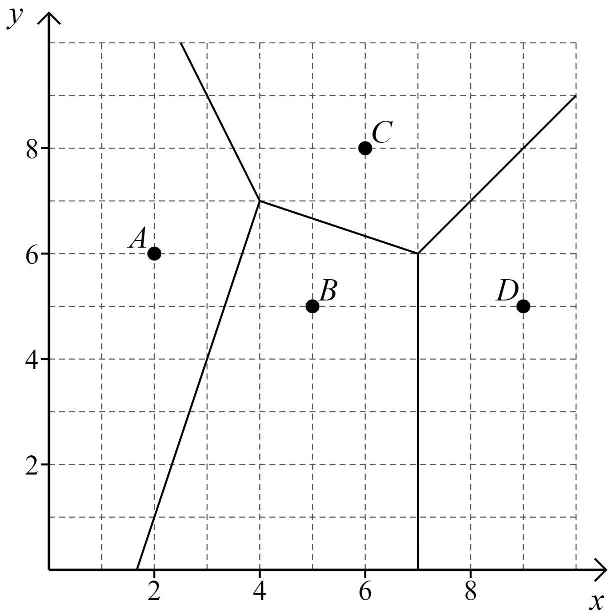
1. [Maximum points: 10]

The Voronoi diagram below shows the locations of four high schools in the suburbs of a city. Units of coordinates are kilometres.



(a) Explain what each cell of the Voronoi diagram represents. [1]

(b) Write down the gradient of the perpendicular bisector of line [2]

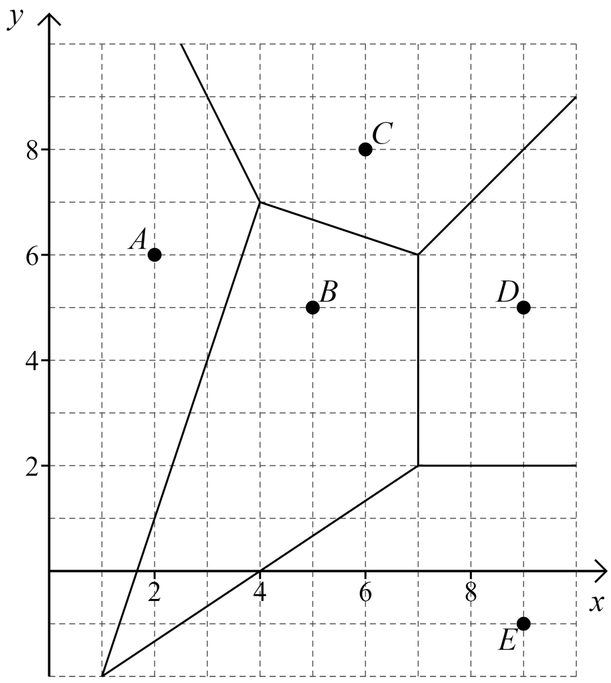
(i) AB

(ii) BC

(c) Show that these two perpendicular bisectors are perpendicular. [1]

The problem continues on the next page

A new school is to be built at $(9,-1)$. The new Voronoi diagram is shown below.

