

Voronoi AI SL [58 marks]

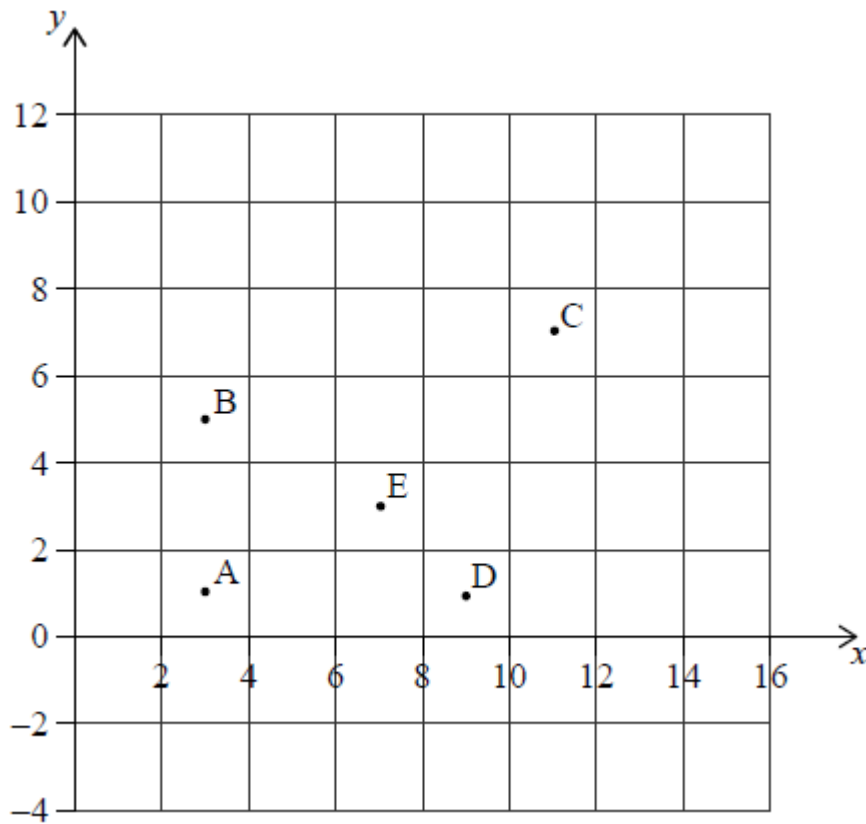
1. [Maximum mark: 6]

SPM.1.SL.TZ0.7

Points A(3, 1), B(3, 5), C(11, 7), D(9, 1) and E(7, 3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

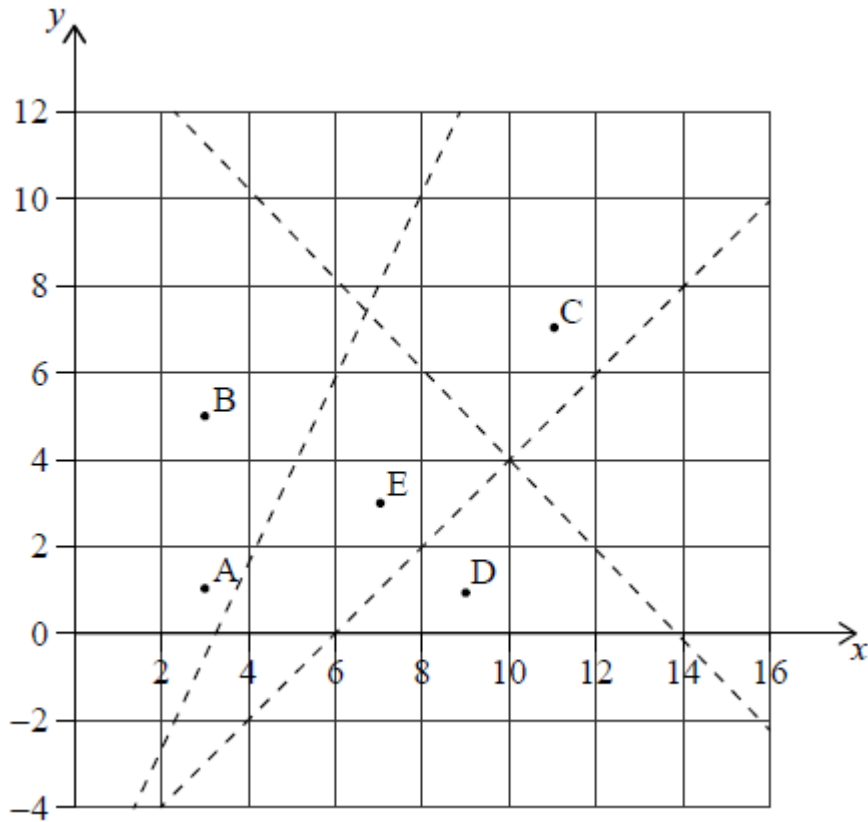
Vertical scale: 1 unit represents 1 km.



