrt{5}\right)\quad \textbf{\textit{A1}}$$

[2 marks]

(e)   Given that the exact length of $[XY]$ is $\sqrt{32}$, find the size of $X\hat{Y}Z$ in degrees.   [4]

Markscheme

METHOD 1 (cosine rule)

length of XZ is $\sqrt{80}\ \left(4\sqrt{5},\ 8.94427\ldots\right)$ *(A1)*

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule   *(M1)*

$$\cos X\hat{Y}Z = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}\ \ (= 0.316227\ldots)\quad \textbf{\textit{(A1)}}$$

Note: Award *A1* for correct substitution of $XZ, YZ, \sqrt{32}$ values in the cos

rule. Exact values do not need to be used in the substitution.

$$(\mathrm{XYZ} =)\ 71.6°\quad (71.5650\dots°)\qquad \textit{A1}$$

Note: Last *A1* mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}\ \left(4\sqrt{5},\ 8.94427\dots\right)\qquad \textit{(A1)}$

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\dfrac{\sqrt{32}}{2\sqrt{80}}\right)\qquad \textit{(M1)(A1)}$

Note: Award *A1* for correct substitution of XZ (or YZ), $\dfrac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$$(\mathrm{XYZ} =)\ 71.6°\quad (71.5650\dots°)\qquad \textit{A1}$$

Note: Last *A1* mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f)   Hence find the area of triangle XYZ.   [2]

A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

(g)   State one criticism of this interpretation.   [1]

5. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6,\ 2),\ B(14,\ 2),\ C(18,\ 6)$ and $D(10.8,\ 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10,\ 6)$ and $Q(13,\ 7)$.
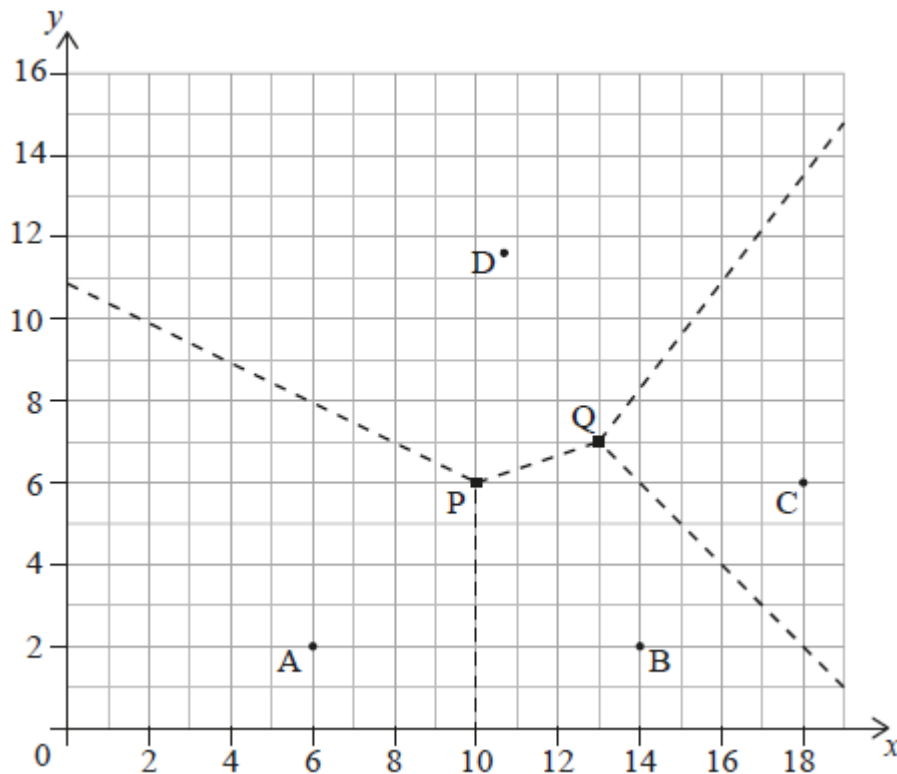
To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

(a)   Show that the new station should be built at P.   [3]