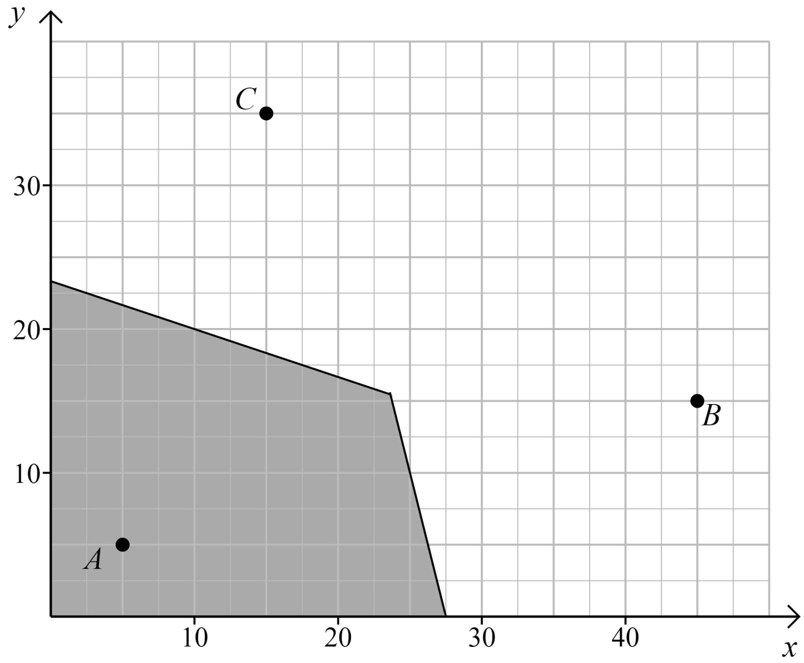


(d) Find the equations of the two new lines created on the diagram with the introduction of point E. [3]

(e) Find the area of the cell which contains point B. [3]

2. [Maximum points: 6]

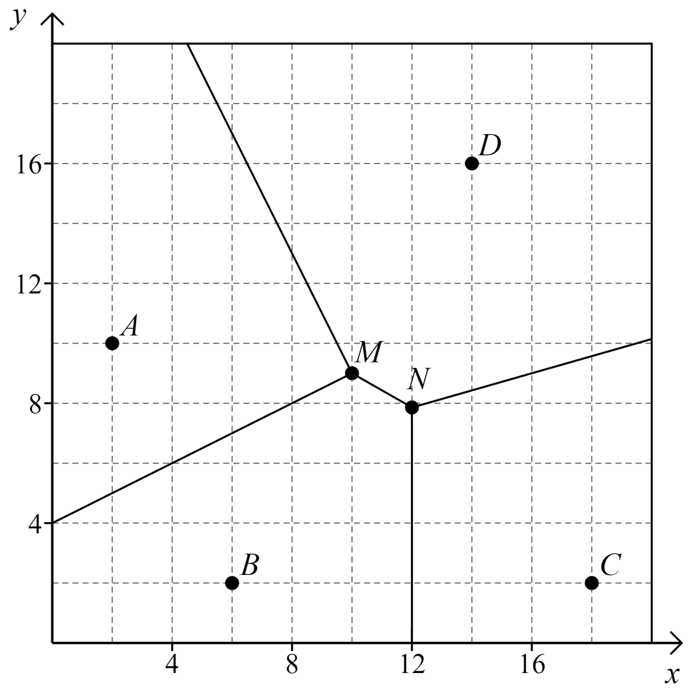
The graph below shows points $A(5,5)$, $B(45,15)$ and $C(15.35)$. The region containing all points which are the closest to point A is shaded.



On the diagram above show the region which contains all points which are the closest to point C

3. [Maximum points: 25]

A square pond has sides of length 20 m. The depth of the pond at points A, B, C and D is measured. Each point has integer coordinates. This is shown in the Voronoi diagram below.



(a) Write down the equation of the perpendicular bisector of line segment [2]

(i) AB

(ii) BC

(b) Find the equation of the perpendicular bisector of the following line segments. Write your answers in the form $y = mx + c$ where $m, c \in \mathbb{Q}$. [6]

(i) AD

(ii) CD

(c) Determine the coordinates of point M. [3]

(d) Show that the coordinates of point N are equal to $\left(12, \frac{55}{7}\right)$. [1]

(e) To two decimal places calculate the area of the cell containing point [8]

(i) A

(ii) B

(iii) C

(f) Hence show the area of the cell containing point D is $141.39\,\text{m}^2$ to two decimal places. [1]

The table below shows the depth of the water at each point.

Point	*A*	*B*	*C*	*D*
Depth	0.8 m	1.1 m	0.5 m	1.6 m

(g) Estimate [4]