(a) Calculate the gradient of the line segment AE.

[2]

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



- (b) Find the equation of the line which would complete the Voronoi cell containing site E.

Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]

- (c) In the context of the question, explain the significance of the Voronoi cell containing site E.

[1]

2. [Maximum mark: 9]

EXM.1.SL.TZ0.4

The diagram below is part of a Voronoi diagram.

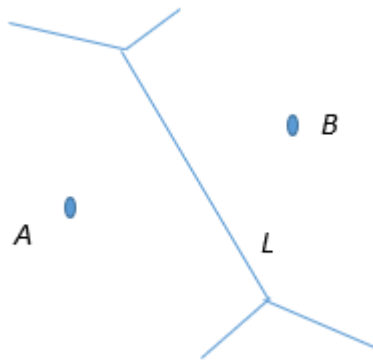


Diagram not to scale

A and B are sites with B having the co-ordinates of $(4, 6)$. L is an edge; the equation of this perpendicular bisector of the line segment from A to B is $y = -2x + 9$

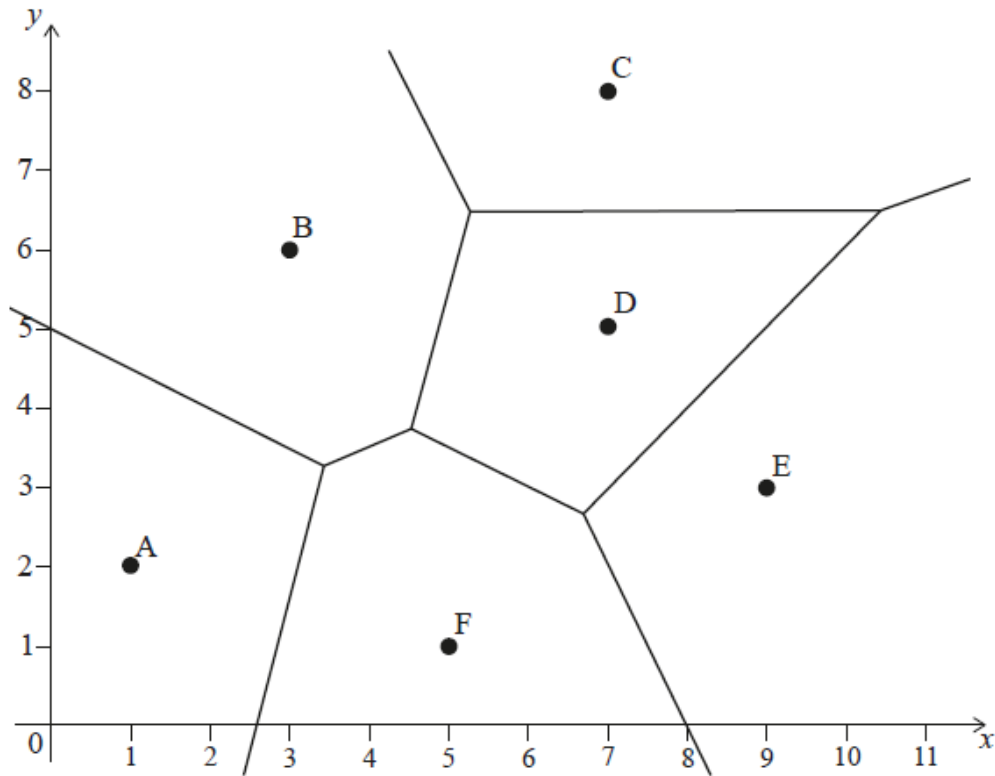
Find the co-ordinates of the point A .