Markscheme

(the best placement is either point P or point Q)

attempt at using the distance formula *(M1)*

$$AP = \sqrt{(10-6)^2 + (6-2)^2} \text{ OR}$$

$$BP = \sqrt{(10-14)^2 + (6-2)^2} \text{ OR}$$

$$DP = \sqrt{(10-10.8)^2 + (6-11.6)^2} \text{ OR}$$

$$BQ = \sqrt{(13-14)^2 + (7-2)^2} \text{ OR}$$

$$CQ = \sqrt{(13-18)^2 + (7-6)^2} \text{ OR}$$

$$DQ = \sqrt{(13-10.8)^2 + (7-11.6)^2}$$

(AP or BP or $DP=$) $\sqrt{32} = 5.66 \ (5.65685\ldots)$ **AND**

(BQ or CQ or $DQ=$) $\sqrt{26} = 5.10 \ (5.09901\ldots)$ *A1*

$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ)   *A1*

point P is the furthest away   *AG*

Note: Follow through from their values provided their AP (or BP or DP) is greater than their BQ (or CQ or DQ).
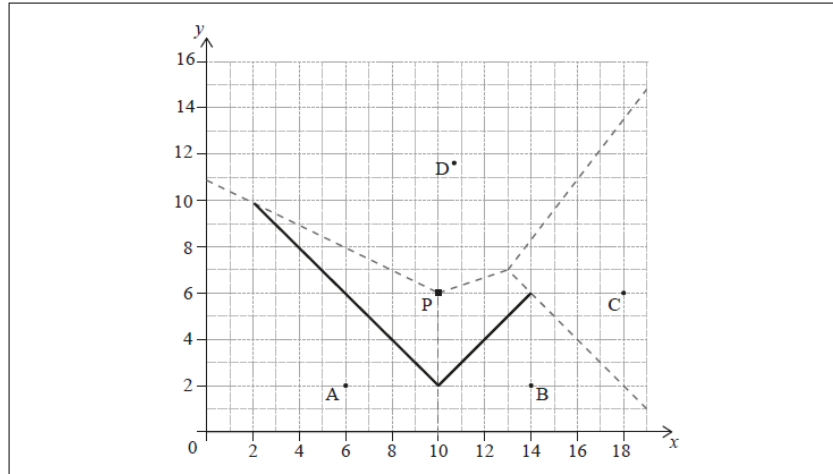
[3 marks]

The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of $[AP]$ and

$[BP]$ have been added to the following diagram.

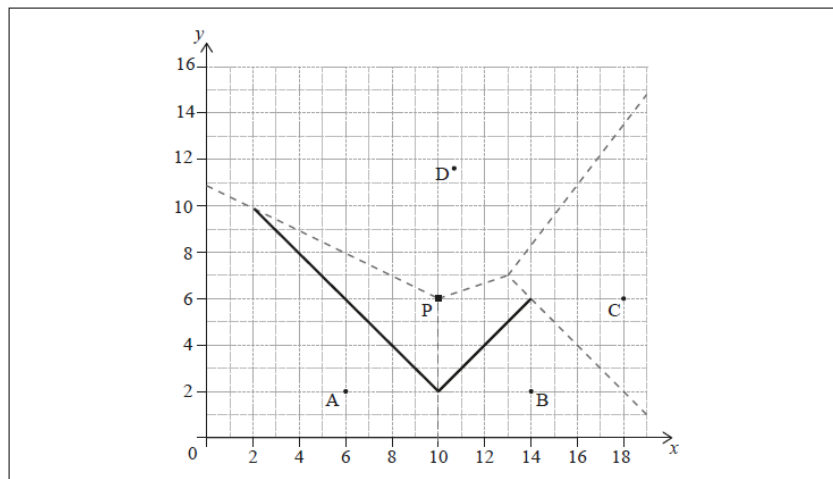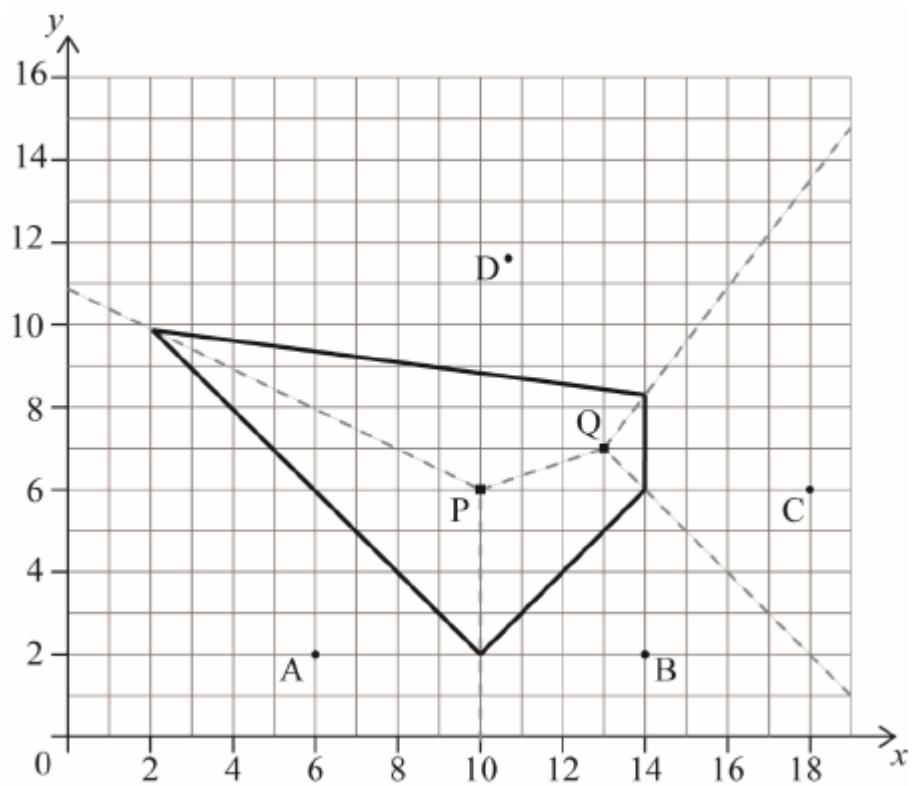(b.i)   Write down the equation of the perpendicular bisector of $[PC]$

