DRV - practice [75 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.13

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

2. [Maximum mark: 6] SPM.1.SL.TZ0.12

Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled −3, −1, 0, 1, 2 and 5.

The score for the game, X, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathrm{P}(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

(c) Jae Hee plays the game twice and adds the two scores together.

Find the probability Jae Hee has a **total** score of −3. [3]

3. [Maximum mark: 6] SPM.1.AHL.TZ0.17

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

In the first month, Mr Burke will teach his class 20 times.

(a) Find the probability he will choose a female student 8 times. [2]

(b) The Head of Year, Mrs Smith, decides to select a student at random from the year group to read the notices in assembly. There are 80 students in total in the year group. Mrs Smith calculates the probability of picking a male student 8 times in the first 20 assemblies is 0.153357 correct to 6 decimal places.

Find the number of male students in the year group. [4]

4. [Maximum mark: 5] 23M.1.SL.TZ2.12

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathbf{P}(X = x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in $\$$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain ($\$$)	-4	-3	-1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

5. [Maximum mark: 7] 22N.1.SL.TZ0.9

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

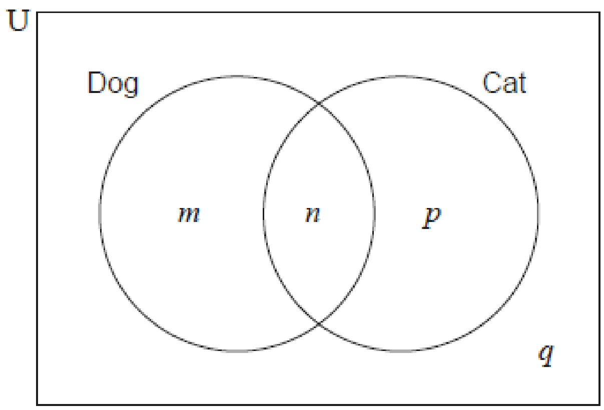
In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

6. [Maximum mark: 16] 22N.2.SL.TZ0.4

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



Find the value of

(a.i) m. [1]

(a.ii) n. [1]

(a.iii) p. [1]

(a.iv) q. [1]

(b) Find the percentage of students who have a dog or a cat or both. [1]

Find the probability that a randomly chosen student

(c.i) has a dog but does not have a cat. [1]

(c.ii) has a dog given that they do not have a cat. [2]

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events "being a school captain" and "having a dog" are independent.

Use Tim's model to find the probability that in the next 10 years

(d.i) 5 school captains have a dog. [2]

(d.ii) more than 3 school captains have a dog. [2]

(d.iii) exactly 9 school captains in succession have a dog. [3]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]

7. [Maximum mark: 7] 22M.1.SL.TZ2.5

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

8. [Maximum mark: 6] 20N.2.SL.TZ0.S_3

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\mathrm{P}(X=x)$	q	$4p^2$	p	$0.7-4p^2$

(a) Find an expression for q in terms of p. [2]

(b.i) Find the value of p which gives the largest value of $\mathrm{E}(X)$. [3]

(b.ii) Hence, find the largest value of $\mathrm{E}(X)$. [1]

9. [Maximum mark: 16] 19M.2.SL.TZ1.S_10

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a.i) Find the probability of rolling exactly one red face. [2]

(a.ii) Find the probability of rolling two or more red faces. [3]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins $10 for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly $10 to his winnings is $\frac{1}{3}$. [5]

The random variable D ($) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where $$w$ represents his winnings in the game so far.

D ($)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c.i) Write down the value of x. [1]

(c.ii) Hence, find the value of y. [2]

(d) Ted will always have another turn if he expects an increase to his winnings.

Find the least value of w for which Ted should end the game instead of having another turn. [3]