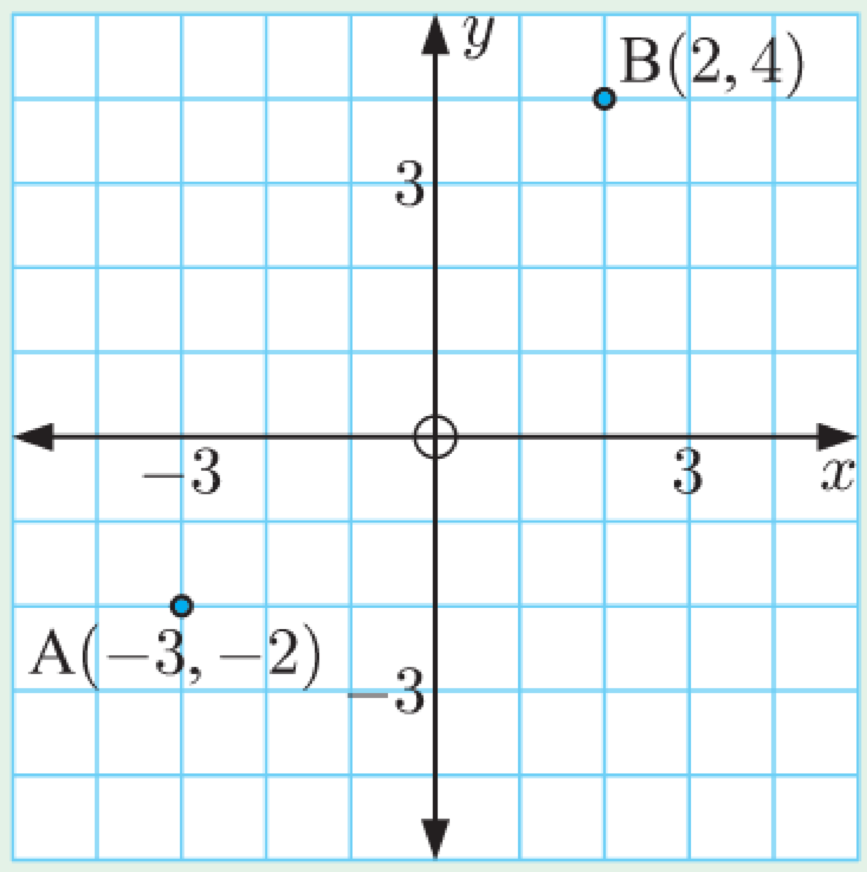
Chapter 1

Straight lines

Contents:

OPENING PROBLEM

A town has two hospitals located at $A(-3, -2)$ and $B(2, 4)$. The grid units are kilometres. In an emergency, an ambulance crew will be sent from the nearest hospital.



Things to think about:

a What is the midpoint between the hospitals?

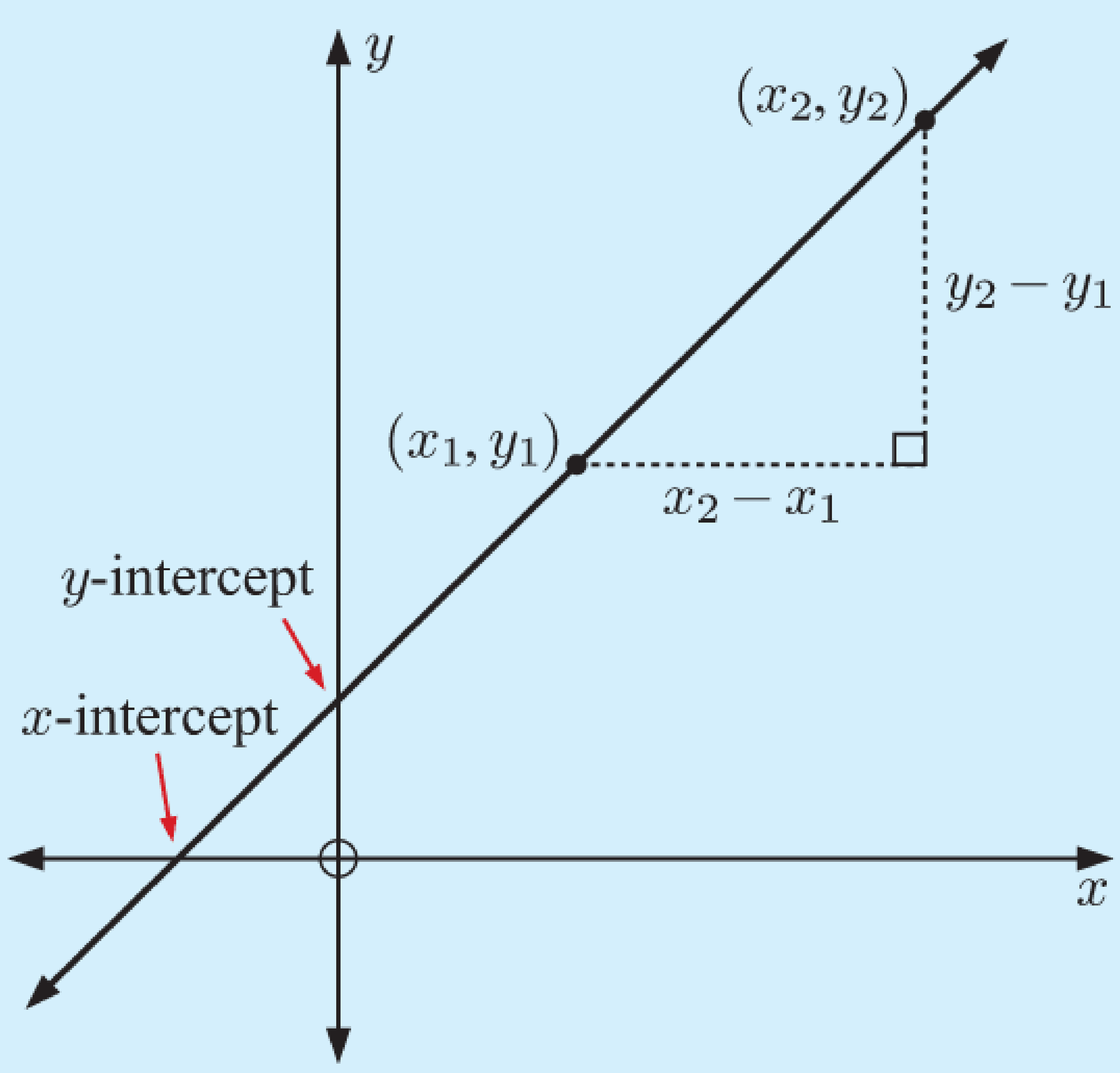
b How can we tell which hospital an ambulance crew should be sent from?

In this Chapter we study the equations and graphs of straight lines in the Cartesian plane. We will consider **perpendicular bisectors** which define the set of points equidistant from two locations, and consider simultaneous linear equations corresponding to the intersection of lines.

A LINES IN THE CARTESIAN PLANE

In previous years you should have seen that:

- The ***x*-intercept** of a line is the value of x where the line cuts the x-axis.
- The ***y*-intercept** of a line is the value of y where the line cuts the y-axis.
- The **gradient** of a line is a measure of its steepness.
  The gradient of the line passing through (x_1, y_1) and (x_2, y_2) is $\dfrac{y\text{-step}}{x\text{-step}} = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Two lines are **parallel** if their gradients are equal.
- Two lines are **perpendicular** if their gradients are negative reciprocals of one another.



THE EQUATION OF A LINE

The **equation of a line** is an equation which connects the x and y values for every point on the line.

Using the gradient formula, the position of a general point (x, y) on a line with gradient m passing through (x_1, y_1), is given by $\dfrac{y - y_1}{x - x_1} = m$.

Rearranging, we find the **equation of the line** is $y - y_1 = m(x - x_1)$.

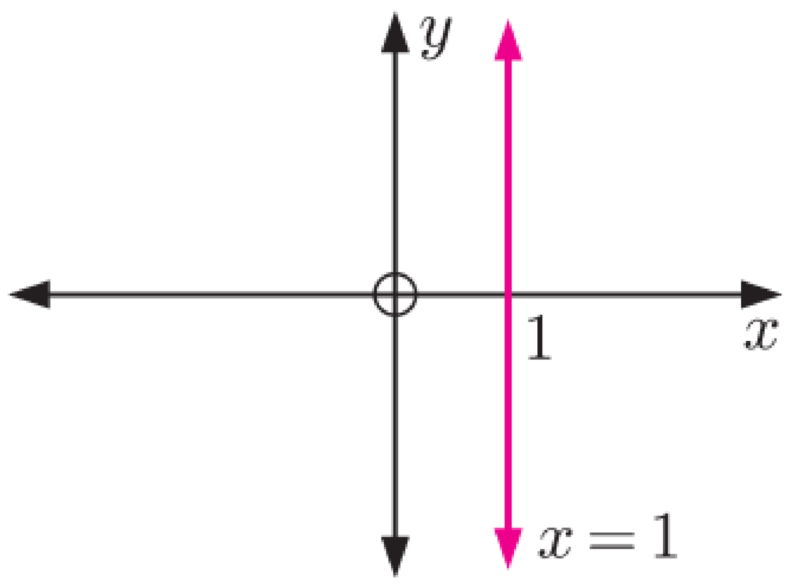
We call this **point-gradient** form. It allows us to quickly write down the equation of a line given its gradient and any point on it.

We can then rearrange the equation into other forms:

- In **gradient-intercept form**, the equation of the line with gradient m and y-intercept c is $y = mx + c$.

- In **general form**, the equation of a line is $ax + by = d$ where a, b, d are constants.

  The general form allows us to write the equations of vertical lines, for which the gradient is undefined.
  For the line $x = 1$ we let $a = 1$, $b = 0$, and $d = 1$.



In examinations you may also be asked to write the equation of a line in the form $ax + by + d = 0$.

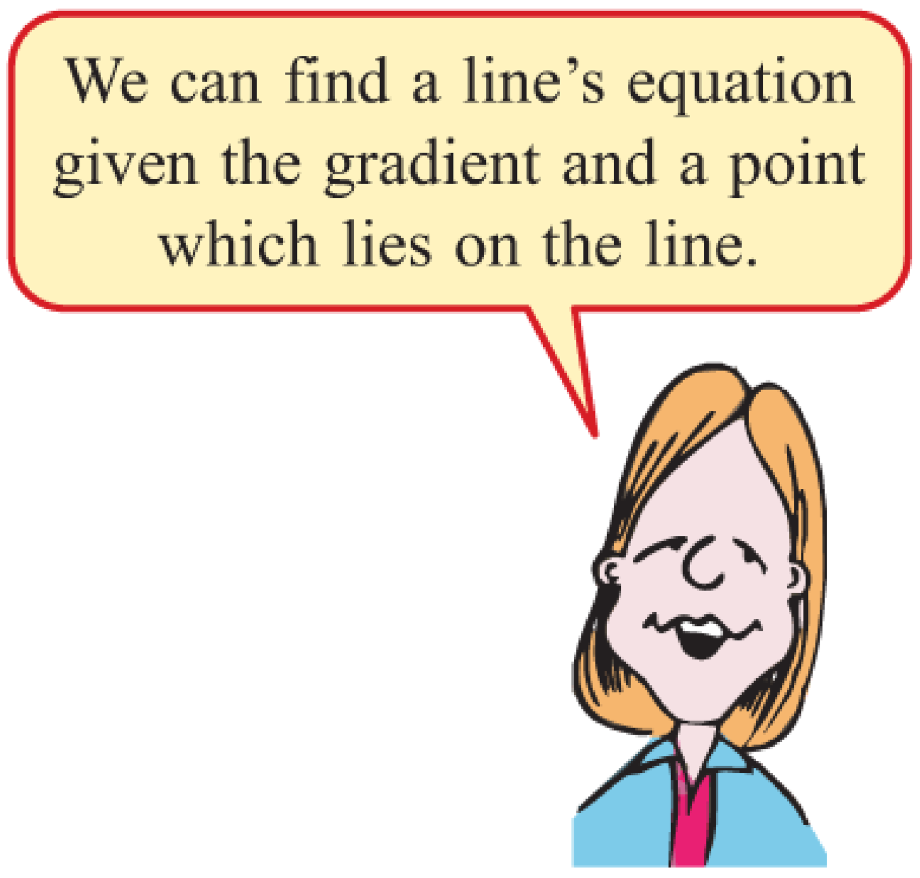
Example 1 **Self Tutor**

Find, in gradient-intercept form, the equation of the line with gradient -3 that passes through $(4, -5)$.

The equation of the line is $y - (-5) = -3(x - 4)$

$$\therefore \quad y + 5 = -3x + 12$$

$$\therefore \quad y = -3x + 7$$



EXERCISE 1A

1 State the gradient and y-intercept of the line with equation:

a $y = 3x + 7$ **b** $y = -2x - 5$ **c** $y = \frac{2}{3}x - \frac{1}{3}$

d $y = \dfrac{7x + 2}{9}$ **e** $y = \dfrac{2x - 3}{6}$ **f** $y = \dfrac{3 - 5x}{8}$

2 Find, in gradient-intercept form, the equation of the line which has:

a gradient 3 and passes through $(4, 1)$ **b** gradient -2 and passes through $(-3, 5)$

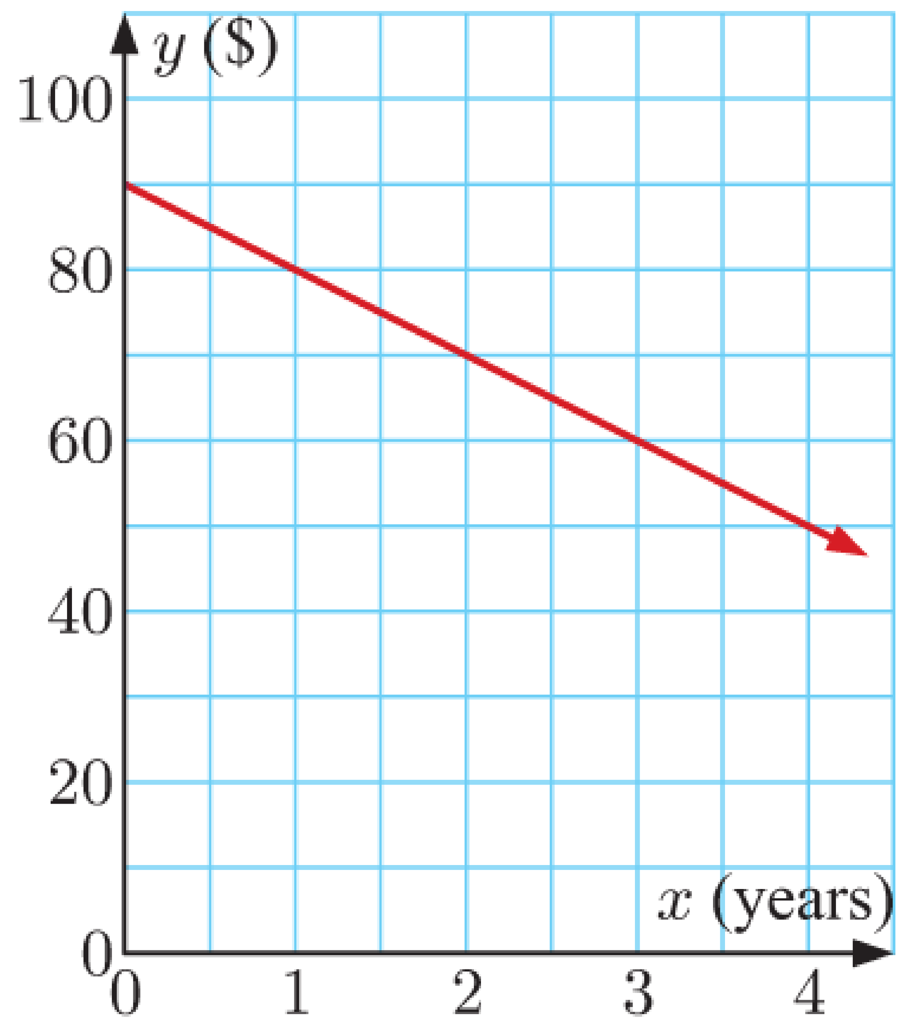
c gradient $\frac{1}{4}$ and passes through $(4, -3)$ **d** gradient $-\frac{3}{4}$ and y-intercept 4.

3 An unused bank account is charged a yearly fee. The graph alongside shows the balance of the account after x years.

a Find the gradient and y-intercept of the line, and interpret your answers.

b Find the equation of the line.