[9]

3. [Maximum mark: 13]

22N.2.SL.TZ0.3

Six restaurant locations (labelled **A**, **B**, **C**, **D**, **E** and **F**) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(a.i) $(2, 7)$. [1]

(a.ii) $(0, 1)$, when restaurant **A** is closed. [1]

Restaurant **C** is at $(7, 8)$ and restaurant **D** is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of **CD**. [2]

Restaurant **B** is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of **BC**. [5]

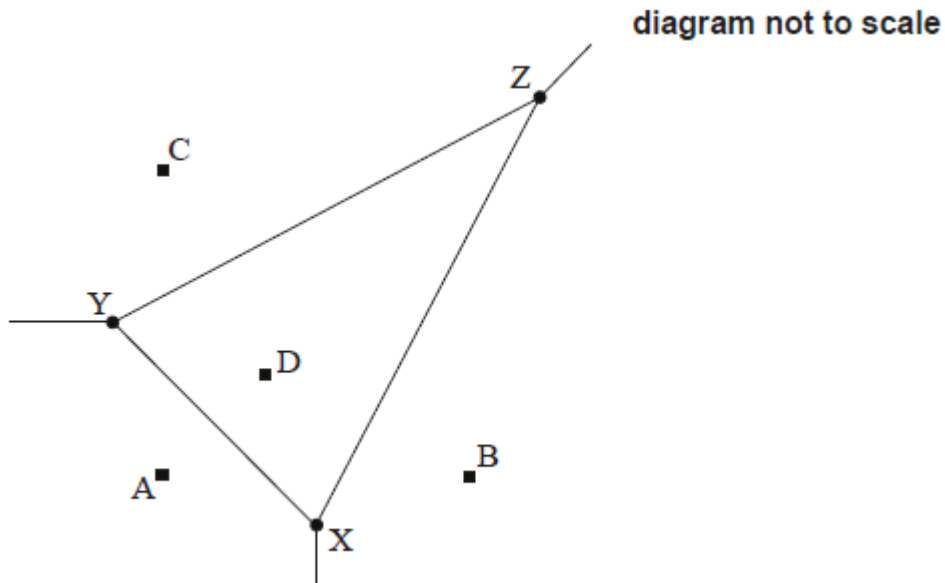
Hence find

- (d.i) the coordinates of the point which is of equal distance from **B**,
C and **D**. [2]
- (d.ii) the distance of this point from **D**. [2]

4. [Maximum mark: 18]

22M.2.SL.TZ2.3

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, 0)$, $B(6, 0)$, $C(0, 6)$ and $D(2, 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.



(a) Find the midpoint of $[BD]$. [2]

(b) Find the equation of (XZ) . [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X . [3]

The coordinates of Y are $(-1, 3)$ and the coordinates of Z are $(7, 7)$.

(d) Determine the exact length of $[YZ]$. [2]

(e) Given that the exact length of $[XY]$ is $\sqrt{32}$, find the size of $\angle XYZ$ in degrees. [4]

(f) Hence find the area of triangle XYZ . [2]

A town planner believes that the larger the area of the Voronoi cell XYZ , the more people will shop at supermarket D .

(g) State one criticism of this interpretation.

