DRV - practice [75 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.13

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

Markscheme

$\frac{6}{15}\left(0.4,\ \frac{2}{5}\right)$ ***A1***

[1 mark]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

Markscheme

$\mathrm{P}(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. eg $\mathrm{P}(X = 8)$, $X \sim \mathrm{B}\left(20,\ \frac{6}{15}\right)$.

0.180 (0.179705…) ***A1***

[2 marks]

(c) Find the probability he will choose a male student at most 9 times. [3]

Markscheme

$\mathrm{P}(\text{male}) = \frac{9}{15}\,(0.6)$ ***A1***

$\mathrm{P}(X \le 9) = 0.128\ (0.127521\ldots)$ ***(M1)A1***

Note: Award ***(M1)*** for evidence of correct approach *eg*, $\mathrm{P}(X \le 9)$.

[3 marks]

2. [Maximum mark: 6] SPM.1.SL.TZ0.12

Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled −3, −1, 0, 1, 2 and 5.

The score for the game, X, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$P(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Markscheme

$\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Markscheme

$-3\times\frac{1}{18}+(-1)\times\frac{4}{18}+0\times\frac{3}{18}+\ldots+5\times\frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$=1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) Jae Hee plays the game twice and adds the two scores together.

Find the probability Jae Hee has a **total** score of −3. [3]

Markscheme

$2\times\frac{1}{18}\times\frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18}\times\frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$=\frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

3. [Maximum mark: 6] SPM.1.AHL.TZ0.17

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

In the first month, Mr Burke will teach his class 20 times.

(a) Find the probability he will choose a female student 8 times. [2]

Markscheme

P(X = 8) ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. *eg,* P(X = 8), $X \sim$ B$\left(20,\ \frac{6}{15}\right)$.

= 0.180 (0.179705…) ***A1***

[2 marks]

(b) The Head of Year, Mrs Smith, decides to select a student at random from the year group to read the notices in assembly. There are 80 students in total in the year group. Mrs Smith calculates the probability of picking a male student 8 times in the first 20 assemblies is 0.153357 correct to 6 decimal places.

Find the number of male students in the year group. [4]

Markscheme

let x be the number of male students

recognize that probability of selecting a male is equal to $\frac{x}{80}$ ***(A1)***

$\left(\text{set up equation } {}^{20}\mathrm{C}_8\left(\frac{x}{80}\right)^8\left(\frac{80-x}{80}\right)^{12} =\right) 0.153357$ ***(M1)***

number of male students = 37 ***(M1)A1***

Note: Award ***(M1)A0*** for 27.

[4 marks]

4. [Maximum mark: 5] 23M.1.SL.TZ2.12

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathrm{P}(X=x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

Markscheme

$0.15+0.2+k+0.16+2k+0.25=1$ *(M1)*

$k=0.08$ ***A1***

[2 marks]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in $\$$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain ($\$$)	-4	-3	-1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

Markscheme

$(-4\times0.15)+(-3\times0.2)+(-1\times0.08)+(0\times0.16)+(1\times0.16)+(4\times0.25)$ *(M1)*

$=-0.12$ ***A1***

$\mathrm{E}(X)\neq0$ therefore the game is not fair ***R1***

Note: Do not award ***A0R1*** without an explicit value for $\mathrm{E}(X)$ seen. The ***R1*** can be awarded for comparing **their** $\mathrm{E}(X)$ to zero provided working is shown.

[3 marks]

5. [Maximum mark: 7] 22N.1.SL.TZ0.9

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

Markscheme

$0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

Markscheme

$0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

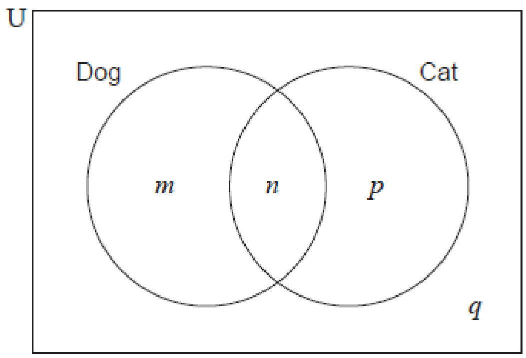
$(k =)\ 24$ (points) ***A1***

[3 marks]

6. [Maximum mark: 16] 22N.2.SL.TZ0.4

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



Find the value of

(a.i) m. [1]

Markscheme

$(m =)\ 54\%$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

(a.ii) n. [1]

Markscheme

$(n =)\ 14\%$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

(a.iii) p. [1]

Markscheme

$(p =)\ 22\%$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

(a.iv) q. [1]

Markscheme

$(q =)\ 10\%$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

(b) Find the percentage of students who have a dog or a cat or both. [1]

Markscheme

$90\,(\%)$ ***A1***

Note: Award ***A0*** for a decimal answer.

[1 mark]

Find the probability that a randomly chosen student

(c.i) has a dog but does not have a cat. [1]

Markscheme

$0.54\ \left(\frac{54}{100},\ \frac{27}{50},\ 54\%\right)$ ***A1***

[1 mark]

(c.ii) has a dog given that they do not have a cat. [2]

Markscheme

$\frac{54}{64}\ \left(0.844,\ \frac{27}{32},\ 84.4\%,\ 0.84375\right)$ ***A1A1***

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[2 marks]

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events "being a school captain" and "having a dog" are independent.