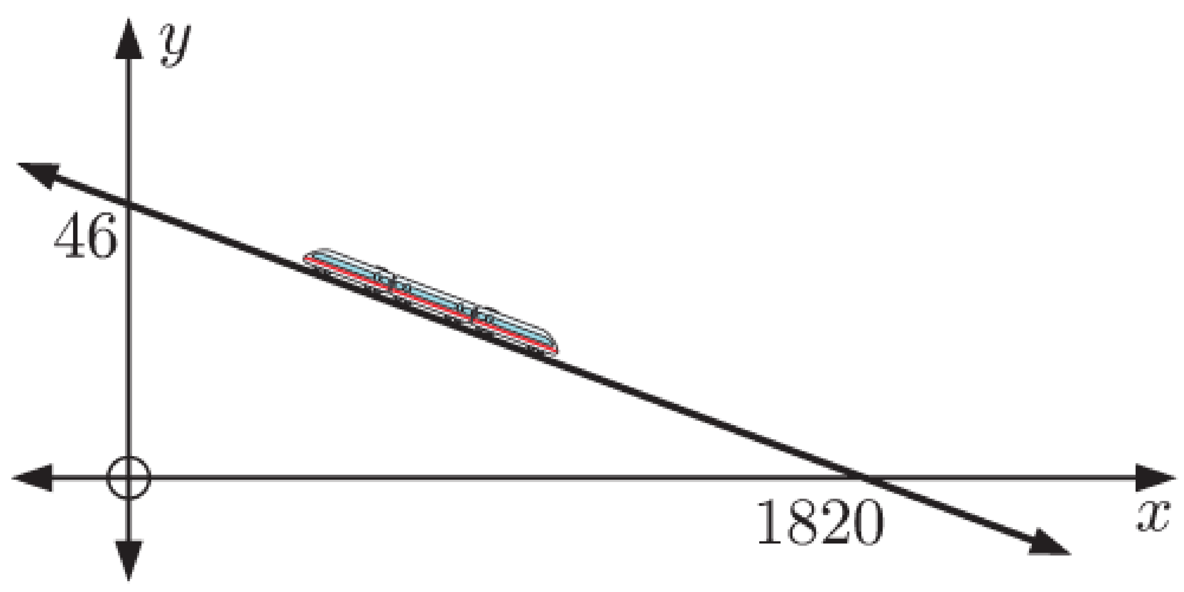
c How long will it take for the account to run out of money?



4 The graph alongside shows the descent of a train down a hill. The units are metres.

a Calculate the gradient of the train's descent.

b Find the equation of the train line.



5 The height of a helicopter above sea level t minutes after taking off is $H = 150 + 120t$ metres.

a What height above sea level did the helicopter take off from?

b Interpret the value 120 in the equation.

c Find the height of the helicopter above sea level after 2 minutes.

d How long will it take for the helicopter to be 650 m above sea level?

Example 2 — Self Tutor

Write the equation:

a $y = -\frac{2}{3}x + 2$ in general form

b $3x - 4y = -2$ in gradient-intercept form.

a
$$y = -\tfrac{2}{3}x + 2$$
$$\therefore\ 3y = -2x + 6$$
$$\therefore\ 2x + 3y = 6$$

b
$$3x - 4y = -2$$
$$\therefore\ -4y = -3x - 2$$
$$\therefore\ y = \tfrac{3}{4}x + \tfrac{1}{2}$$

6 Write in general form:

a $y = -4x + 6$ **b** $y = 5x - 3$ **c** $y = -\frac{3}{4}x + \frac{5}{4}$ **d** $y = \frac{3}{5}x - \frac{1}{5}$

7 Write in gradient-intercept form:

a $5x + y = 2$ **b** $3x + 7y = -2$ **c** $2x - y = 6$ **d** $3x - 13y = -4$

8 Explain why the gradient of the line with general form $ax + by = d$ is $-\frac{a}{b}$.

Example 3 — Self Tutor

Find, in general form, the equation of the line with gradient $\frac{2}{3}$ that passes through $(-2, -1)$.

Since the line has gradient $\frac{2}{3}$, the general form of its equation is $2x - 3y = d$

Using the point $(-2, -1)$, the equation is $2x - 3y = 2(-2) - 3(-1)$

which is $2x - 3y = -1$

9 Find, in general form, the equation of the line which has:

a gradient -4 and passes through $(1, 2)$

b gradient $\frac{1}{2}$ and passes through $(3, -5)$

c gradient $-\frac{5}{3}$ and passes through $(-2, 6)$

d gradient $\frac{7}{6}$ and passes through $(-1, -4)$.

Example 4 — Self Tutor

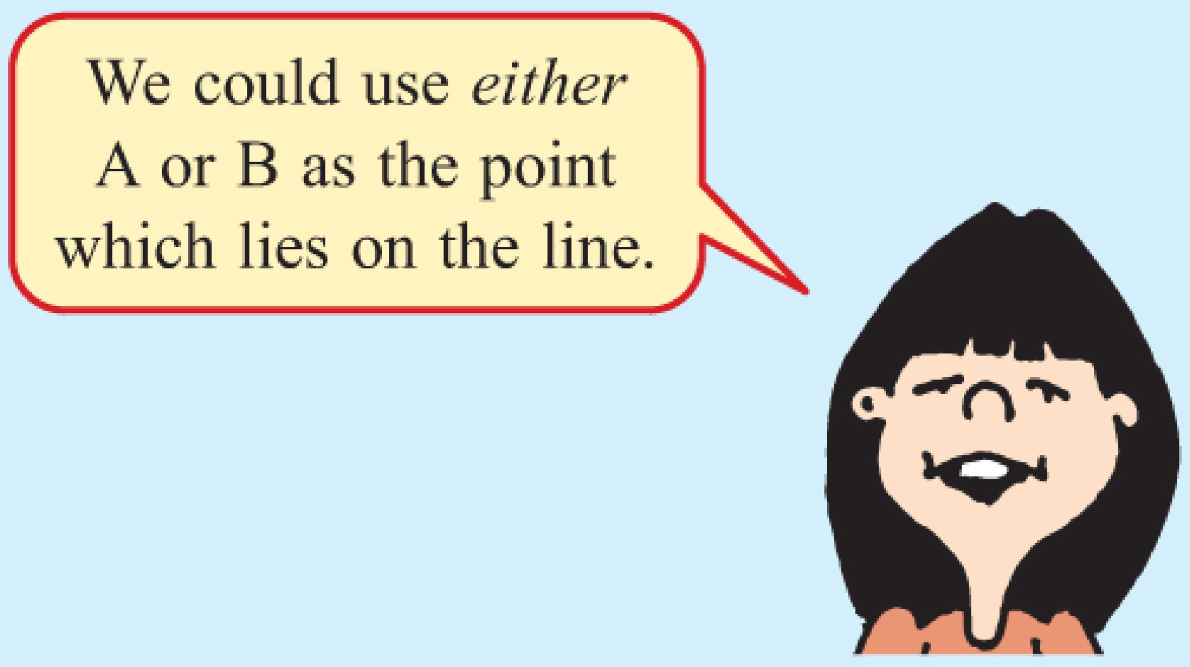
Find, in gradient-intercept form, the equation of the line which passes through A(3, 2) and B(5, −1).

The line has gradient $= \frac{-1-2}{5-3} = \frac{-3}{2} = -\frac{3}{2}$,

and passes through the point A(3, 2).

$\therefore$ the equation of the line is

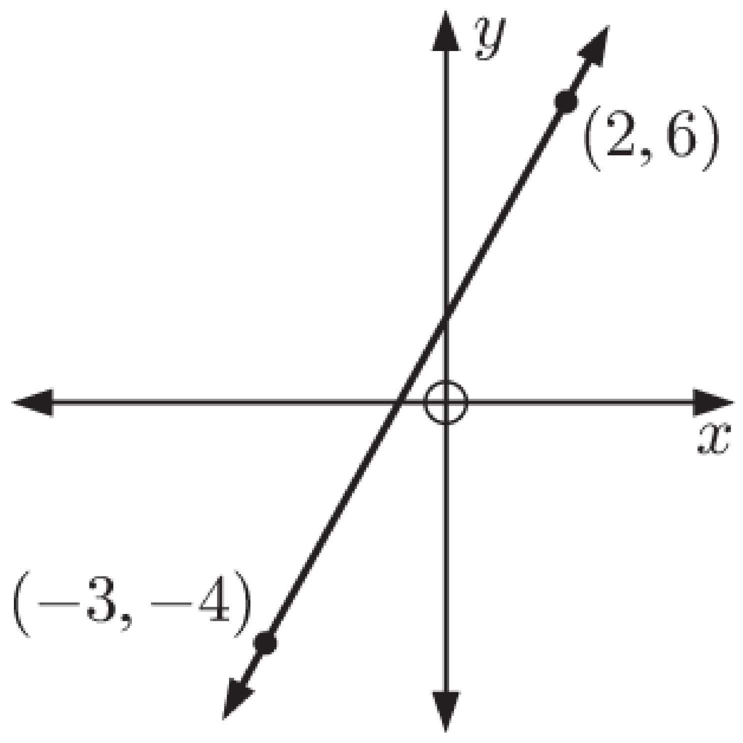
$$y - 2 = -\tfrac{3}{2}(x-3)$$
$$\therefore\ y - 2 = -\tfrac{3}{2}x + \tfrac{9}{2}$$
$$\therefore\ y = -\tfrac{3}{2}x + \tfrac{13}{2}$$

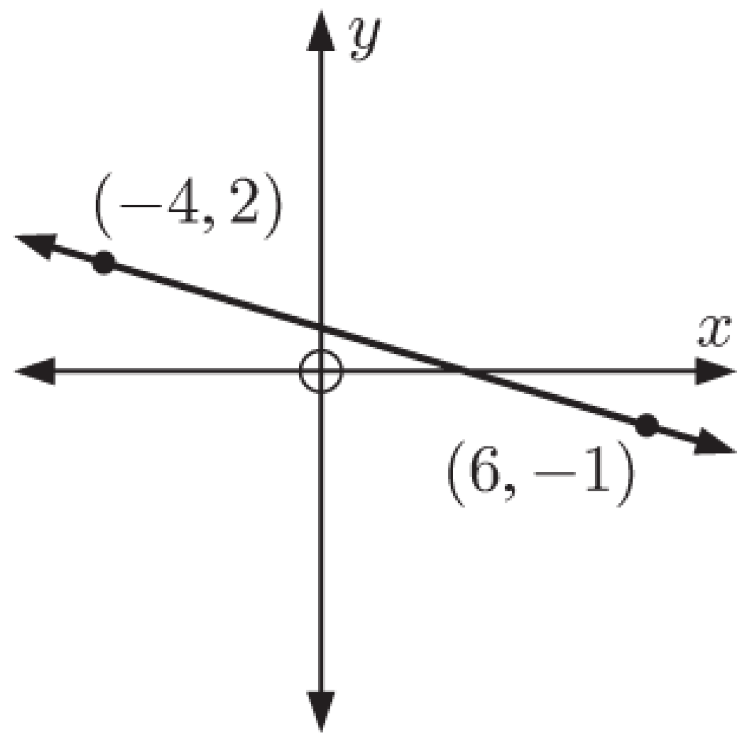


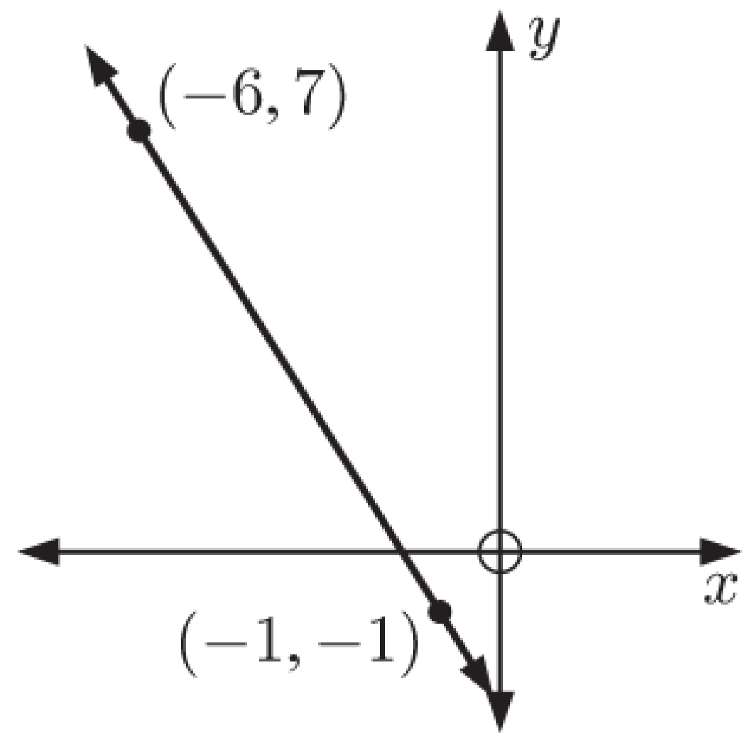
10 Find, in gradient-intercept form, the equation of the line which passes through:

a A(−2, 1) and B(3, 11)
b A(7, 2) and B(4, 5)
c M(−2, −5) and N(3, 2)
d R(5, −1) and S(−7, 9).

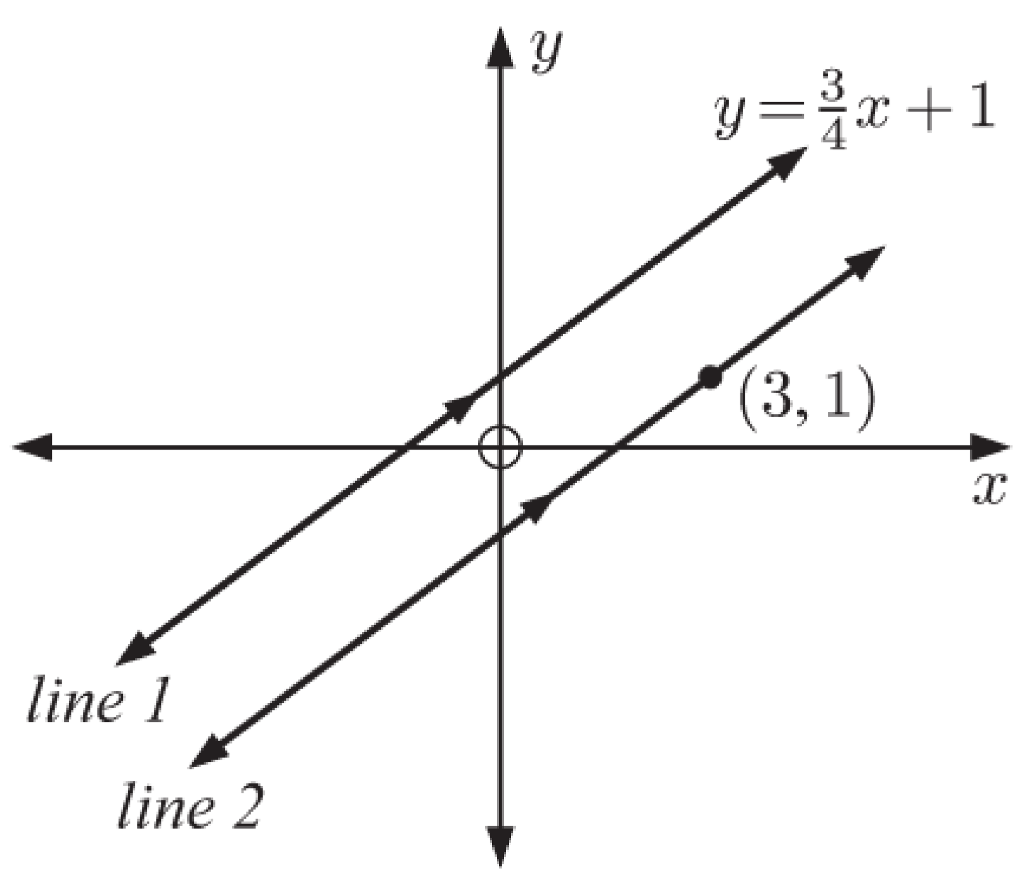
11 Find, in general form, the equation of each line:

a


b


c


12 **a** Find, in gradient-intercept form, the equation of *line 2*.
b Hence find the y-intercept of *line 2*.



13 Find the equation of the line which is:

a parallel to $y = 3x - 2$ and passes through (1, 4)
b parallel to $2x - y = -3$ and passes through (3, −1)
c perpendicular to $y = -2x + 1$ and passes through (−1, 5)
d perpendicular to $x + 2y = 6$ and passes through (−2, −1).

14 *Line 1* passes through A(−2, −1) and B(4, 3). *Line 2* is perpendicular to *line 1* and passes through A. Find the equation of each line.

Example 5 **Self Tutor**

a Find m given that $(-2, 3)$ lies on the line with equation $y = mx + 7$.

b Find k given that $(3, k)$ lies on the line with equation $x + 4y = -9$.

a Substituting $x = -2$ and $y = 3$ into the equation gives

$$3 = m(-2) + 7$$
$$\therefore\ 2m = 4$$
$$\therefore\ m = 2$$

b Substituting $x = 3$ and $y = k$ into the equation gives

$$3 + 4k = -9$$
$$\therefore\ 4k = -12$$
$$\therefore\ k = -3$$

15 Determine whether:

a $(3, 11)$ lies on the line with equation $y = 4x - 1$

b $(-6, -2)$ lies on the line with equation $y = \frac{2}{3}x - 6$

c $(-4, -8)$ lies on the line with equation $7x - 3y = -4$

d $(-\frac{1}{2}, 2)$ lies on the line with equation $6x + 10y = 17$.

16 **a** Find c given that $(2, 15)$ lies on the line with equation $y = 4x + c$.

b Find m given that $(\frac{1}{2}, 3)$ lies on the line with equation $y = mx - \frac{5}{2}$.

c Find t given that $(t, 4)$ lies on the line with equation $y = \frac{2}{3}x - \frac{4}{3}$.