.

[1]

(b.ii)   Hence draw the missing boundaries of the region around P on the following diagram.

Markscheme

A1A1

**Note:** Award *A1* for each correct straight line. Do not *FT* from their part (b)(i).

[1 mark]

The Voronoi diagram below shows three identical cellular phone towers, $T1,\ T2$ and $T3$. A fourth identical cellular phone tower, $T4$ is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents $1\,\text{km}$.
Vertical scale: 1 unit represents $1\,\text{km}$.
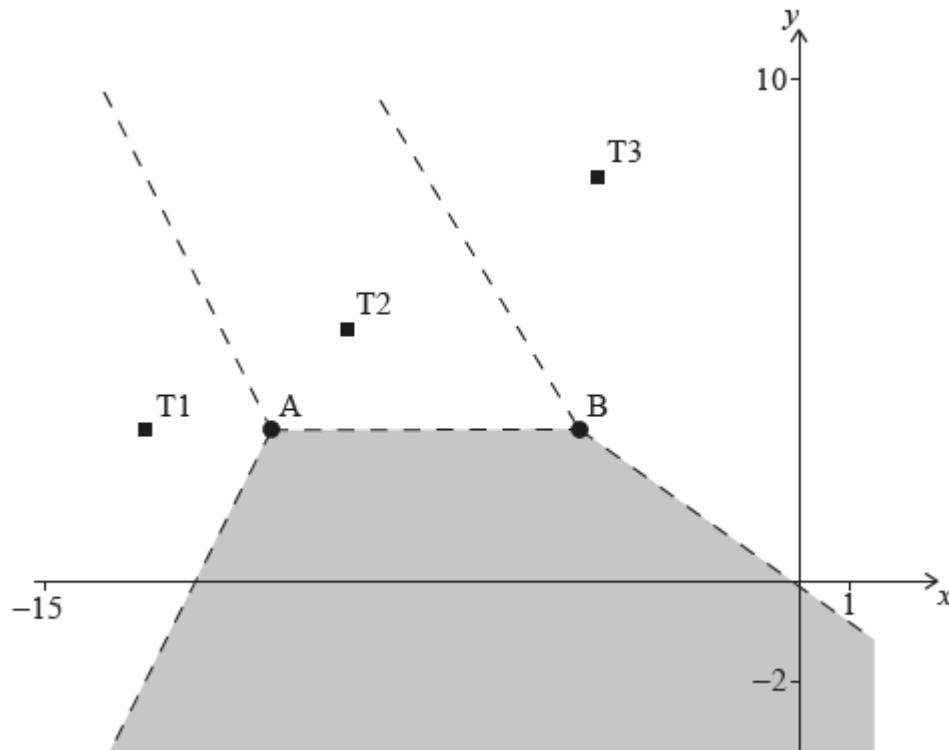
Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower $T4$. [1]

Markscheme
every point in the shaded region is closer to tower $T4$ *R1*

Tower $T2$ has coordinates $(-9,\ 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower $T4$. [2]

Markscheme

$(-9,\ 1)$ *A1A1*

Note: Award *A1* for each correct coordinate. Award at most *A0A1* if parentheses are missing.

[2 marks]

(c) Tower $T1$ has coordinates $(-13,\ 3)$.

Find the gradient of the edge of the Voronoi diagram between towers $T1$ and $T2$. [3]

Markscheme

correct use of gradient formula   *(M1)*

$$\text{e.g. } (m =) \frac{5-3}{-9--13} \left(= \frac{1}{2}\right)$$

taking negative reciprocal of **their** m (at any point)   *(M1)*

edge gradient $= -2$ *A1*

[3 marks]