Use Tim's model to find the probability that in the next 10 years

(d.i) 5 school captains have a dog. [2]

Markscheme

recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10,\ 0.68)$

$(\mathrm{P}(X = 5) =)\ 0.123\ \ (0.122940\ldots,\ 12.3\%)$ ***A1***

[2 marks]

(d.ii) more than 3 school captains have a dog. [2]

Markscheme

$1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

$0.984 \ (0.984497\ldots, \ 98.4\%)$ ***A1***

[2 marks]

(d.iii) exactly 9 school captains in succession have a dog. [3]

Markscheme

$(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199 \ (0.0198957\ldots, \ 1.99\%)$ ***A1***

[3 marks]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]

Markscheme

EITHER

the probability is not constant ***A1***

OR

the events are not independent ***A1***

OR

the events should be modelled by the hypergeometric distribution instead ***A1***

[1 mark]

7. [Maximum mark: 7] 22M.1.SL.TZ2.5

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

Markscheme

$(\mathrm{E}(X) =)\ 10 \times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

Markscheme

recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

Markscheme

0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

8. [Maximum mark: 6] 20N.2.SL.TZ0.S_3

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X=x)$	q	$4p^2$	p	$0.7-4p^2$

(a) Find an expression for q in terms of p. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of summing probabilities to 1 ***(M1)***

eg $q+4p^2+p+0.7-4p^2=1,\ \ 1-4p^2-p-0.7+4p^2$

$q=0.3-p$ ***A1 N2***

[2 marks]

(b.i) Find the value of p which gives the largest value of $\mathrm{E}(X)$. [3]

Markscheme

correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0\times(0.3-p)+1\times4p^2+2\times p+3\times\left(0.7-4p^2\right)$

valid approach to find when $\mathrm{E}(X)$ is a maximum ***(M1)***

eg max on sketch of $\mathrm{E}(X)$, $8p+2+3\times(-8p)=0$, $\frac{-b}{2a}=\frac{-2}{2\times(-8)}$

$p=\frac{1}{8}\ (=0.125)$ (exact) (accept $x=\frac{1}{8}$) ***A1 N3***

[3 marks]

(b.ii) Hence, find the largest value of $\mathrm{E}(X)$. [1]

Markscheme

2.225

$\frac{89}{40}$ (exact), 2.23 ***A1 N1***

[1 mark]

9. [Maximum mark: 16] 19M.2.SL.TZ1.S_10

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a.i) Find the probability of rolling exactly one red face. [2]

Markscheme

valid approach to find P(one red) (M1)

eg ${}_nC_a \times p^a \times q^{n-a},\ \mathrm{B}\,(n, p),\ 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2,\ \binom{3}{1}$

listing all possible cases for exactly one red (may be indicated on tree diagram)

P(1 red) = 0.444 $\left(= \frac{4}{9}\right)$ [0.444, 0.445] ***A1 N2***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

(a.ii) Find the probability of rolling two or more red faces. [3]

Markscheme

valid approach ***(M1)***

eg P($X = 2$) + P($X = 3$), 1 − P($X \le 1$), binomcdf$\left(3,\ \frac{1}{3},\ 2,\ 3\right)$

correct working ***(A1)***

eg $\frac{2}{9} + \frac{1}{27}$, 0.222 + 0.037, $1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9}$

0.259259

P(at least two red) = 0.259 $\left(= \frac{7}{27}\right)$ ***A1 N3***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins $10 for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly $10 to his winnings is $\frac{1}{3}$. [5]

Markscheme

recognition that winning \$10 means rolling exactly one green ***(M1)***

recognition that winning \$10 also means rolling at most 1 red ***(M1)***

eg "cannot have 2 or more reds"

correct approach ***A1***

eg P(1G ∩ 0R) + P(1G ∩ 1R), P(1G) − P(1G ∩ 2R),

"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this ***A1*** for purely numerical expressions.

one correct probability for their approach ***(A1)***

eg $3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2,\ \frac{6}{27}, 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2,\ \frac{1}{9},\ \frac{2}{9}$

correct working leading to $\frac{1}{3}$ ***A1***

eg $\frac{3}{27}+\frac{6}{27},\ \frac{12}{27}-\frac{3}{27},\ \frac{1}{9}+\frac{2}{9}$

probability = $\frac{1}{3}$ ***AG N0***

[5 marks]

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where \$$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c.i) Write down the value of x. [1]

Markscheme

$x=\frac{7}{27}$, 0.259 (check ***FT*** from (a)(ii)) ***A1 N1***

[1 mark]

(c.ii) Hence, find the value of y. [2]

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $\sum = 1$, $x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1$, $1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$

0.148147 (0.148407 if working with **their** x value to 3 sf)

$y = \frac{4}{27}$ (exact), 0.148 ***A1 N2***

[2 marks]

(d) Ted will always have another turn if he expects an increase to his winnings.

Find the least value of w for which Ted should end the game instead of having another turn. [3]

Markscheme

correct substitution into the formula for expected value ***(A1)***

eg $-w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27}$

correct critical value (accept inequality) ***A1***

eg $w = 34.2857$ $\left(= \frac{240}{7}\right)$, $w > 34.2857$

$40 ***A1 N2***

[3 marks]