[1]

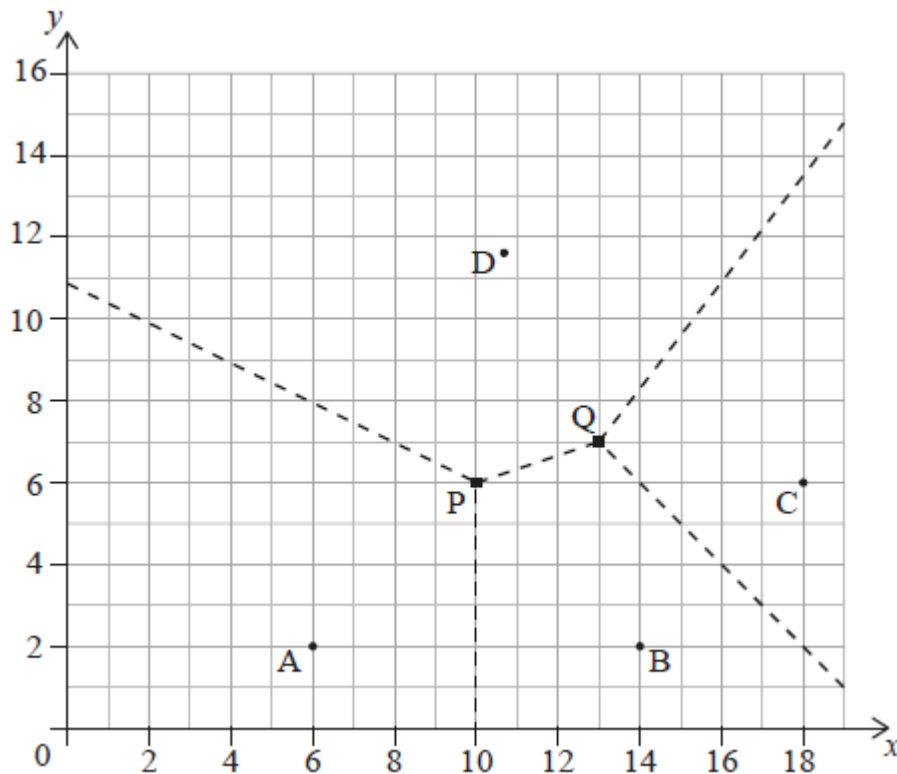
5. [Maximum mark: 6]

21N.1.SL.TZ0.7

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



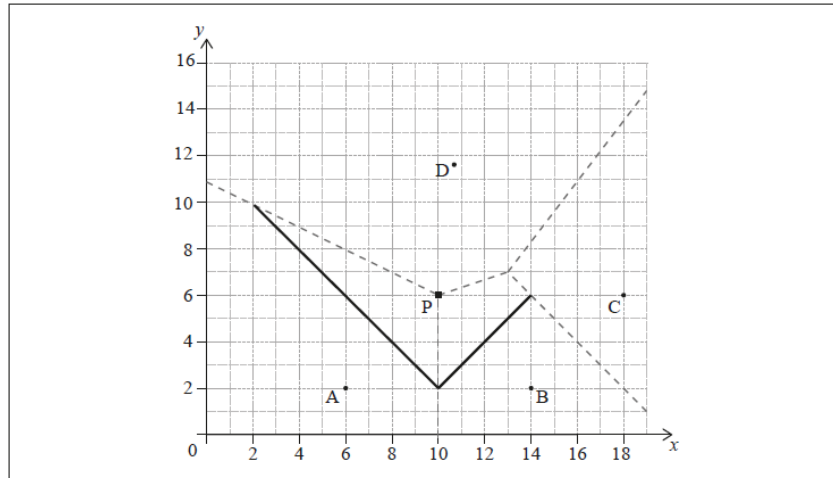
To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P .