17 Find k given that:

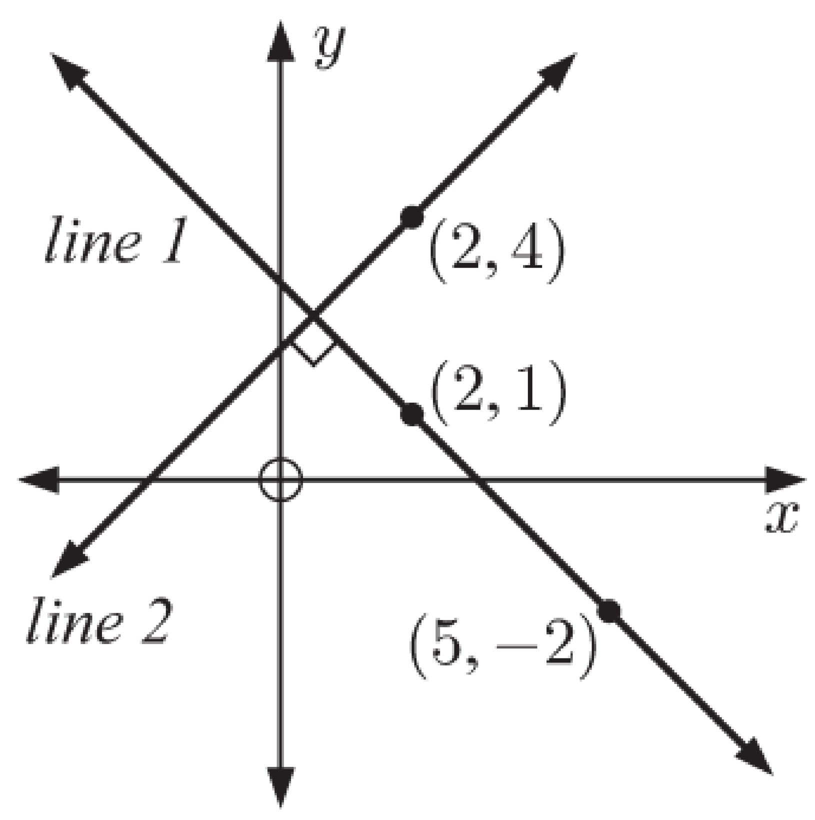
a $(6, -3)$ lies on the line with equation $2x + 5y = k$

b $(-8, -5)$ lies on the line with equation $7x - y = k$

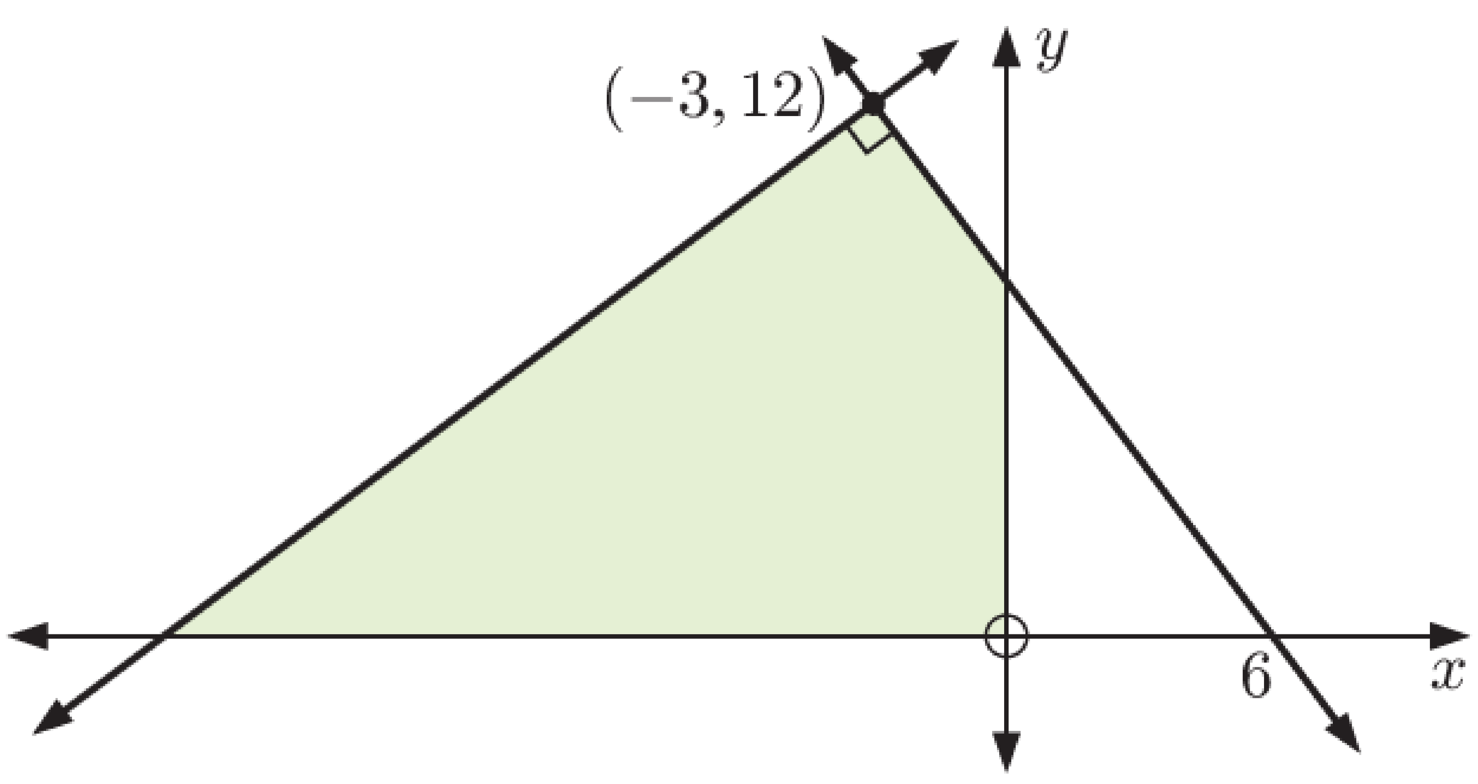
c $(k, 0)$ lies on the line with equation $3x - 4y = -36$.

18 **a** Find the equation of *line 2*. Write your answer in the form $ax + by + d = 0$.

b Find the x-intercept of *line 2*.



19 Find the shaded area:

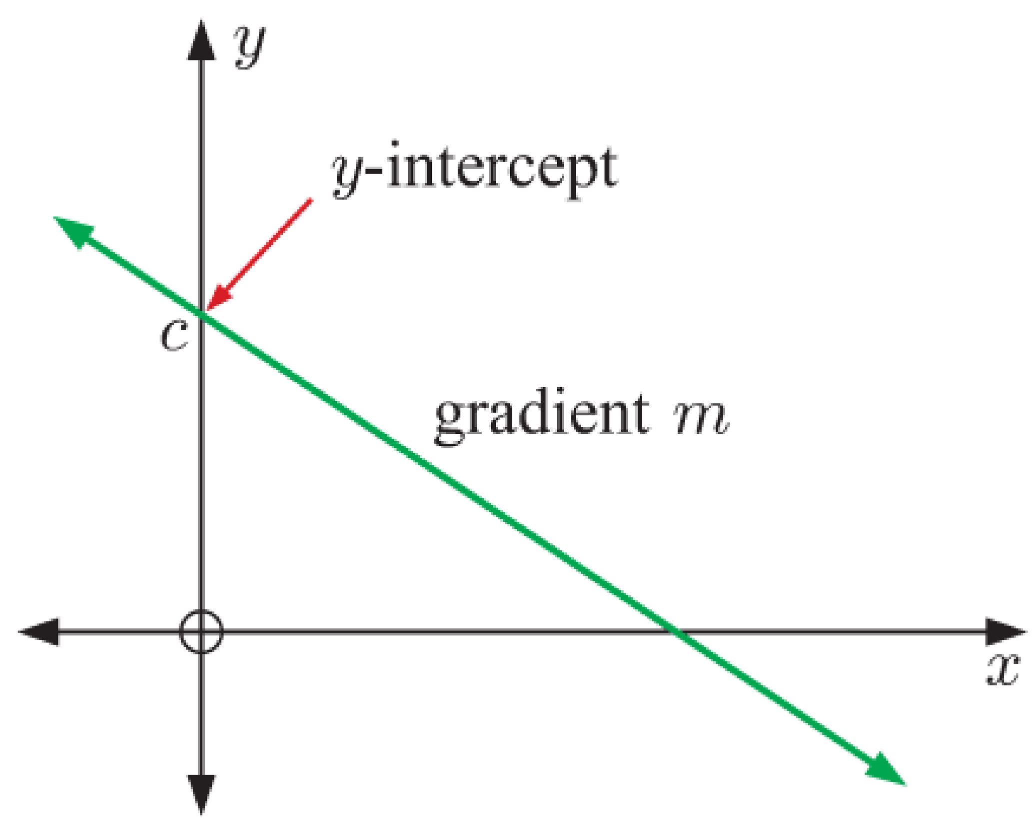


B GRAPHING A STRAIGHT LINE

LINES IN GRADIENT-INTERCEPT FORM

To draw the graph of $y = mx + c$ we:

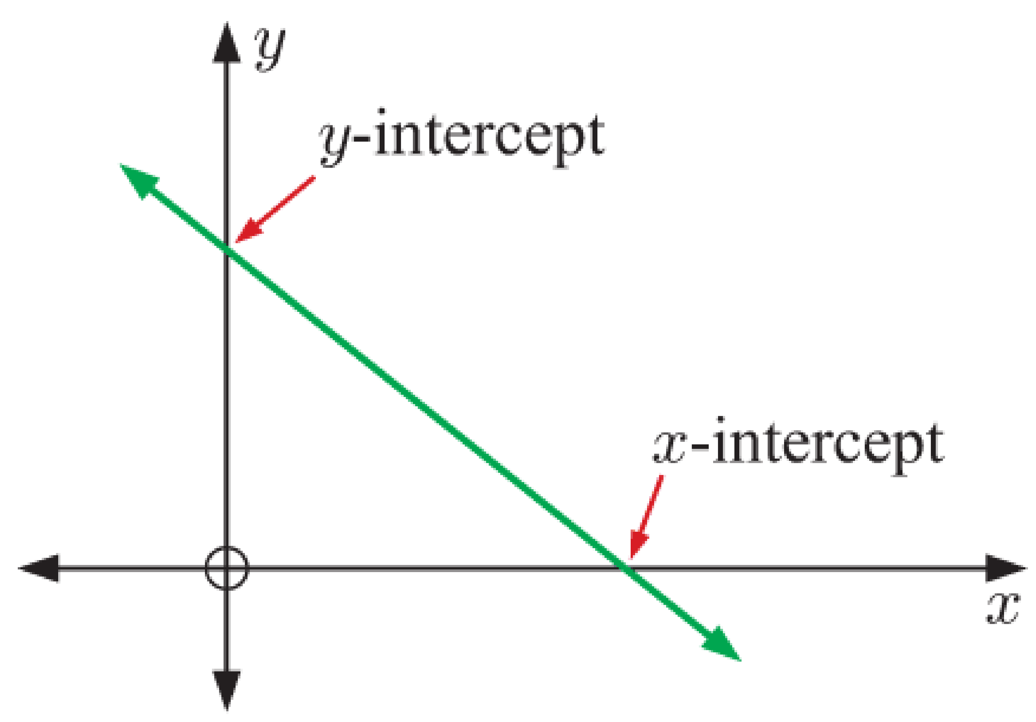
- Use the y-intercept c to plot the point $(0, c)$.
- Use x and y-steps from the gradient m to locate another point on the line.
- Join the two points and extend the line in either direction.



LINES IN GENERAL FORM

To draw the graph of $ax + by = d$ we:

- Find the y-intercept by letting $x = 0$.
- Find the x-intercept by letting $y = 0$.
- Join the points where the line cuts the axes and extend the line in either direction.



If $d = 0$ then the graph passes through the origin. In this case we plot $y = -\frac{a}{b}x$ using its gradient.

Example 6 — Self Tutor

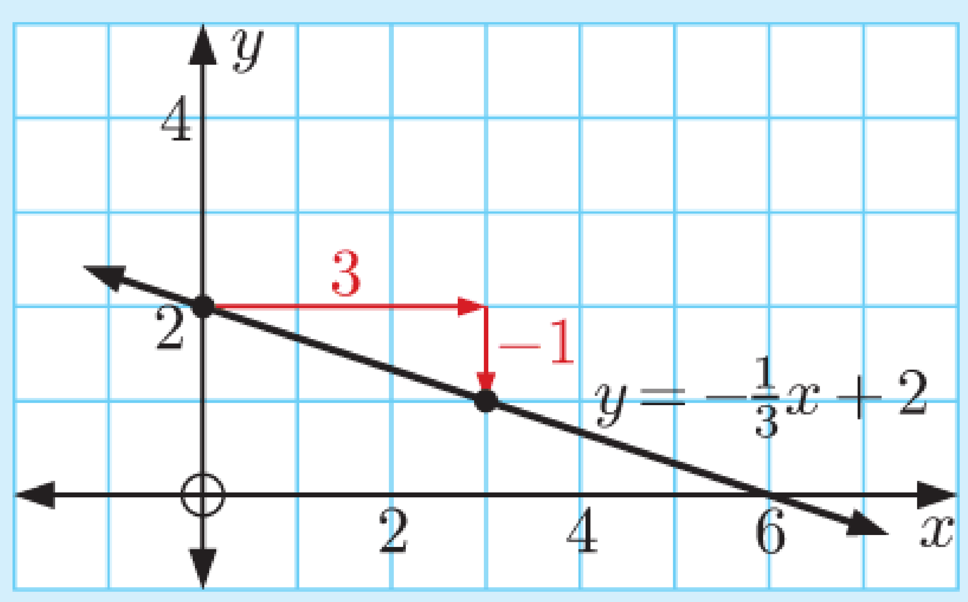
Draw the graph of:

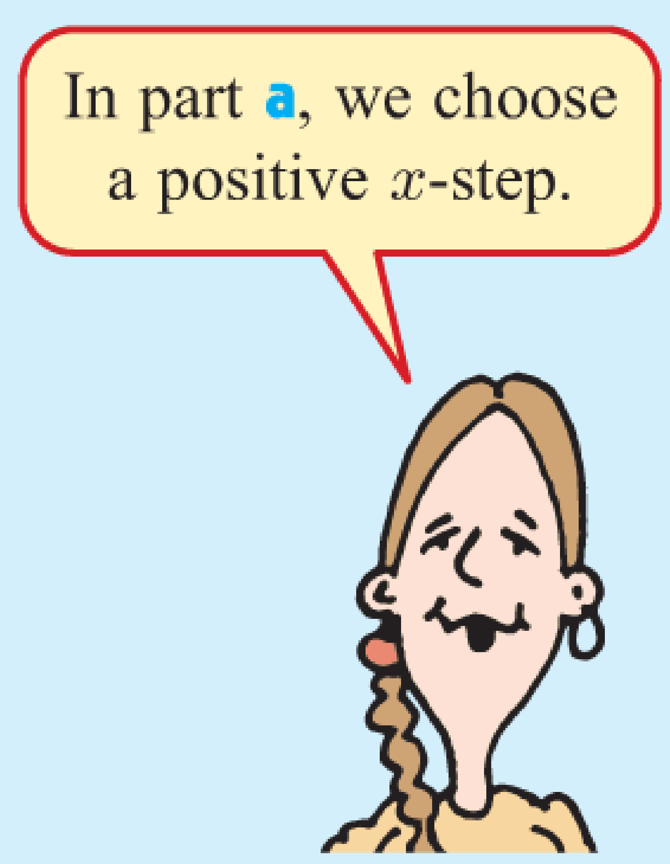
a $y = -\frac{1}{3}x + 2$ **b** $2x - 4y = 12$

a For $y = -\frac{1}{3}x + 2$:

- the y-intercept is $c = 2$
- the gradient is

$m = \frac{-1}{3}$ ← y-step, ← x-step





b When $x = 0$, $-4y = 12$

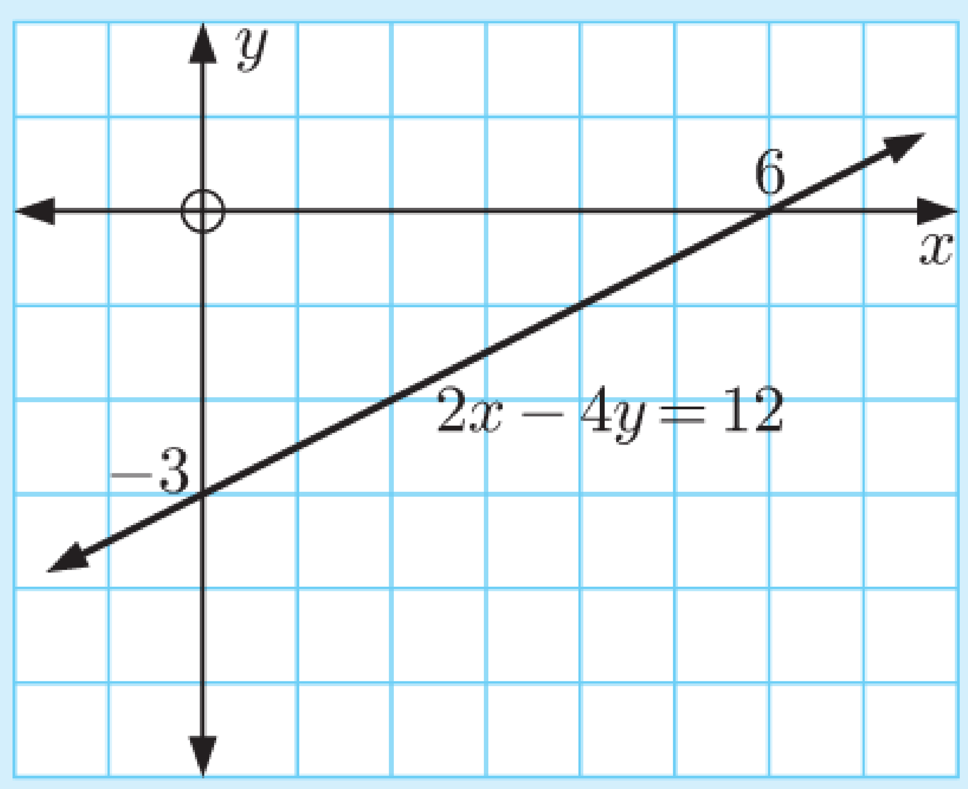
$\therefore\ y = -3$

So, the y-intercept is -3.