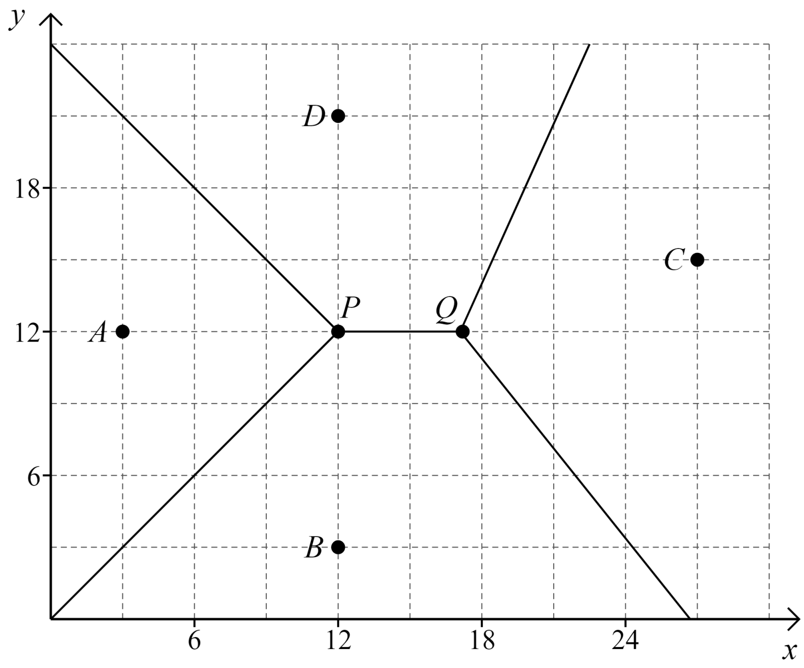
(i) the volume of water in the pond

(ii) the average depth of the pond

4. [Maximum points: 15]

Four trees are positioned in a $30\,\text{m} \times 24\,\text{m}$ rectangular garden at points $A(3,12)$, $B(12,3)$, $C(27,15)$ and $D(12,21)$ where units of coordinates are metres.

These points are used to create a Voronoi diagram. This is shown below where points P and Q represent vertices of the Voronoi cells.



(a) Write down the equation of the perpendicular bisector of line segment BD. [1]

(b) Find the equation of the perpendicular bisector of line segment BC. Write your answer in the form $Ax + By + D = 0$ where $A, B, D \in \mathbb{Z}$. [4]

(c) Write down the coordinates of point P. [1]

(d) Find the coordinates of point Q. [2]

A new tree is to be planted in the garden so that it is the furthest possible distance from any existing trees.

(e) Determine whether it should be planted at point P or Q. [4]

(f) Show that there is a more suitable location for the tree. [3]

1. (a) Each cell contains all points whose closest school is the school in the same cell. A1

(b)

(i) 3 A1

(ii) $-\frac{1}{3}$ A1

(c) $3 \times \left(-\frac{1}{3}\right) = -1$ A1

(d) The horizontal line has the equation $y = 2$. A1

The gradient of the other line is $\frac{4}{6} = \frac{2}{3}$. M1

The equation is therefore

$$y - 2 = \frac{2}{3}(x - 7)$$ A1

Or

$$y = \frac{2x}{3} - \frac{8}{3}$$

(e) Use the area of a triangle M1

$$6 \times 9 - \frac{3 \times 9}{2} - \frac{3 \times 1}{2} - \frac{4 \times 6}{2} = 27\,\text{km}^2$$ A1A1

2. Determine the perpendicular bisector of line BC using any method e.g.

The midpoint of line BC is $\left(\frac{15+45}{2},\frac{35+15}{2}\right)=(30,25)$. A1

