

# Trigonometric modelling

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If you understand graphs of trig functions and their transformations, this should all be very easy.

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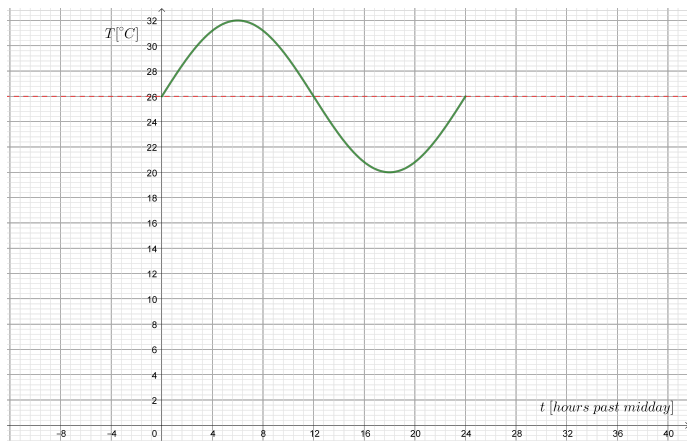
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We want to sketch the graph for  $0 \leq t \leq 24$ , so for one full period.

# Exercise 17D question 1 (a)

We get the following graph:



## Exercise 17D question 1 (b)

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i. at midnight:  $T(12) = 6 \sin\left(\frac{\pi}{12} \times 12\right) + 26 = 6 \sin \pi + 26 = 26^\circ \text{C}.$



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- ii. at 2 pm.:  $T(2) = 6 \sin\left(\frac{\pi}{12} \times 2\right) + 26 = 6 \sin \frac{\pi}{6} + 26 = 29^\circ\text{C}$ .

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this gives  $t = 6$ .

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this gives  $t = 6$ . So the maximum temperature of  $32^\circ\text{C}$  occurs at 6 pm.

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$$h(t) = 6 \cos\left(\frac{\pi}{6} \times t\right)$$

## Exercise 17D question 9 (b)

We want the function  $d(t)$ , where  $h$  is the horizontal displacement in *cm* of the tip of the minute hand relative to the centre of the clock and  $t$  is time in hours after midnight.

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$$d(t) = 12 \sin(2\pi t)$$

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# Trigonometric model

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## Sunrise and sunset by month (Warsaw)

Month	Sunrise	Sunset	Hours of daylight
Januar	07:36 am	03:54 pm	8:18 hours
Februar	06:50 am	04:49 pm	9:59 hours
März	05:47 am	05:42 pm	11:55 hours
April	05:35 am	07:36 pm	14:00 hours
Mai	04:38 am	08:26 pm	15:49 hours
Juni	04:11 am	09:01 pm	16:50 hours
Juli	04:31 am	08:52 pm	16:21 hours
August	05:18 am	08:02 pm	14:44 hours
September	06:09 am	06:52 pm	12:43 hours
Oktober	07:00 am	05:43 pm	10:43 hours
November	06:55 am	03:45 pm	8:50 hours
Dezember	07:37 am	03:25 pm	7:49 hours

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$$\frac{\pi}{6}(6 - c) = \frac{\pi}{2}$$

which gives  $c = 3$ .

# Trigonometric model

We have the following equation:

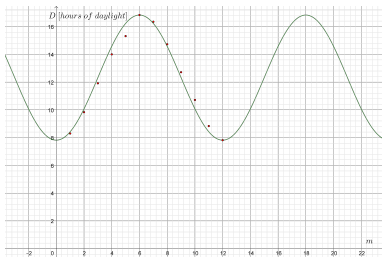
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Which gives the following graph (together with all the points):

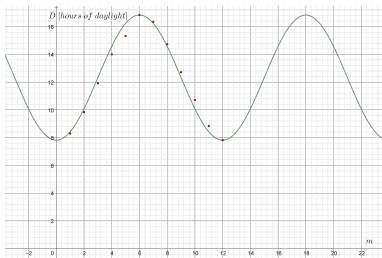


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Looks like we could've done a slightly better job with horizontal shift by choosing a different point.



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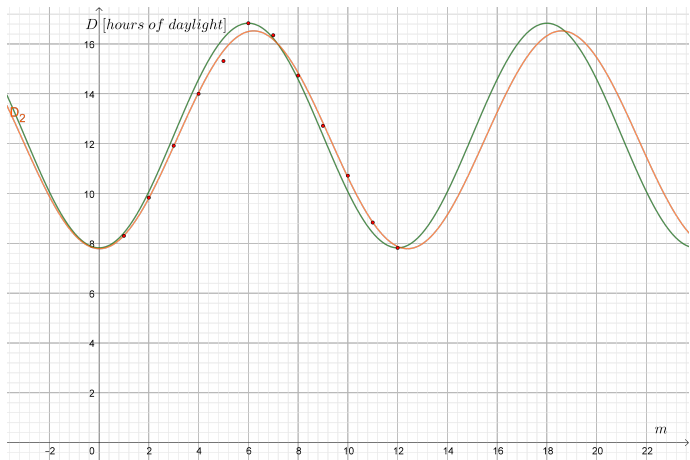
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The function we got is:

$$D(m) = 4.3711 \sin(0.5075m - 1.5814) + 12.1548$$

Graphs of both functions:



Admittedly the GDC has done a slightly better job.

In case of any questions you can email me at [T.J.Lechowski@gmail.com](mailto:T.J.Lechowski@gmail.com).