When $y = 0$, $2x = 12$

$\therefore\ x = 6$

So, the x-intercept is 6.



EXERCISE 1B

GRAPHICS CALCULATOR INSTRUCTIONS

GRAPHING PACKAGE

1 Draw the graph of:

a $y = \frac{1}{2}x + 1$ **b** $y = 3x - 2$ **c** $y = -\frac{3}{2}x + 4$

d $y = -4x$ **e** $y = \frac{6}{5}x - 3$ **f** $y = -\frac{5}{3}x - 1$

Check your answers using technology.

2 Draw the graph of:

a $3x + 2y = 12$ **b** $4x - y = 8$ **c** $3x - 4y = -24$

d $2x + 5y = 15$ **e** $6x + 4y = -36$ **f** $7x + 4y = 42$

3 Consider the line with equation $y = -\frac{3}{4}x + 2$.

a Find the gradient and y-intercept of the line.

b Determine whether the following points lie on the line:

i $(8, -4)$ **ii** $(1, 3)$ **iii** $(-2, \frac{7}{2})$

c Draw the graph of the line, showing the results you have found.

4 Consider the line with equation $2x - 3y = 18$.

a Find the axes intercepts of the line.

b Determine whether the following points lie on the line:

i $(3, -4)$ **ii** $(7, -2)$

c Find c such that $(-3, c)$ lies on the line.

d Draw the graph of the line, showing the results you have found.

5 At a sushi restaurant, *nigiri* costs \$4.50 per serve and *sashimi* costs \$9.00 per serve. Hiroko spent a total of \$45 buying x serves of *nigiri* and y serves of *sashimi*.

a Explain why $4.5x + 9y = 45$.

b If Hiroko bought 4 serves of *nigiri*, how much *sashimi* did she buy?

c If Hiroko bought 1 serve of *sashimi*, how much *nigiri* did she buy?

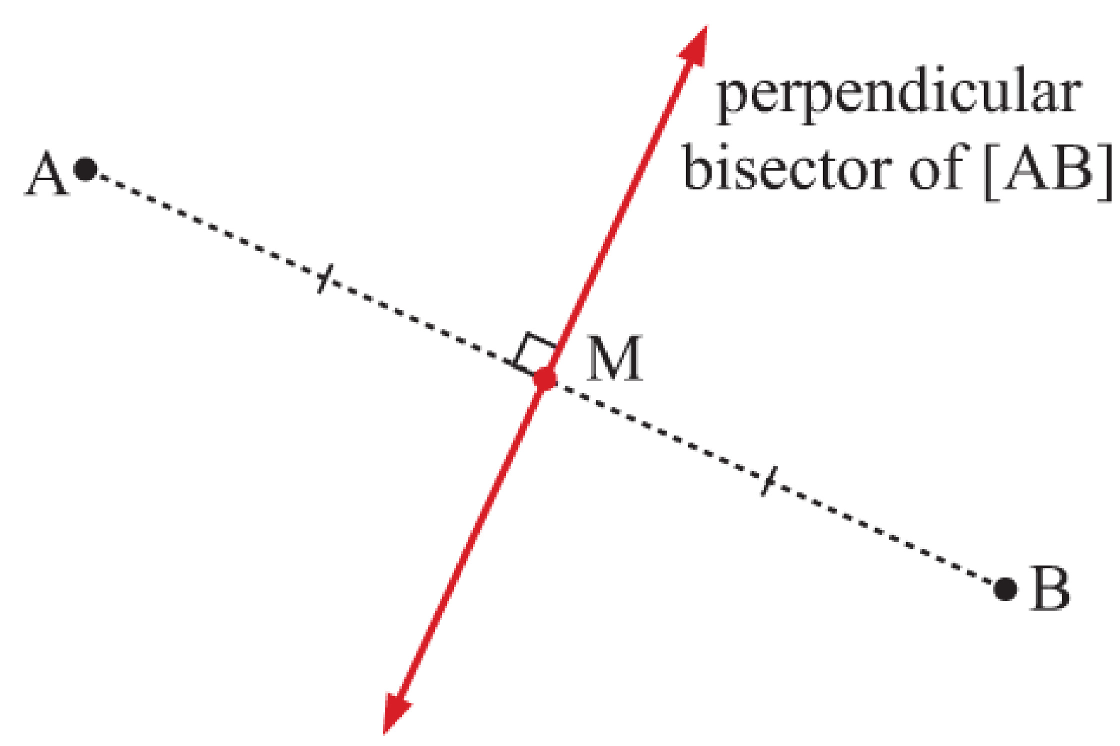
d Draw the graph of $4.5x + 9y = 45$. Mark two points on your graph to indicate your answers to **b** and **c**.

C PERPENDICULAR BISECTORS

The **perpendicular bisector** of a line segment [AB] is the line perpendicular to [AB] which passes through its midpoint.

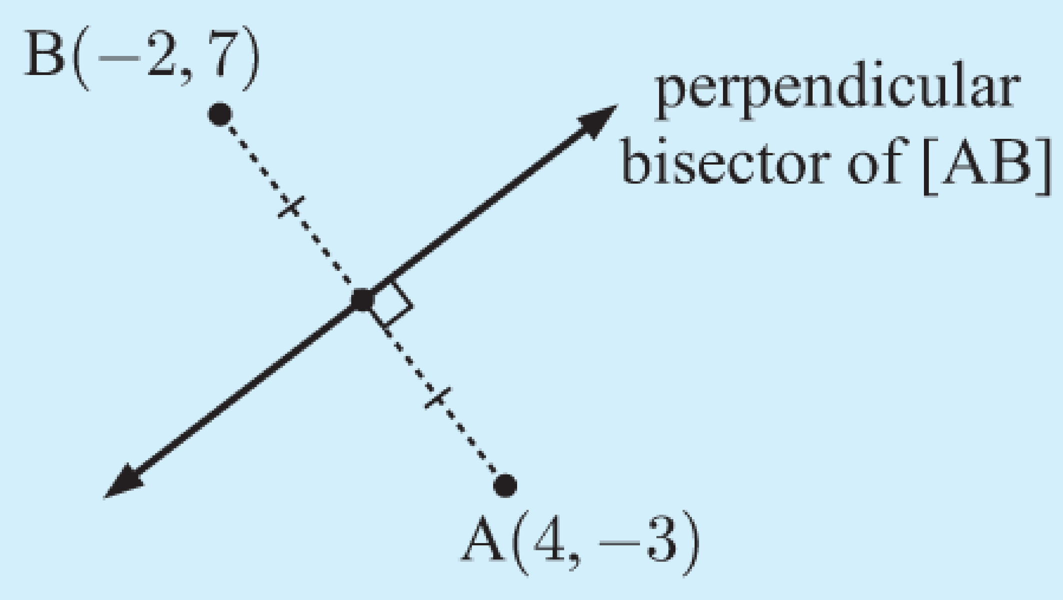
Notice that:

- Points on the perpendicular bisector are equidistant from A and B.
- The perpendicular bisector divides the number plane into two regions. On one side of the line are points that are closer to A than to B, and on the other side are points that are closer to B than to A.



Example 7 **Self Tutor**

Given A(4, −3) and B(−2, 7), find the equation of the perpendicular bisector of [AB].



The midpoint M of [AB] is $\left(\frac{4+-2}{2}, \frac{-3+7}{2}\right)$

or (1, 2).

The gradient of [AB] is $\frac{7--3}{-2-4} = \frac{10}{-6} = -\frac{5}{3}$

$\therefore$ the gradient of the perpendicular bisector is $\frac{3}{5}$.

$\therefore$ the equation of the perpendicular bisector is $3x - 5y = 3(1) - 5(2)$

which is $3x - 5y = -7$.

EXERCISE 1C

1 Consider the points A(3, 1) and B(5, 7).

- **a** Find the midpoint of [AB].
- **b** Find the gradient of [AB].
- **c** Hence state the gradient of the perpendicular bisector.
- **d** Find the equation of the perpendicular bisector.

2 Find the equation of the perpendicular bisector of:

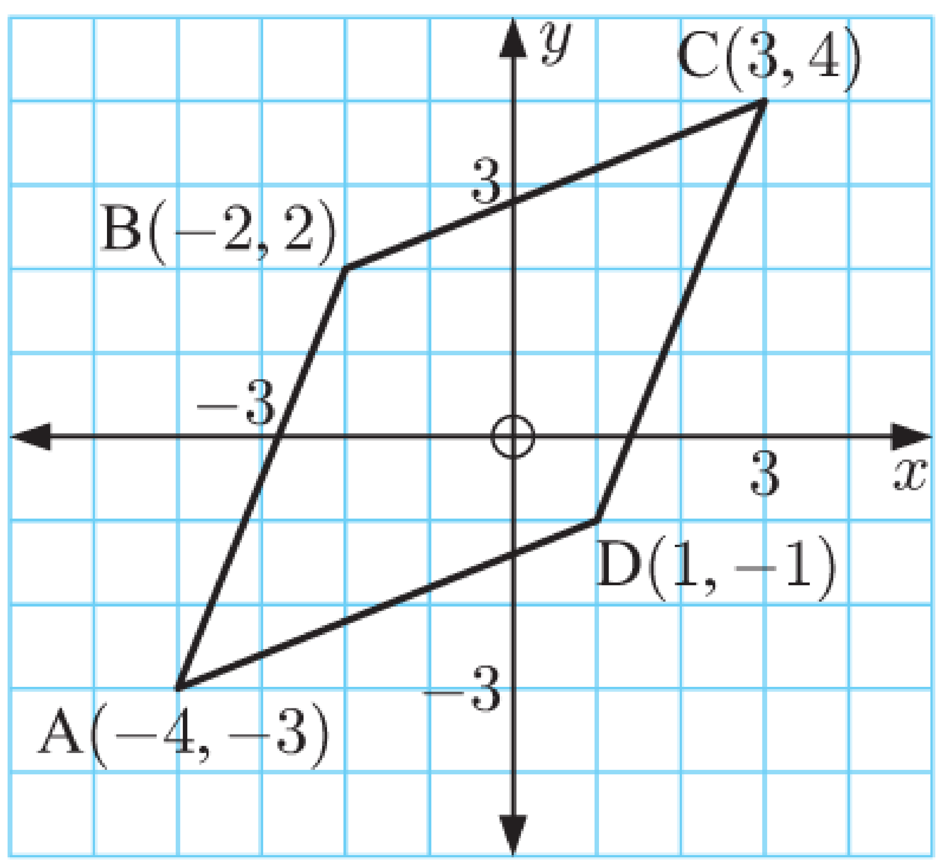
- **a** A(5, 2) and B(1, 4)
- **b** A(−1, 5) and B(5, 3)
- **c** M(6, −3) and N(2, 1)
- **d** M(7, 2) and N(−1, 6)
- **e** O(0, 0) and P(9, 0)
- **f** A(3, 6) and B(−1, 3).

3 Suppose P is (6, −1) and Q is (2, 5).

- **a** Find the equation of the perpendicular bisector of [PQ].
- **b** Show that R(1, 0) lies on the perpendicular bisector.
- **c** Show that R is equidistant from P and Q.

4 Consider the quadrilateral ABCD.

- **a** Use side lengths to show that ABCD is a rhombus.
- **b** Find the equation of the perpendicular bisector of [AC].
- **c** Show that B and D both lie on this perpendicular bisector.



5 A line segment has equation $3x - 2y + 1 = 0$. Its midpoint is (3, 5).

- **a** State the gradient of:
  - **i** the line segment
  - **ii** its perpendicular bisector.
- **b** State the equation of the perpendicular bisector. Write your answer in the form $ax + by + d = 0$.

6 Answer the **Opening Problem** on page **20**.

7 Consider the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

a Show that the equation of the perpendicular bisector of [AB] is

$$(x_2 - x_1)x + (y_2 - y_1)y = \frac{(x_2^2 + y_2^2) - (x_1^2 + y_1^2)}{2}.$$

b Suggest one advantage of writing this equation in general form.

8 Consider three points $A(1, 2)$, $B(4, 5)$, and $C(2, -1)$.

a Find the equation of the perpendicular bisector of: **i** [AB] **ii** [AC] **iii** [BC].

b Graph the three perpendicular bisectors on the same set of axes. Discuss your observations.

c Describe how to find the centre of the circle which passes through three non-collinear points.

9 Three post offices are located in a small city at $P(-8, -6)$, $Q(1, 5)$, and $R(4, -2)$.

a Find the equation of the perpendicular bisector of: **i** [PQ] **ii** [PR] **iii** [QR].

b Graph the three post offices and the three perpendicular bisectors on the same set of axes. Use these lines to locate the point that is equidistant from all three post offices. Shade regions of your graph in different colours according to their closest post office.

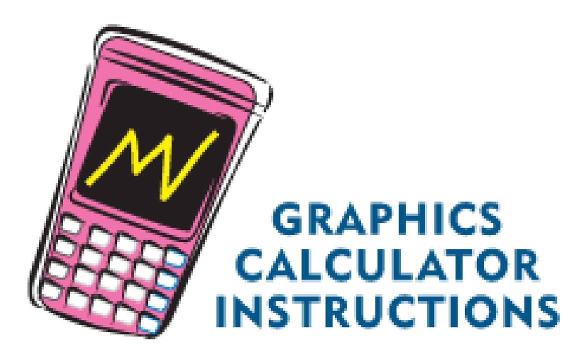
D SIMULTANEOUS EQUATIONS

In previous years you should have seen how the intersection of two straight lines corresponds to the simultaneous solution of their equations.

A system of two equations in two unknowns can be solved by:

- **graphing** the straight lines on the same set of axes
- algebra using **substitution** or **elimination**
- technology.

To use the **equation solver function** on your calculator, you will need to write each equation in the form $ax + by = d$.



Example 8 **Self Tutor**

Solve simultaneously: $\begin{cases} y = x - 3 \\ 2x + 3y = 16 \end{cases}$

Illustrate your answer.

$y = x - 3$ .... (1)

$2x + 3y = 16$ .... (2)

Substituting (1) into (2) gives $2x + 3(x - 3) = 16$

$\therefore 2x + 3x - 9 = 16$

$\therefore 5x = 25$

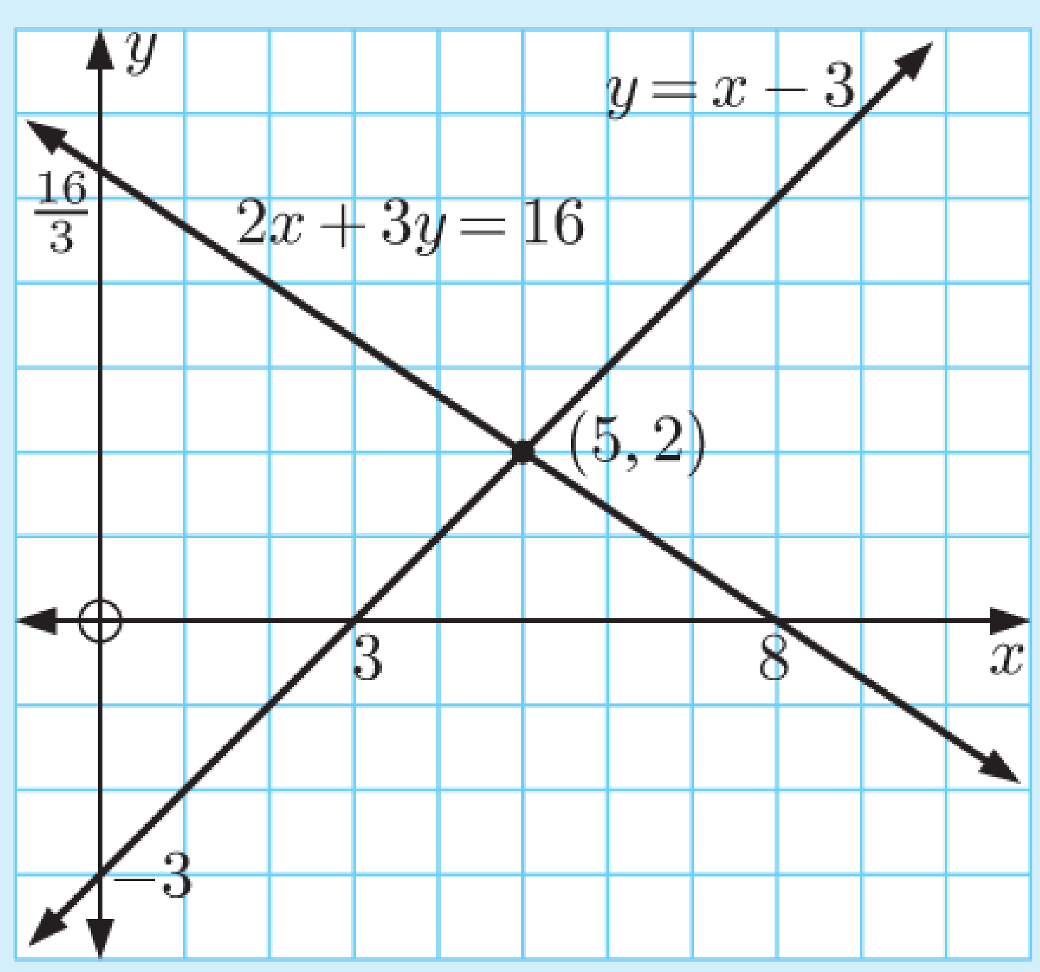
$\therefore x = 5$

Substituting $x = 5$ into (1) gives $y = 5 - 3$

$\therefore y = 2$

The solution is $x = 5$, $y = 2$.