The gradient of line BC is $\frac{35-15}{15-45}=-\frac{2}{3}$. A1

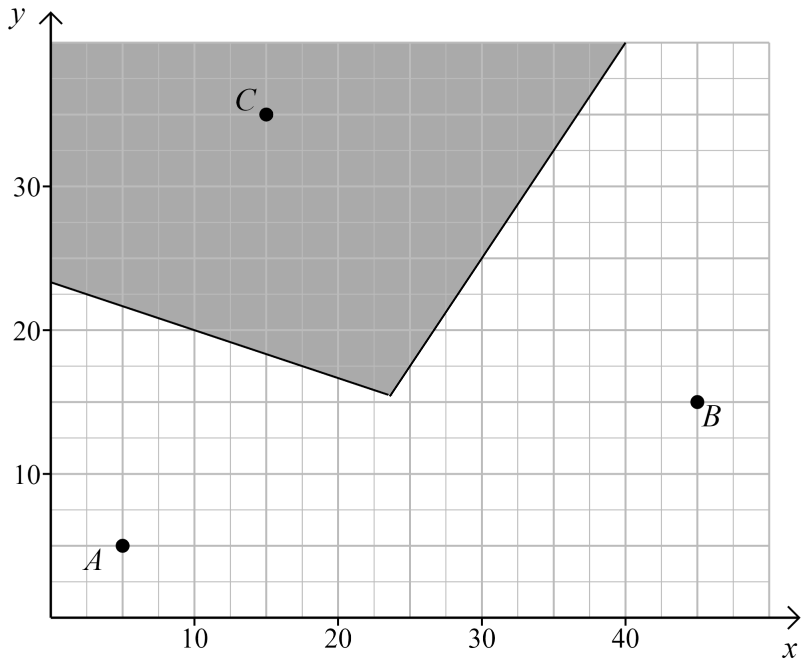
So the equation is

$$y-25=\frac{3}{2}(x-30)$$ M1

Giving

$$y=\frac{3}{2}x-20$$ A1

Add this line to the graph and shade the region shown below. A1A1



3. (a)

(i) $y = \frac{x}{2} + 4$ A1

(ii) $x = 12$ A1

(b)

(i) The gradient is -2 and a point on the line is $(8,13)$. A1

So the equation is of the form

$$y - 13 = -2(x - 8)$$ M1

Giving

$$y = -2x + 29$$ A1

(ii) The gradient is $\frac{2}{7}$ and a point on the line is $\left(\frac{18+14}{2}, \frac{2+16}{2}\right) = (16,9)$. A1

So the equation is of the form

$$y - 9 = \frac{2}{7}(x - 16)$$ M1

Giving

$$y = \frac{2x}{7} + \frac{31}{7}$$ A1

(c) We have

$$-2x + 29 = \frac{x}{2} + 4$$ M1

Giving

$$-\frac{5x}{2} = -25$$ A1

So $x = 10$ and $y = 9$. A1

(d) For point N we have $x = 12$ so $y = \frac{2(12)}{7} + \frac{31}{7} = \frac{55}{7}$. A1

(e)

(i) The vertex of the cell on the top edge has an x-coordinate of 4.5. A1

So the area is

$$\frac{5 \times 10}{2} + \frac{(10 + 4.5) \times 11}{2} = 104.75\,\text{m}^2$$ M1A1

(ii) $\frac{(4+9) \times 10}{2} + \frac{(9 + 55/7) \times 2}{2} = 81.86\,\text{m}^2$ M1A1

(iii) The vertex on the cell on the far right has a y-coordinate of $\frac{71}{7}$. A1

So the area is

$$\frac{(71/7 + 55/7) \times 8}{2} = 72\,\text{m}^2$$ M1A1

(f) $20 \times 20 - 104.75 - 81.86 - 72 = 141.39\,\text{m}^2$ A1

(g)

(i) $104.75 \times 0.8 + 81.86 \times 1.1 + 72 \times 0.5 + 141.39 \times 1.6 = 436\,\text{m}^3$ M1A1

(ii) $\frac{436}{400} = 1.09\,\text{m}$ M1A1

4. (a) $y = 12$ A1

(b) The gradient is

$$-\frac{27-12}{15-3} = -\frac{5}{4}$$ A1

The midpoint is

$$\left(\frac{12+27}{2}, \frac{3+15}{2}\right) = \left(\frac{39}{2}, 9\right)$$ A1

The equation is then

$$y - 9 = -\frac{5}{4}\left(x - \frac{39}{2}\right)$$ M1

This gives

$$10x + 8y - 267 = 0$$ A1

(c) $(12,12)$ A1

(d) We have $y = 12$ so

$$10x + 96 - 267 = 0$$ M1

Giving

$$x = 17.1$$ A1

In summary the coordinates are $(17.1,12)$.

(e) Use the distance formula M1

$$BQ = \sqrt{(17.1-12)^2 + (12-3)^2} = 10.3$$ A1

$$PB = 9$$ A1

Since $10.3 > 9$ it should be planted at point Q. A1

(f) For example the point $(26.7,0)$ is a distance of A1

$$\sqrt{(26.7-12)^2 + (0-3)^2} = 14.02$$ A1

from points B and C and a distance of

$$\sqrt{(26.7-17.1)^2 + (0-12)^2} = 15.4$$ A1

from point Q.