[3]

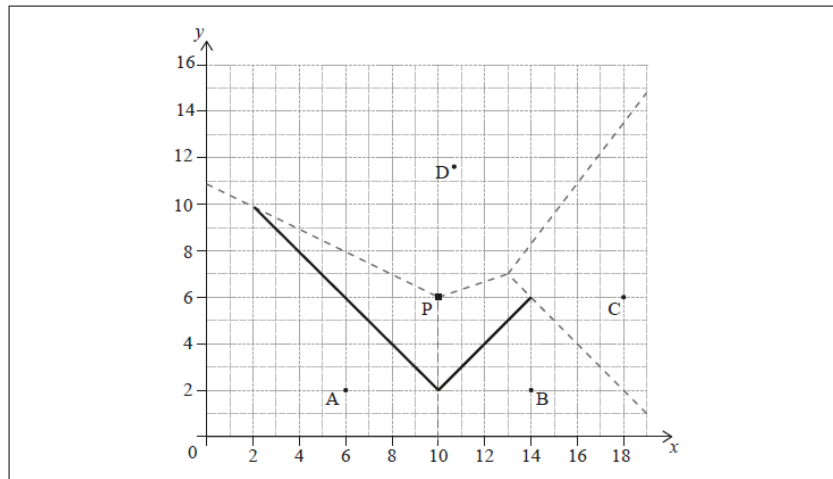
The Voronoi diagram is to be updated to include the region around the new station at P . The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

(b.i) Write down the equation of the perpendicular bisector of $[PC]$



[1]

(b.ii) Hence draw the missing boundaries of the region around P on the following diagram.



[2]

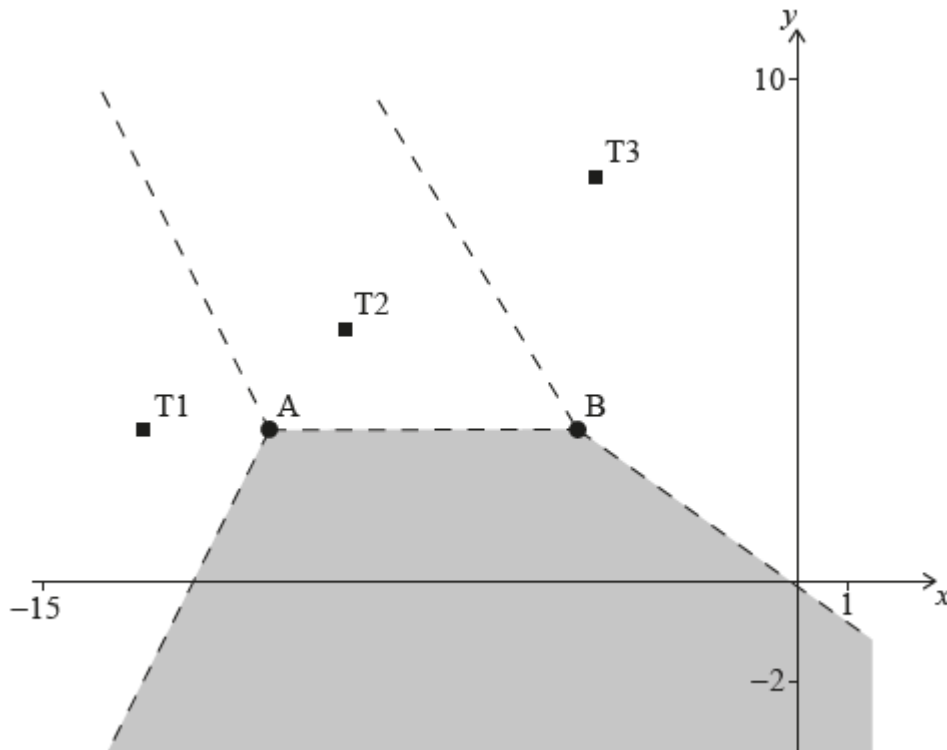
6. [Maximum mark: 6]

21M.1.SL.TZ1.5

The Voronoi diagram below shows three identical cellular phone towers, T_1 , T_2 and T_3 . A fourth identical cellular phone tower, T_4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

- (a) Explain why Tim will receive the strongest signal from tower T_4 .

[1]

Tower T_2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

- (b) Write down the coordinates of tower T_4 .

[2]

(c) Tower **T1** has coordinates $(-13, 3)$.

Find the gradient of the edge of the Voronoi diagram between towers **T1** and **T2**.

[3]