Check: Substituting into (2), $2(5) + 3(2) = 10 + 6 = 16$ ✓



EXERCISE 1D

1 Solve the following simultaneous equations graphically:

a $\begin{cases} y = 3x + 2 \\ y = x - 2 \end{cases}$ **b** $\begin{cases} y = -4x + 1 \\ y = 3x - 6 \end{cases}$ **c** $\begin{cases} y = 2x - 5 \\ y = \frac{1}{2}x + 4 \end{cases}$

d $\begin{cases} y = x - 1 \\ 2x + 3y = 12 \end{cases}$ **e** $\begin{cases} x + 3y = 9 \\ x - 2y = 4 \end{cases}$ **f** $\begin{cases} 3x - 2y = 30 \\ 4x + y = -4 \end{cases}$

2 Solve by substitution:

a $\begin{cases} y = x + 2 \\ 2x + 3y = 21 \end{cases}$ **b** $\begin{cases} y = 2x - 3 \\ 4x - 3y = 7 \end{cases}$ **c** $\begin{cases} 5x + 3y = 19 \\ y = 6 - 2x \end{cases}$

d $\begin{cases} x = y - 3 \\ 5x - 2y = 9 \end{cases}$ **e** $\begin{cases} 3x + 4y = -13 \\ x = 8y - 2 \end{cases}$ **f** $\begin{cases} x = -5y - 2 \\ 7x + 4y = -10 \end{cases}$

g $\begin{cases} y = \frac{1}{2}x + 5 \\ 3x + 4y = 5 \end{cases}$ **h** $\begin{cases} x = -\frac{3}{4}y \\ 4x - 5y = -24 \end{cases}$ **i** $\begin{cases} 3x + 7y = 6 \\ x = \frac{5}{3}y - 1 \end{cases}$

Example 9 **Self Tutor**

Solve by elimination: $\begin{cases} 3x + 4y = 2 \\ 2x - 3y = 7 \end{cases}$

$3x + 4y = 2$ .... (1)
$2x - 3y = 7$ .... (2)

To make the coefficients of y the same size but opposite in sign, we multiply (1) by 3 and (2) by 4.

$\therefore\ 9x + 12y = 6$ $\{(1) \times 3\}$
$8x - 12y = 28$ $\{(2) \times 4\}$

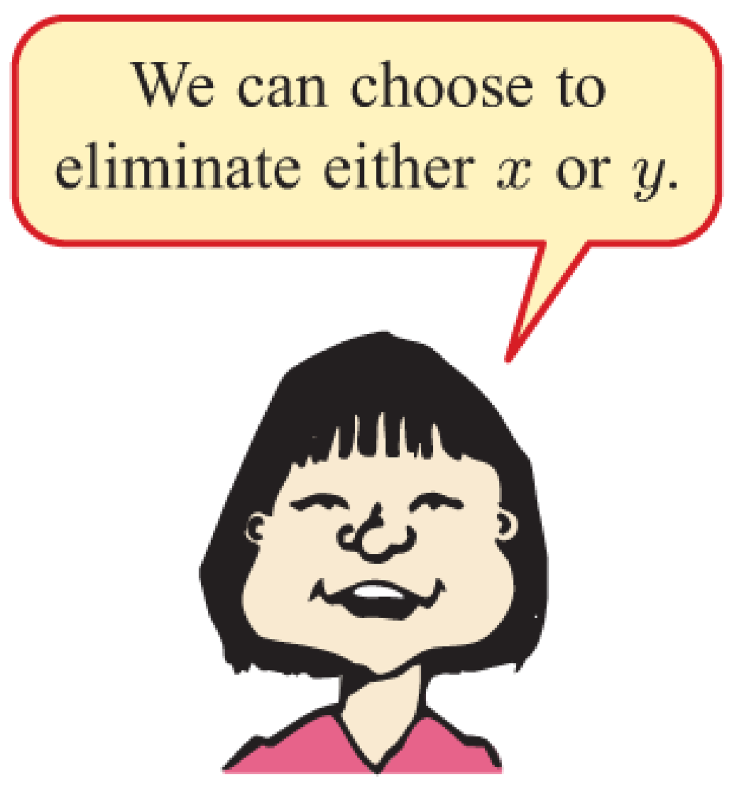
Adding, $17x = 34$

$\therefore\ x = 2$

Substituting $x = 2$ into (1) gives $3(2) + 4y = 2$
$\therefore\ 6 + 4y = 2$
$\therefore\ 4y = -4$
$\therefore\ y = -1$

The solution is $x = 2,\ y = -1$.

Check: In (2): $2(2) - 3(-1) = 4 + 3 = 7$ ✓



3 Solve by elimination:

a $\begin{cases} 3x - y = 5 \\ 4x + y = 9 \end{cases}$ **b** $\begin{cases} 5x - 2y = 17 \\ 3x + 2y = 7 \end{cases}$ **c** $\begin{cases} 3x + y = 16 \\ 7x - 2y = 7 \end{cases}$

d $\begin{cases} 3x - 7y = -27 \\ -6x + 5y = 18 \end{cases}$ **e** $\begin{cases} 3x - 7y = -8 \\ 9x + 11y = 16 \end{cases}$ **f** $\begin{cases} 4x + 3y = 14 \\ 3x - 4y = 23 \end{cases}$

g $\begin{cases} 2x - 3y = 6 \\ 5x - 4y = 1 \end{cases}$ **h** $\begin{cases} 4x + 2y = -23 \\ 5x - 7y = -5 \end{cases}$ **i** $\begin{cases} 4x - 7y = 9 \\ 5x - 8y = -2 \end{cases}$

4 Find the area of the triangle defined by:

a $y = x + 2$, $x + y = 9$, and $y = 2$

b $5x - 2y = 18$, $2x + 5y = 13$, and $8x - 9y = 11.4$

5 Consider the simultaneous equations $\begin{cases} y = 4x + 7 \\ 2y - 8x = 1 \end{cases}$.

a Graph each line on the same set of axes. What do you notice?

b Try to solve the simultaneous equations using:

i substitution **ii** elimination **iii** technology.

c How many solutions does this system of simultaneous equations have?

6 Consider the simultaneous equations $\begin{cases} y = -2x + 5 \\ 4x + 2y = 10 \end{cases}$.

a Graph each line on the same set of axes. What do you notice?

b Try to solve the simultaneous equations using:

i substitution **ii** elimination **iii** technology.

c How many solutions does this system of simultaneous equations have?

7 Consider the system of simultaneous equations $\begin{cases} 3x - 2y = 12 \\ y = mx - 6 \end{cases}$.

a Find the gradient of each line.

b Hence determine the value of m for which the simultaneous equations do *not* have a unique solution. Explain what is happening in this case.

c For any other value of m, the system has a unique solution. Find this solution.

8 Consider the system of simultaneous equations $\begin{cases} 12 = 4x - cy \\ 2x + 6 = 3y \end{cases}$.

a Find the gradient of each line.

b Hence determine the value of c for which the simultaneous equations do *not* have a unique solution. Explain what is happening in this case.

c For any other value of c, the system has a unique solution. Find this solution in terms of c.

REVIEW SET 1A

1 Consider the table of values alongside.

x	0	1	2	3	4
y	20	17	14	11	8

a Draw a graph of y against x.

b Are the variables linearly related? Explain your answer.

c Find the gradient and y-intercept of the graph.

d Find the equation connecting x and y.

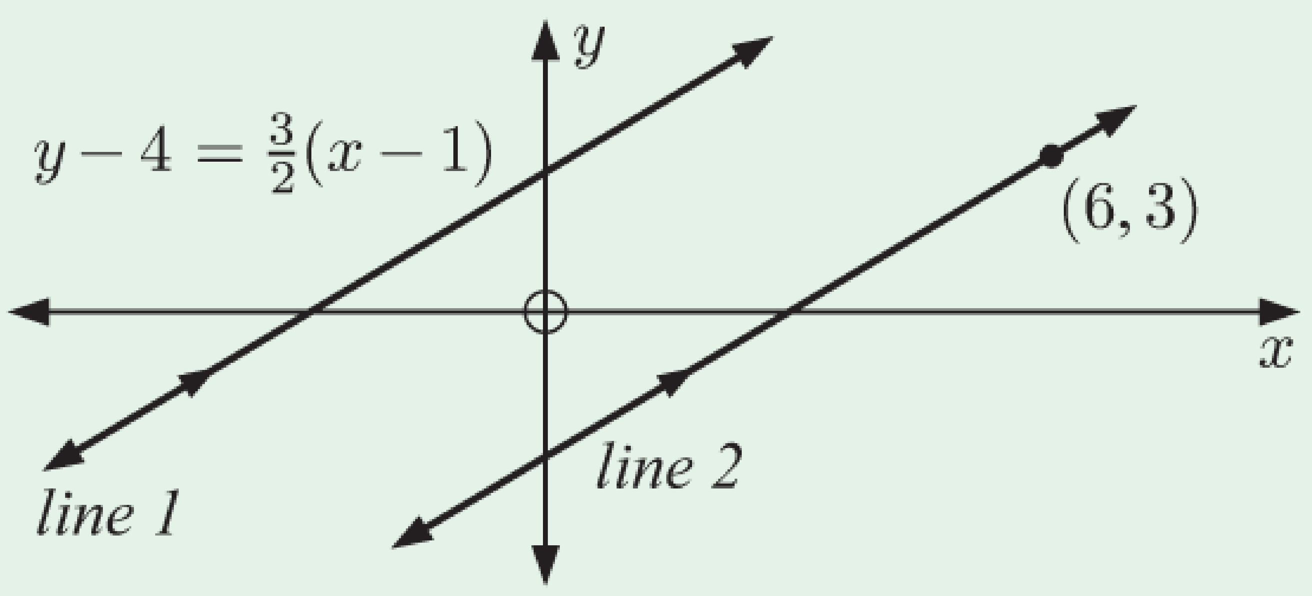
e Find the value of y when $x = 7$.

2 **a** Find, in gradient-intercept form, the equation of the line which has gradient $-\frac{1}{3}$ and passes through $(6, 2)$.

b Write the equation of the line in the form $ax + by + d = 0$.

3 **a** Find, in general form, the equation of *line 2*.

b Hence find the x-intercept of *line 2*.



4 Determine whether:

a $(5, -2)$ lies on the line with equation $y=-x+3$

b $(-3, \frac{1}{2})$ lies on the line with equation $3x+8y=-5$.

5 Draw the graph of:

a $y=-\frac{3}{4}x+1$ **b** $3x-4y=72$ **c** $2x+5y=-20$

6 Find the equation of the perpendicular bisector of:

a A(5, 2) and B(5, −4) **b** A(8, 1) and B(2, 5).

7 Quadrilateral ABCD has vertices A(3, 2), B(2, −4), C(−4, −3), and D(−3, 3).

a Find the equation of the perpendicular bisector of: **i** [AC] **ii** [BD].

b Classify quadrilateral ABCD.

8 Solve graphically: **a** $\begin{cases} y=3x+1 \\ x-y=3 \end{cases}$ **b** $\begin{cases} 2x+y=6 \\ x-2y=8 \end{cases}$

9 Solve by substitution: **a** $\begin{cases} y=3x+4 \\ 2x-y=-5 \end{cases}$ **b** $\begin{cases} x=2y-5 \\ 3x+4y=5 \end{cases}$

10 Solve by elimination: **a** $\begin{cases} 3x+2y=7 \\ 5x-2y=17 \end{cases}$ **b** $\begin{cases} 2x+7y=13 \\ -4x+3y=25 \end{cases}$

11 Consider the system of simultaneous equations $\begin{cases} x=k-2y \\ y=-\frac{1}{2}x+2 \end{cases}$.

a Find the gradient of each line.

b Find the value(s) of k such that the system has:

i no solutions **ii** infinitely many solutions.

Interpret these cases geometrically.

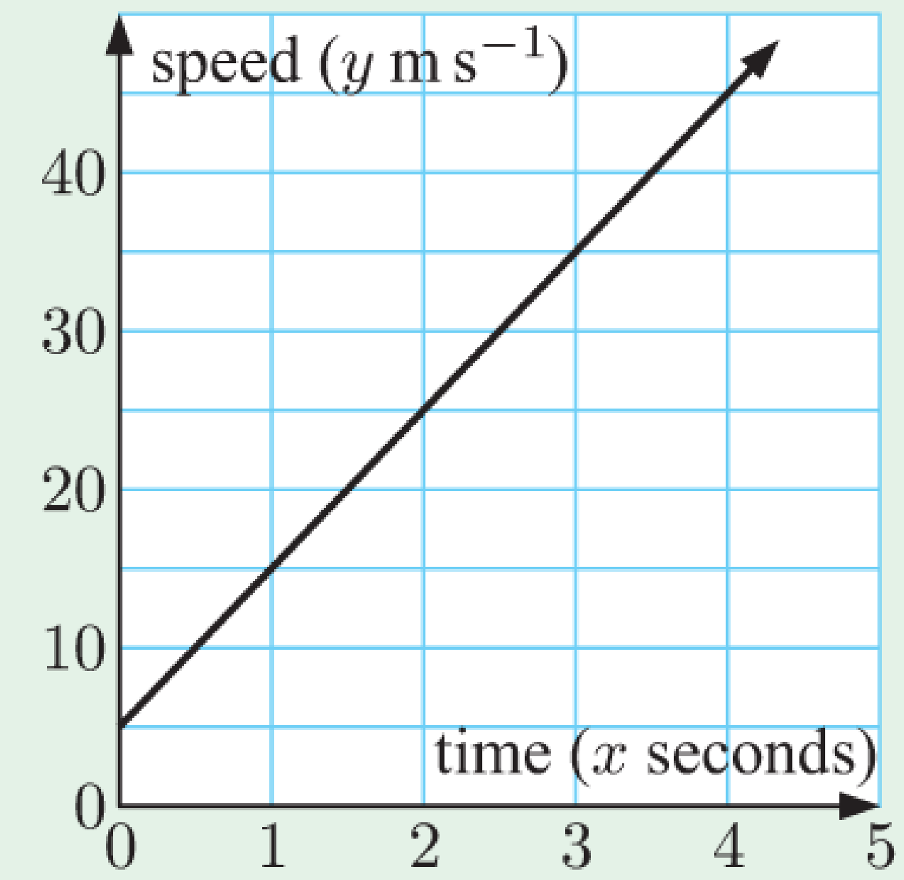
REVIEW SET 1B

1 The speed of a pebble thrown from the top of a cliff is shown alongside.

a Find the gradient and y-intercept of the line, and explain what these values mean.

b Find the equation of the line.

c Find the speed of the pebble after 8 seconds.



2 Find the equation of the line which is:

a parallel to $y = 3x - 8$ and passes through $(2, 7)$

b perpendicular to $2x + 5y = 7$ and passes through $(-1, -1)$.

3 Find k given that:

a $(2, k)$ lies on the line with equation $y = 5x - 3$

b $(\frac{1}{2}, -\frac{3}{2})$ lies on the line with equation $5x + 9y = k$.

4 Draw the graph of:

a $3x + 2y = 30$ **b** $y = -2x + 5$ **c** $y = -\frac{1}{3}x + \frac{4}{3}$

5 Consider the line with equation $y = \frac{2}{3}x - \frac{8}{3}$.

a Find the gradient of the line.

b Determine whether: **i** $(-2, -4)$ **ii** $(4, 5)$ lie on the line.

c Draw the graph of the line, showing your results from **a** and **b**.

6 *Line 1* has equation $2x + 3y = -24$. *Line 2* is perpendicular to *line 1*, and meets *line 1* at R$(3, -10)$. *Line 1* and *line 2* meet the x-axis at P and Q respectively. Find the area of triangle PQR.

7 A line segment has equation $x - 5y + 6 = 0$. Its midpoint is $(4, 2)$.

a State the gradient of:

i the line segment **ii** its perpendicular bisector.

b State the equation of the perpendicular bisector.

8 Triangle ABC has vertices A$(3, 6)$, B$(-1, 4)$, and C$(1, 0)$.

a Find the equation of the perpendicular bisector of:

i [AB] **ii** [AC] **iii** [BC].

b Graph the three perpendicular bisectors on the same set of axes. Discuss your observations.

9 Solve by substitution: **a** $\begin{cases} y = 6x + 2 \\ 3x - 2y = -7 \end{cases}$ **b** $\begin{cases} y = \frac{1}{2}x + 5 \\ 4x + 3y = 4 \end{cases}$

10 Solve by elimination: **a** $\begin{cases} 3x + 2y = 8 \\ 5x - 4y = 17 \end{cases}$ **b** $\begin{cases} 4x + 6y = -15 \\ 3x - 5y = 22 \end{cases}$

11 Consider the system of simultaneous equations $\begin{cases} ax + 4y = 6 \\ x - 2y = -2 \end{cases}$.

a Find the gradient of each line.

b Hence determine the value of a for which there is *not* a unique solution. Explain what is happening in this case.

c For any other value of a, the system has a unique solution. Find this solution in terms of a.

EXERCISE 1A

1 **a** $m = 3,\ c = 7$ **b** $m = -2,\ c = -5$
c $m = \frac{2}{3},\ c = -\frac{1}{3}$ **d** $m = \frac{7}{9},\ c = \frac{2}{9}$
e $m = \frac{1}{3},\ c = -\frac{1}{2}$ **f** $m = -\frac{5}{8},\ c = \frac{3}{8}$

2 **a** $y = 3x - 11$ **b** $y = -2x - 1$ **c** $y = \frac{1}{4}x - 4$
d $y = -\frac{3}{4}x + 4$

3 **a** The gradient is -10 which means that the balance in the account decreases by $10 each year.
The y-intercept is 90 which means that the initial balance was $90.
b $y = -10x + 90$ **c** 9 years

4 **a** $-\frac{23}{910}$ **b** $y = -\frac{23}{910}x + 46$

5 **a** 150 metres
b The height of the helicopter above sea level increases by 120 metres each minute after taking off.
c 390 metres **d** 4 minutes 10 seconds

6 **a** $4x + y = 6$ **b** $5x - y = 3$ **c** $3x + 4y = 5$
d $3x - 5y = 1$

7 **a** $y = -5x + 2$ **b** $y = -\frac{3}{7}x - \frac{2}{7}$ **c** $y = 2x - 6$
d $y = \frac{3}{13}x + \frac{4}{13}$

8 $ax + by = d$ can be written as $y = -\frac{a}{b}x + \frac{d}{b}$ which has the form $y = mx + c$. $\therefore\ m = -\frac{a}{b}$

9 **a** $4x + y = 6$ **b** $x - 2y = 13$ **c** $5x + 3y = 8$
d $7x - 6y = 17$

10 **a** $y = 2x + 5$ **b** $y = -x + 9$ **c** $y = \frac{7}{5}x - \frac{11}{5}$
d $y = -\frac{5}{6}x + \frac{19}{6}$

11 **a** $2x - y = -2$ **b** $3x + 10y = 8$ **c** $8x + 5y = -13$

12 **a** $y = \frac{3}{4}x - \frac{5}{4}$ **b** $-\frac{5}{4}$

13 **a** $y = 3x + 1$ **b** $2x - y = 7$ **c** $y = \frac{1}{2}x + \frac{11}{2}$
d $2x - y = -3$

14 *Line 1*: $y = \frac{2}{3}x + \frac{1}{3}$, *Line 2*: $y = -\frac{3}{2}x - 4$

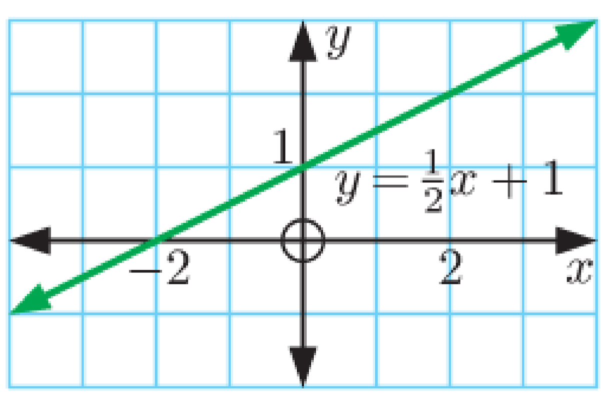
15 **a** yes **b** no **c** yes **d** yes

16 **a** $c = 7$ **b** $m = 11$ **c** $t = 8$

17 **a** $k = -3$ **b** $k = -51$ **c** $k = -12$

18 **a** $x - y + 2 = 0$ **b** -2 **19** 126 units^2

EXERCISE 1B

1 **a**


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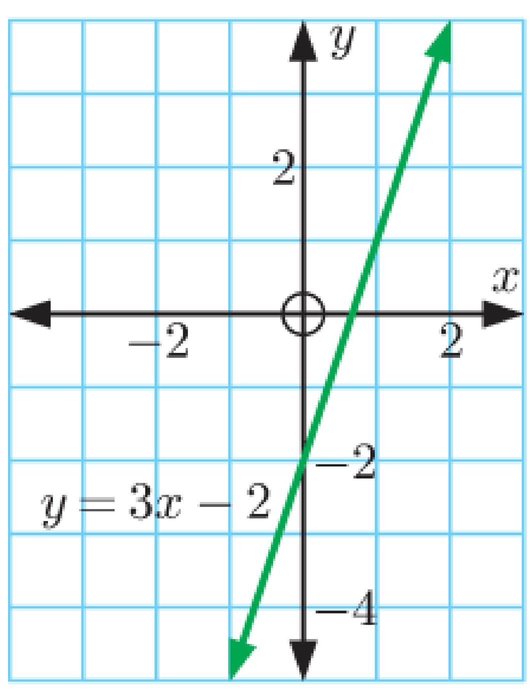
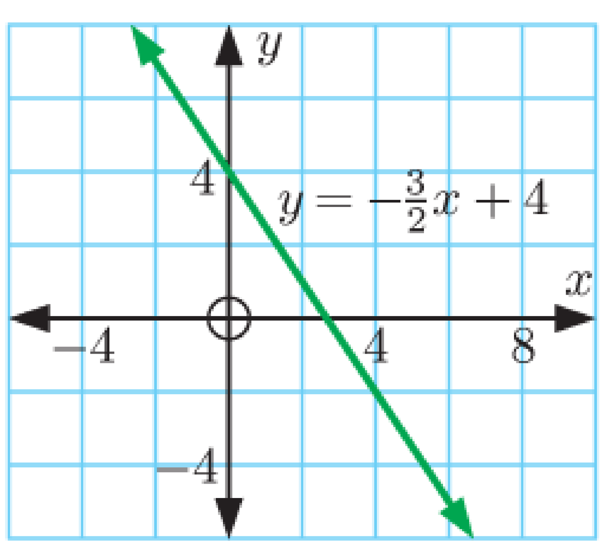


c


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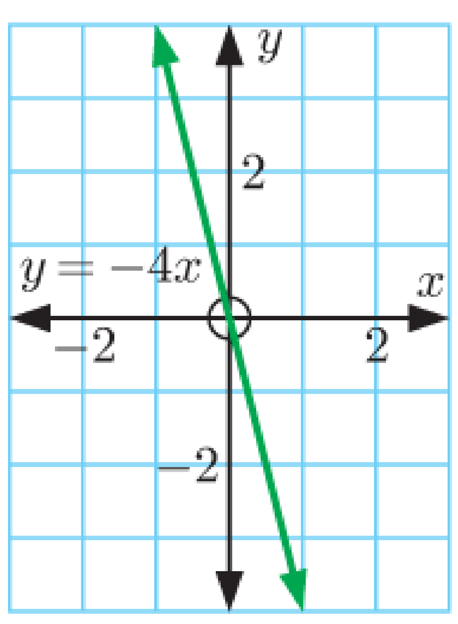

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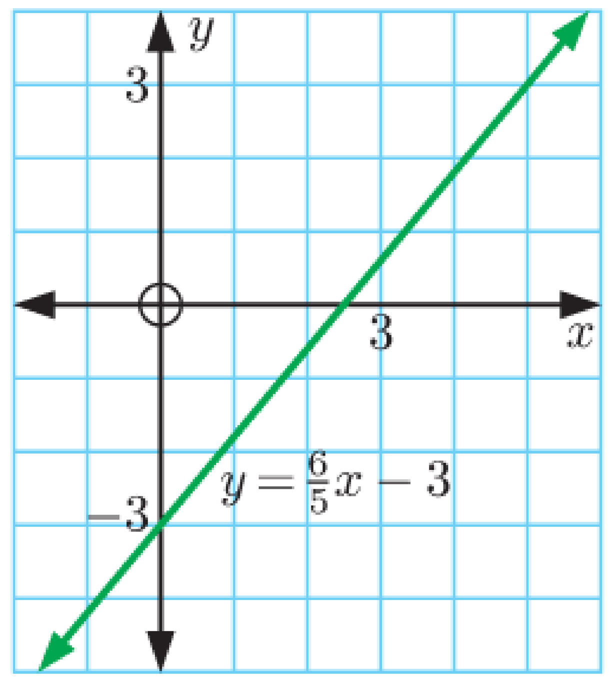

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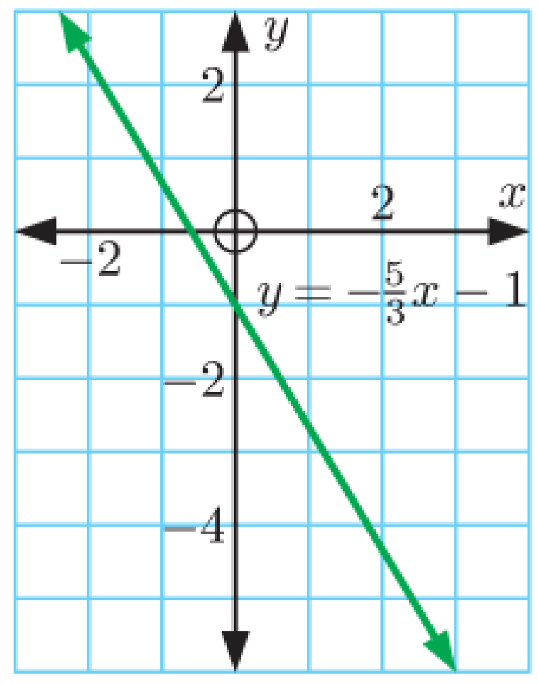
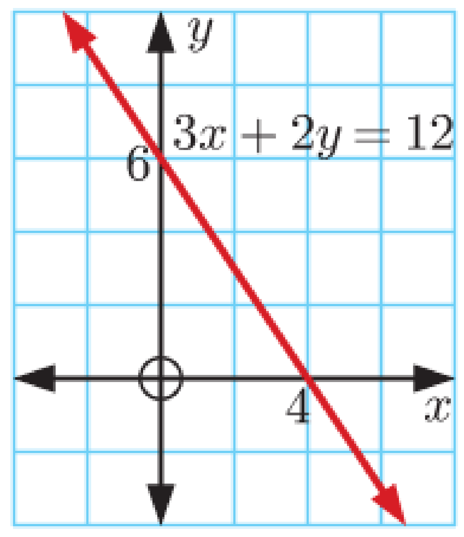


2 **a**


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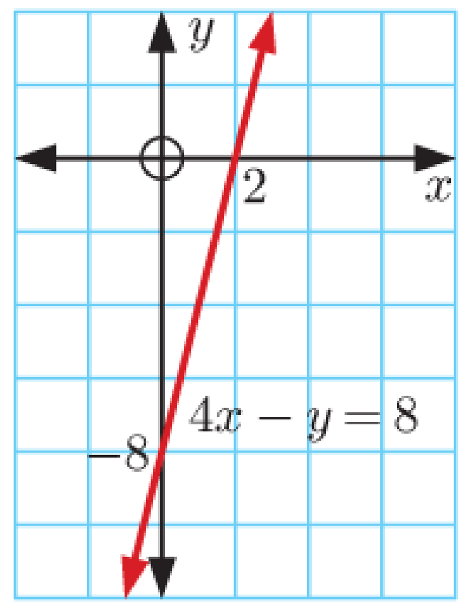

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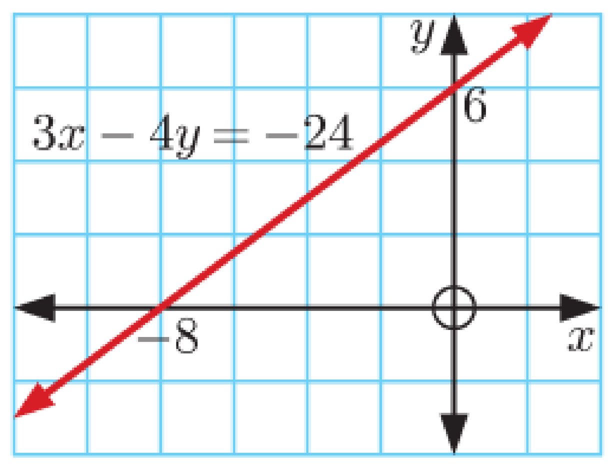

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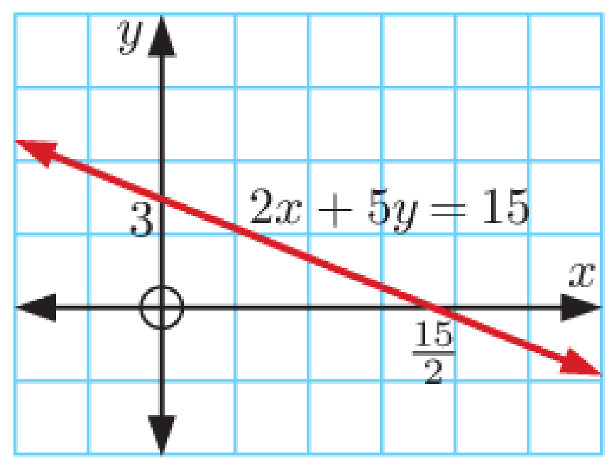

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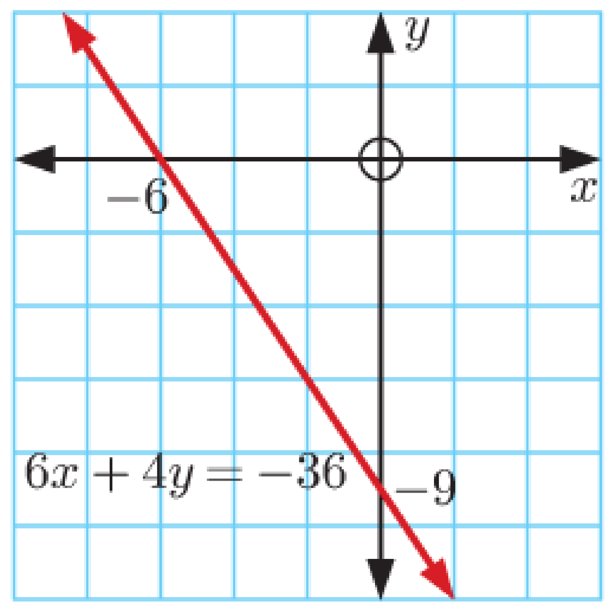

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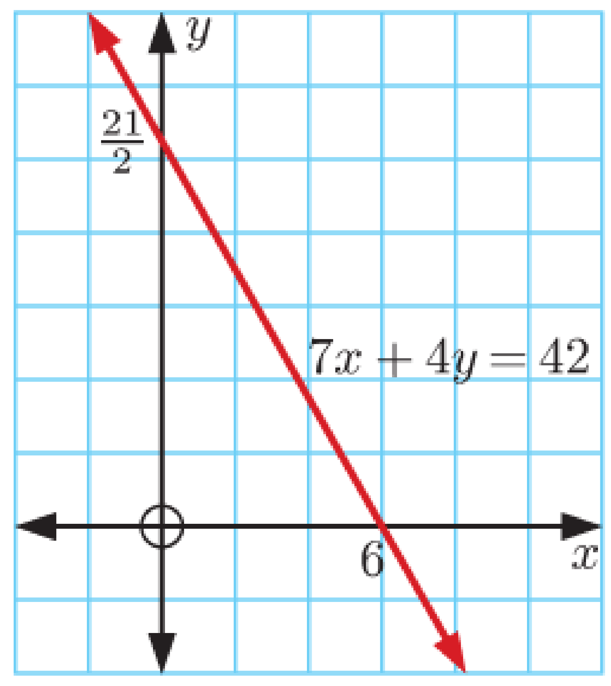
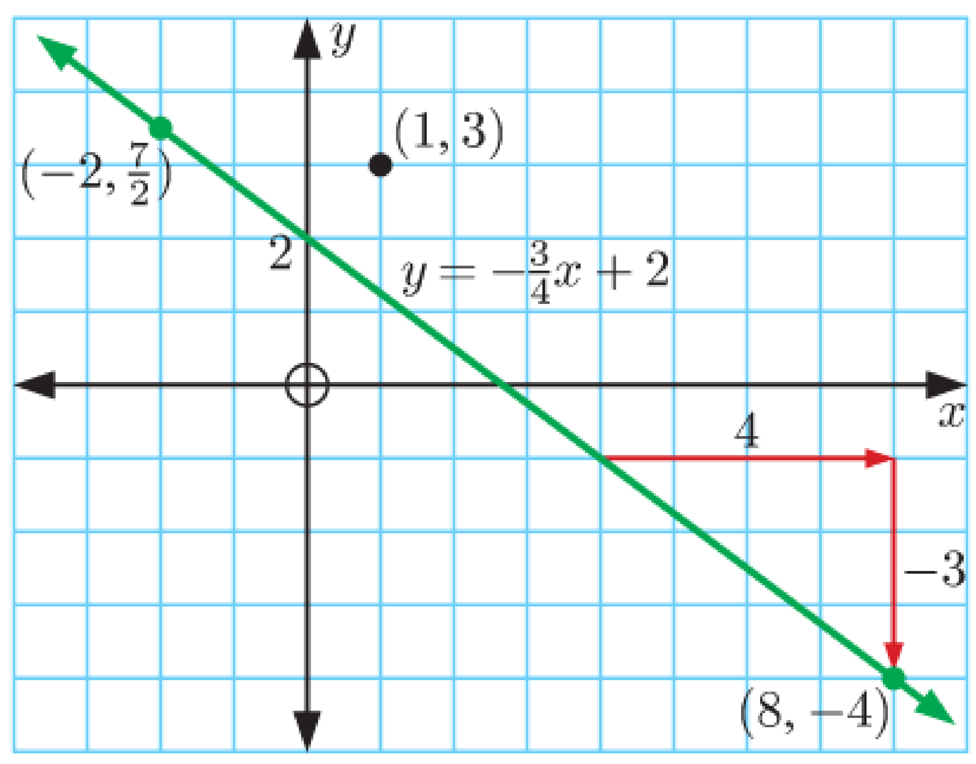
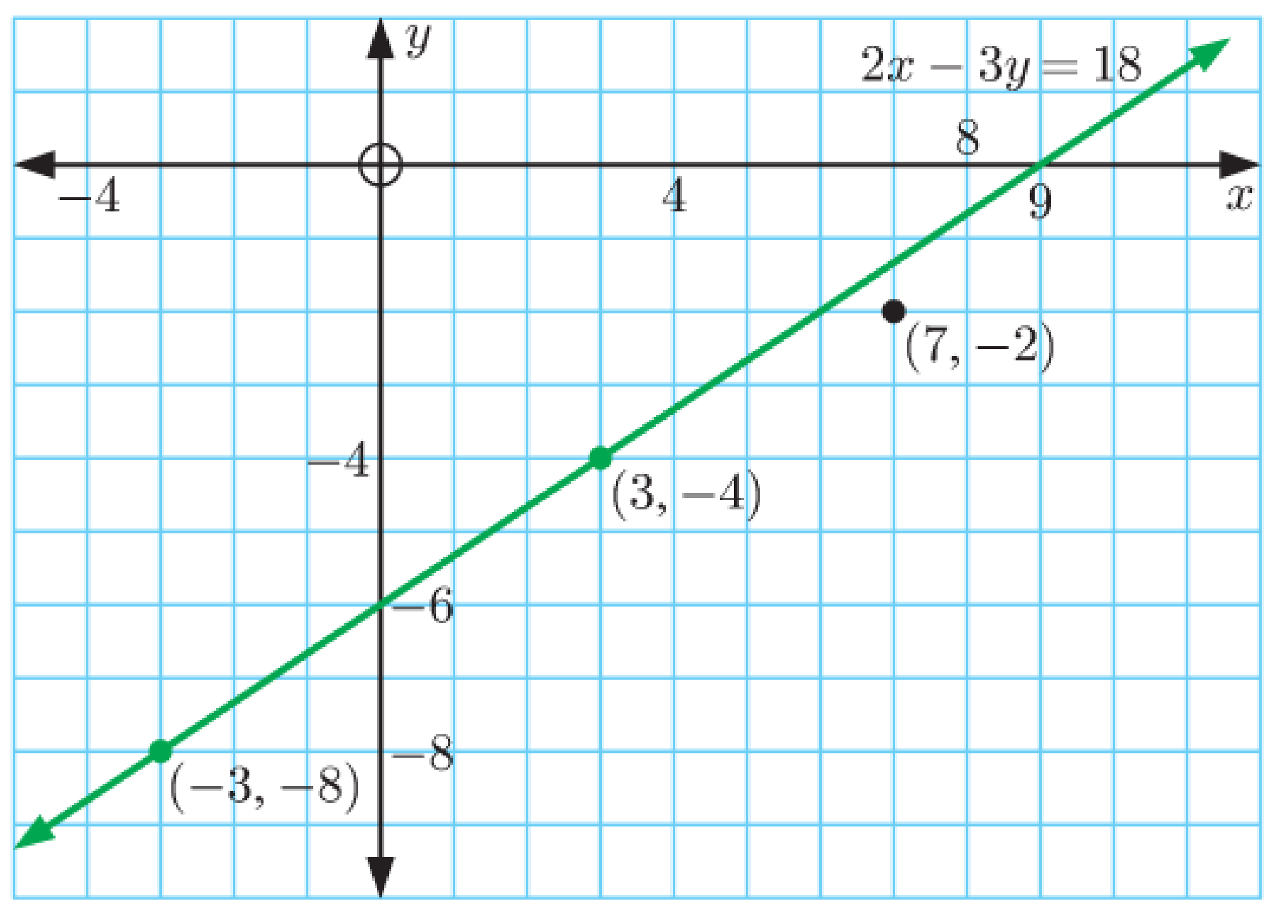


3 **a** $m = -\frac{3}{4},\ c = 2$ **b** **i** yes **ii** no **iii** yes
c


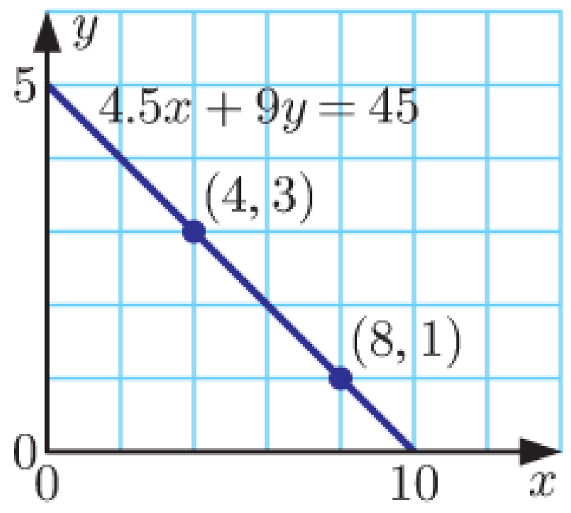
4 **a** x-intercept 9, y-intercept -6
b **i** yes **ii** no **c** $c = -8$
d


5 **a** x serves of nigiri at \$4.50 each and y serves of sashimi at \$9 each adds up to a total of \$45. $\therefore$ $4.5x + 9y = 45$

b 3 serves of sashimi

c 8 serves of nigiri

d



EXERCISE 1C

1 **a** $(4, 4)$ **b** 3 **c** $-\frac{1}{3}$ **d** $x + 3y = 16$

2 **a** $2x - y = 3$ **b** $3x - y = 2$ **c** $x - y = 5$ **d** $2x - y = 2$ **e** $x = \frac{9}{2}$ **f** $8x + 6y = 35$

3 **a** $2x - 3y = 2$ **b** $2(1) - 3(0) = 2$ ✓ **c** $\text{PR} = \text{QR} = \sqrt{26}$ units

4 **a** $\text{AB} = \text{BC} = \text{CD} = \text{AD} = \sqrt{29}$ units $\therefore$ ABCD is a rhombus.
b $y = -x$ **c** B: $2 = -(-2)$ ✓ D: $-1 = -(1)$ ✓

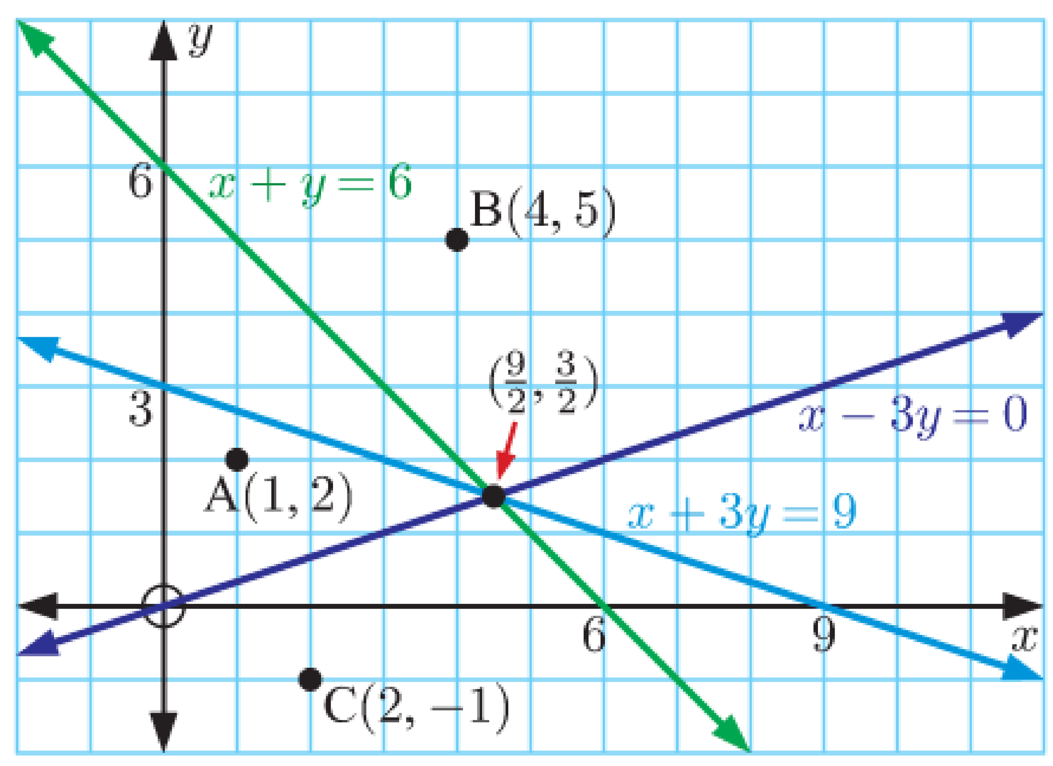
5 **a** **i** $\frac{3}{2}$ **ii** $-\frac{2}{3}$ **b** $2x + 3y - 21 = 0$

6 **a** $(-\frac{1}{2}, 1)$
b The perpendicular bisector of the line joining the two hospitals is $10x + 12y = 7$. An ambulance crew should be sent from A to locations below this line, and from B to locations above this line.

7 **a** **Hint:** Start by finding the gradient and midpoint of [AB].
b We can find the perpendicular bisector of any two points $\text{A}(x_1, y_1)$ and $\text{B}(x_2, y_2)$ by substituting in the values of x_1, x_2, y_1, and y_2.

8 **a** **i** $x + y = 6$ **ii** $x - 3y = 0$ **iii** $x + 3y = 9$

b

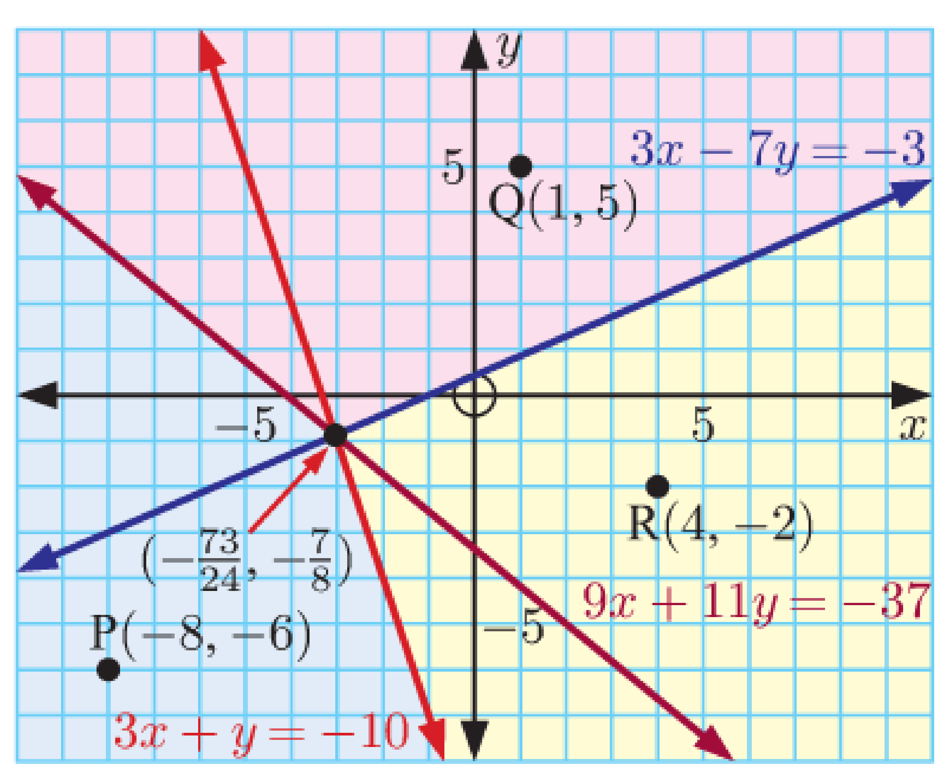


The perpendicular bisectors all intersect at $(\frac{9}{2}, \frac{3}{2})$.
A, B, and C are all equidistant from this point.

c The perpendicular bisectors of each pair of points will meet at a single point. As the three points are equidistant from the point of intersection, a circle centred at the point of intersection that passes through one of them will pass through all of them.

9 **a** **i** $9x + 11y = -37$ **ii** $3x + y = -10$ **iii** $3x - 7y = -3$

b



EXERCISE 1D

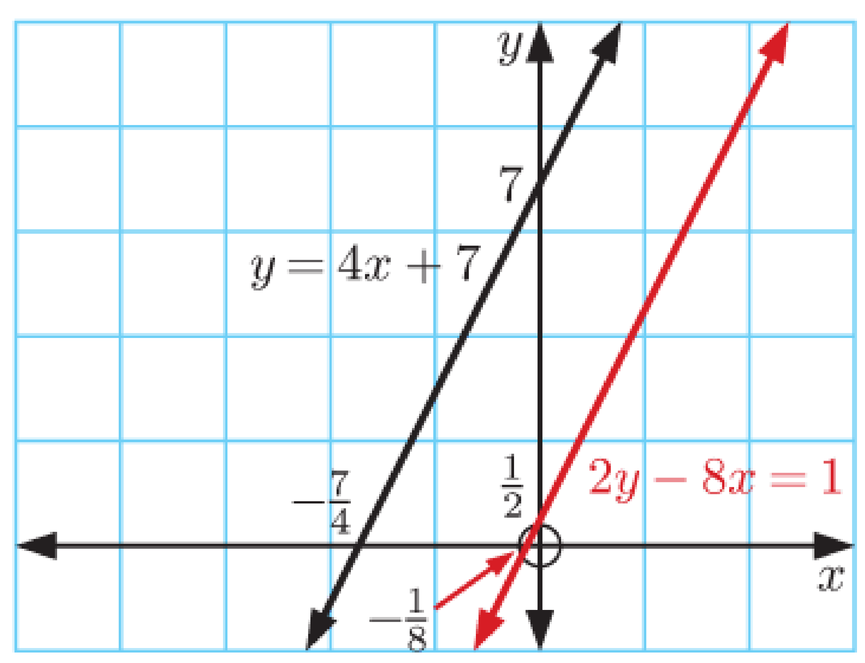
1 **a** $x = -2,\ y = -4$ **b** $x = 1,\ y = -3$ **c** $x = 6,\ y = 7$ **d** $x = 3,\ y = 2$ **e** $x = 6,\ y = 1$ **f** $x = 2,\ y = -12$

2 **a** $x = 3,\ y = 5$ **b** $x = 1,\ y = -1$ **c** $x = -1,\ y = 8$ **d** $x = 5,\ y = 8$ **e** $x = -4,\ y = -\frac{1}{4}$ **f** $x = -1\frac{11}{31},\ y = -\frac{4}{31}$ **g** $x = -3,\ y = 3\frac{1}{2}$ **h** $x = -2\frac{1}{4},\ y = 3$ **i** $x = \frac{1}{4},\ y = \frac{3}{4}$

3 **a** $x = 2,\ y = 1$ **b** $x = 3,\ y = -1$ **c** $x = 3,\ y = 7$ **d** $x = \frac{1}{3},\ y = 4$ **e** $x = \frac{1}{4},\ y = 1\frac{1}{4}$ **f** $x = 5,\ y = -2$ **g** $x = -3,\ y = -4$ **h** $x = -4\frac{1}{2},\ y = -2\frac{1}{2}$ **i** $x = -28\frac{2}{3},\ y = -17\frac{2}{3}$

4 **a** $12\frac{1}{4}$ units^2 **b** $1\frac{4}{25}$ units^2

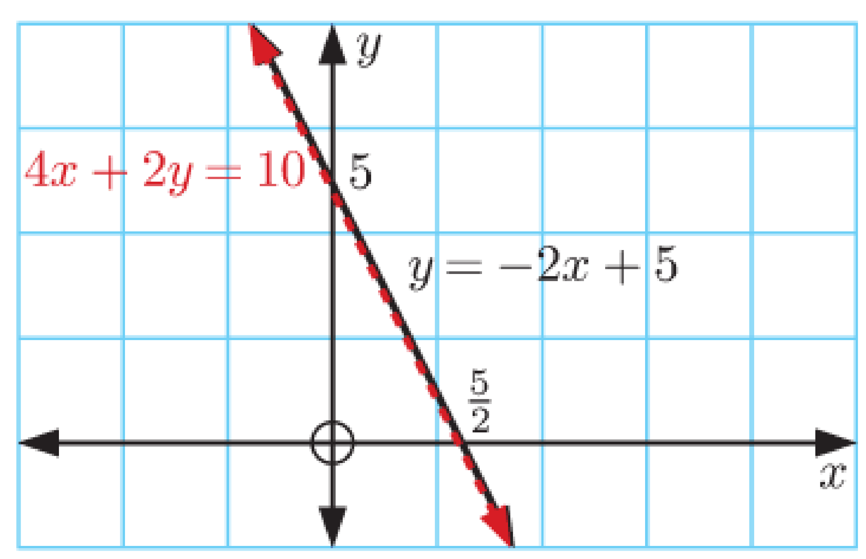
5 **a**



The lines are parallel.

c no solutions

6 **a**



The lines are coincident.

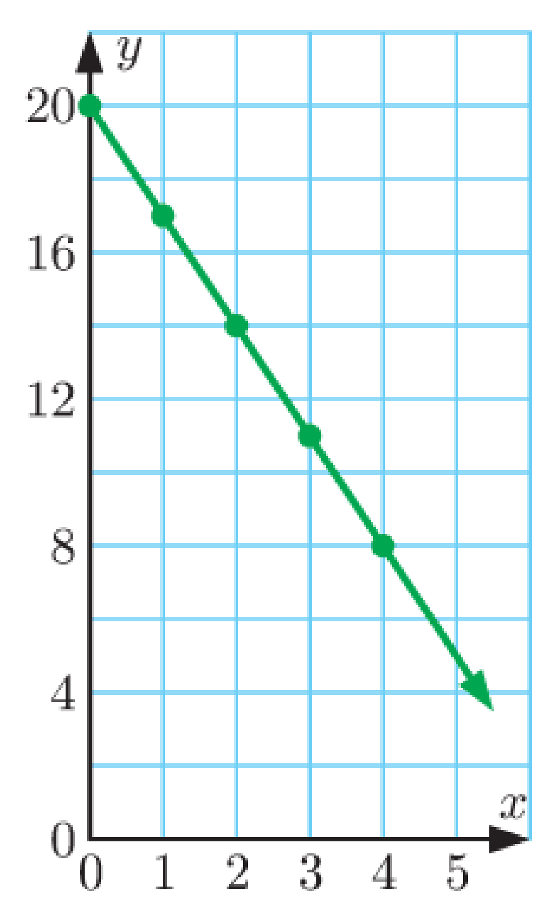
c infinitely many solutions

7 **a** $\frac{3}{2}$ and m
b $m = \frac{3}{2}$, in this case the lines are coincident and hence there are infinitely many solutions.
c $x = 0,\ y = -6$

8 **a** $\dfrac{4}{c}$ and $\frac{2}{3}$
b $c = 6$, in this case the lines are parallel and hence there are no solutions.
c $x = \dfrac{18 + 3c}{6 - c}$, $y = \dfrac{24}{6 - c}$

REVIEW SET 1A

1 **a**



b Yes, the variables are linearly related as the points all lie on a straight line.

c gradient is -3, y-intercept is 20

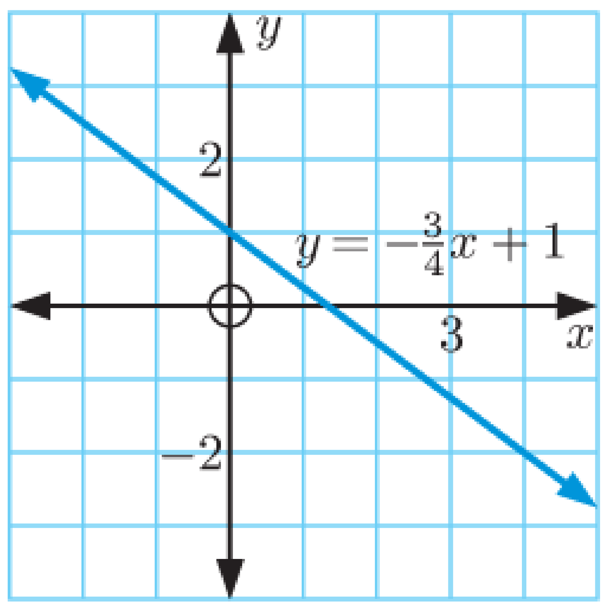
d $y = -3x + 20$

e $y = -1$

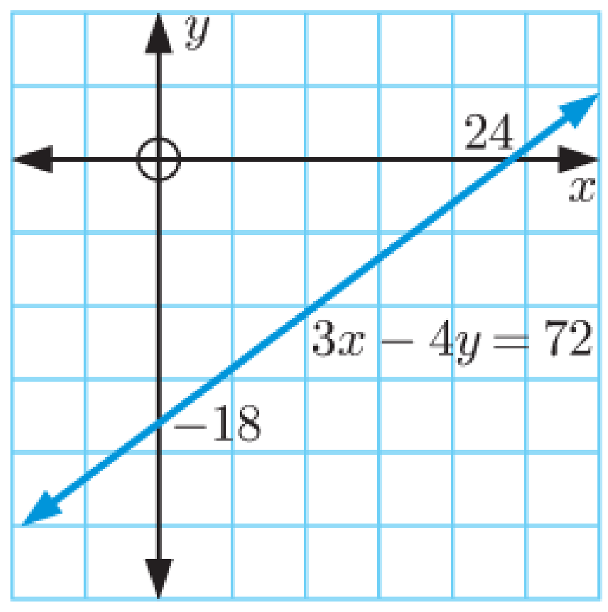
2 **a** $y = -\frac{1}{3}x + 4$ **b** $x + 3y - 12 = 0$

3 **a** $3x - 2y = 12$ **b** 4 **4** **a** yes **b** yes

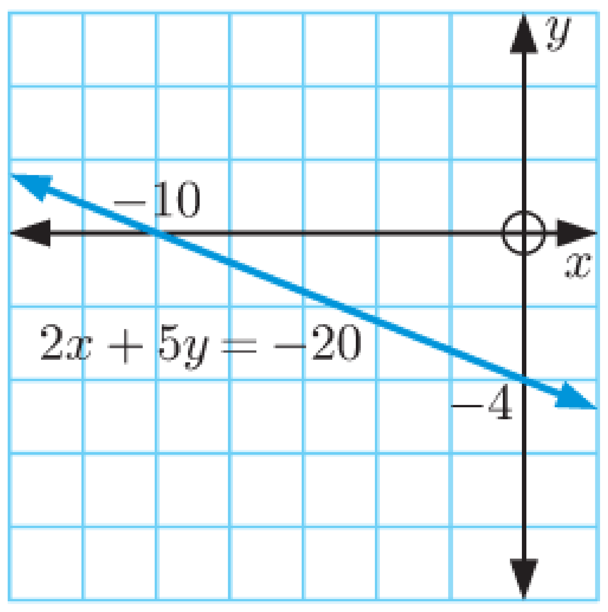
5 **a**



b



c



6 **a** $y = -1$ **b** $3x - 2y = 9$

7 **a** **i** $7x + 5y = -6$ **ii** $5x - 7y = 1$

b ABCD is a square.

8 **a** $x = -2,\ y = -5$ **b** $x = 4,\ y = -2$

9 **a** $x = 1,\ y = 7$ **b** $x = -1,\ y = 2$

10 **a** $x = 3,\ y = -1$ **b** $x = -4,\ y = 3$

11 **a** $-\frac{1}{2}$ and $-\frac{1}{2}$

b **i** $k \neq 4$ **ii** $k = 4$

If $k \neq 4$, the lines are parallel and hence there are no solutions.

If $k = 4$, the lines are coincident and hence there are infinitely many solutions.

REVIEW SET 1B

1 **a** The gradient is 10 which means that the speed increases by 10 m s^{-1} each second.

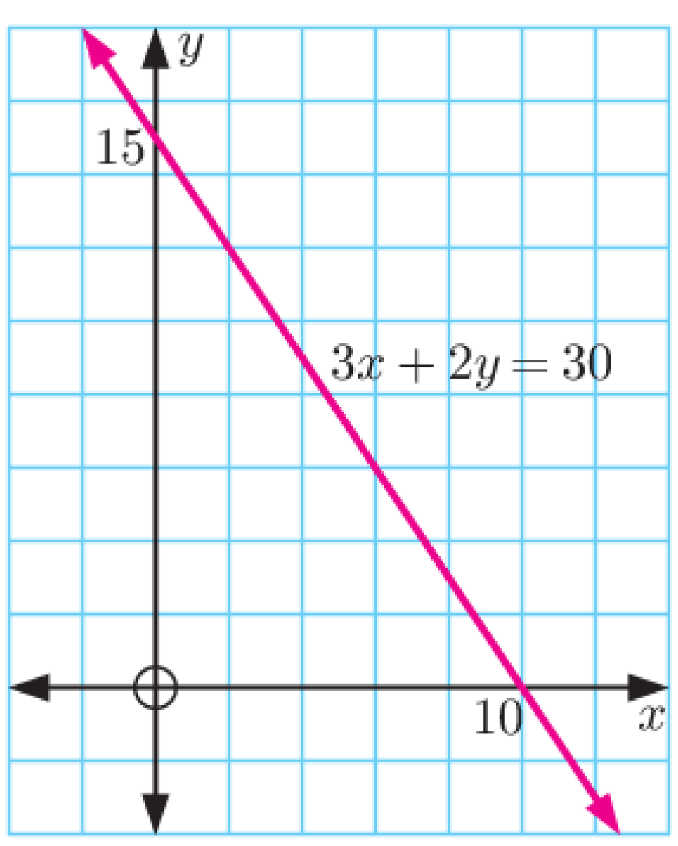
The y-intercept is 5 which means that the initial speed was 5 m s^{-1}.

b $y = 10x + 5$ **c** 85 m s^{-1}

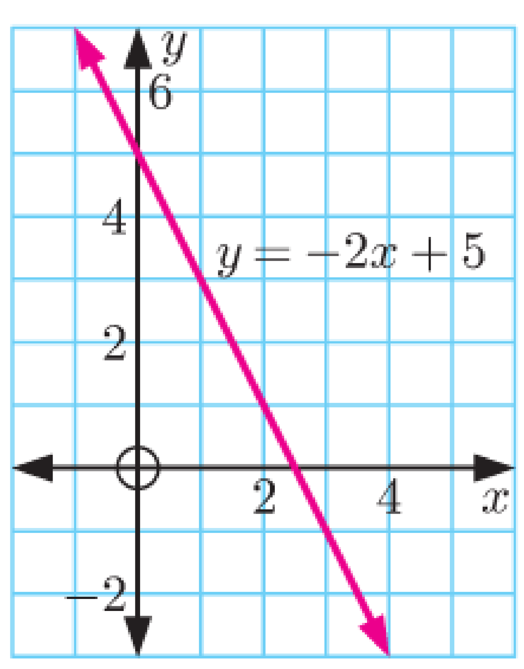
2 **a** $y = 3x + 1$ **b** $5x - 2y = -3$

3 **a** $k = 7$ **b** $k = -11$

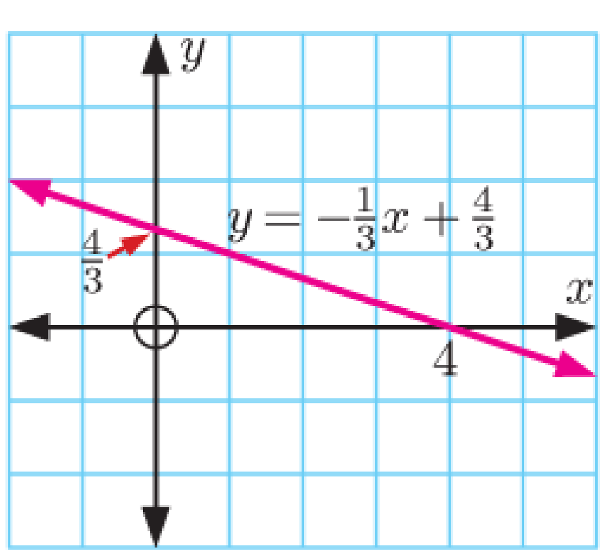
4 **a**



b



c

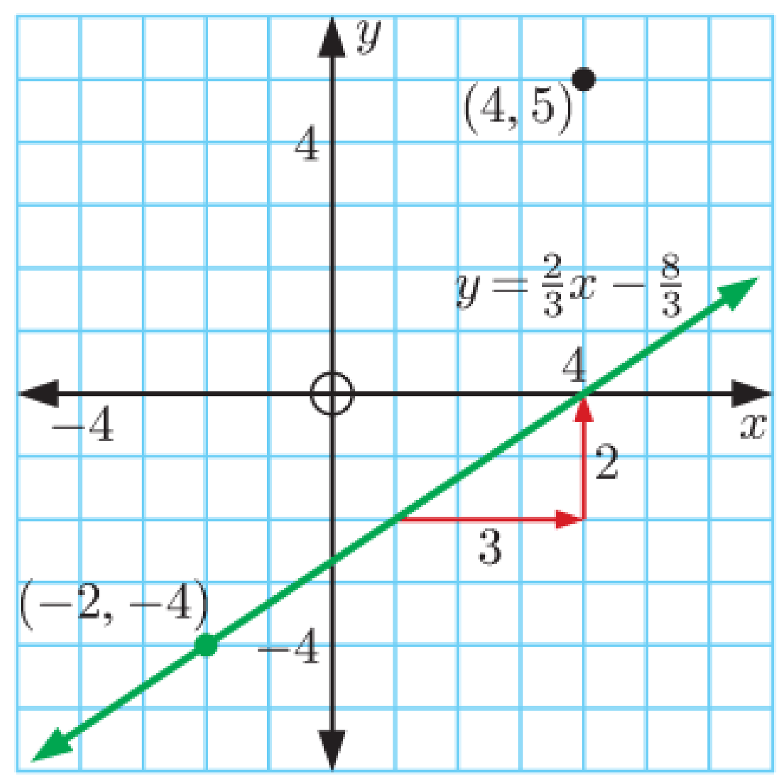


5 **a** $m = \frac{2}{3}$

b **i** yes

ii no

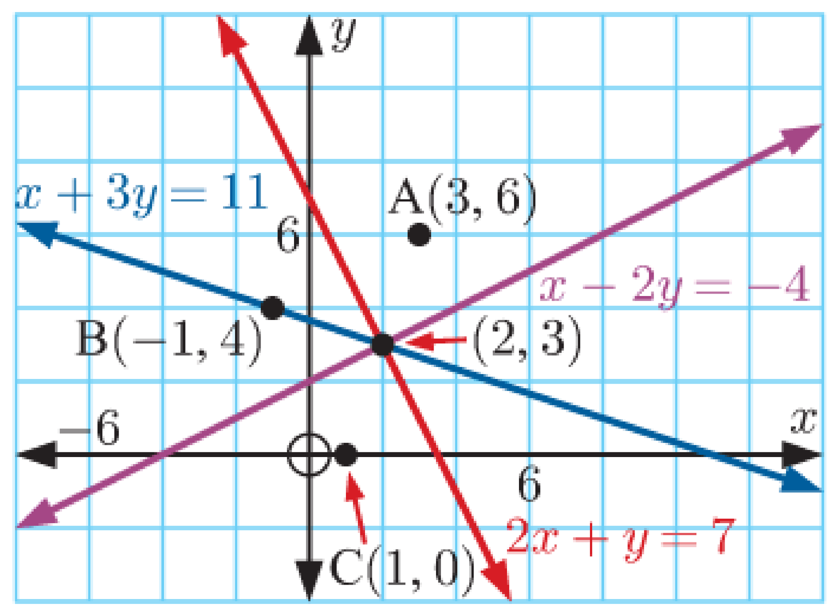
c



6 $108\frac{1}{3}$ units^2 **7** **a** **i** $\frac{1}{5}$ **ii** -5 **b** $5x + y = 22$

8 **a** **i** $2x + y = 7$ **ii** $x + 3y = 11$ **iii** $x - 2y = -4$

b



All three perpendicular bisectors intersect at (2, 3).
A, B, and C are all equidistant from this point.

9 **a** $x = \frac{1}{3},\ y = 4$ **b** $x = -2,\ y = 4$

10 **a** $x = 3,\ y = -\frac{1}{2}$ **b** $x = 1\frac{1}{2},\ y = -3\frac{1}{2}$

11 **a** $-\dfrac{a}{4}$ and $\frac{1}{2}$

b $a = -2$, in this case the lines are parallel and hence there are no solutions.

c $x = \dfrac{2}{a+2},\quad y = \dfrac{a+3}{a+2}$

EXERCISE 2A

1 **a** $A = \{1, 2, 4, 8\},\quad n(A) = 4$

b $A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\},\quad n(A) = 10$

c $A = \{$A, R, D, V, K$\},\quad n(A) = 5$

d $A = \{41, 43, 47\},\quad n(A) = 3$

2 **a** finite **b** infinite **c** infinite

3 **a** **i** 6 **ii** 3

b **i** true **ii** false **iii** true **iv** true **v** true

4 **a** subsets of S: $\varnothing, \{1\}, \{2\}, \{1, 2\}$

subsets of T: $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

b yes **c** $\frac{1}{2}$

5 Each of the four elements can be *included* or *not included* in a subset.

$\therefore$ the set has $2 \times 2 \times 2 \times 2 = 16$ subsets.

6 $x = 3$

7 If $A \subseteq B$, then all elements of A are in B.

If $B \subseteq A$, then all elements of B are in A.

This is only possible if A and B contain exactly the same elements.

$\therefore\ A = B$.

EXERCISE 2B

1 **a** **i** $A \cap B = \{9\}$

ii $A \cup B = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

b **i** $A \cap B = \varnothing$ **ii** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$