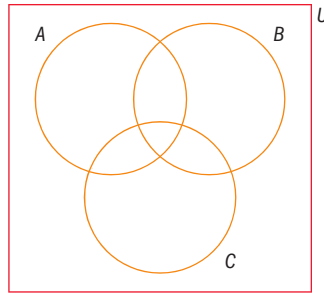


8.3 Extending to three sets

This Venn diagram shows a general three-set problem.



Use the same notation for three sets. But take great care using brackets to describe the sets.

Example 8

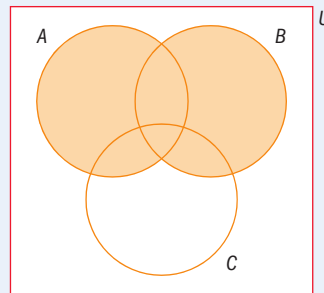
Shade the region on a Venn diagram that shows the sets:

a $(A \cup B) \cap C$

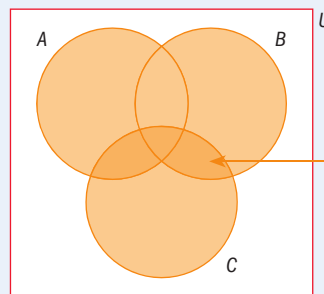
b $A \cup (B \cap C)$

Answers

First shade the region in the brackets $(A \cup B)$:

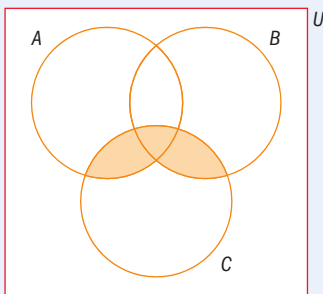


Then shade the other region, C :

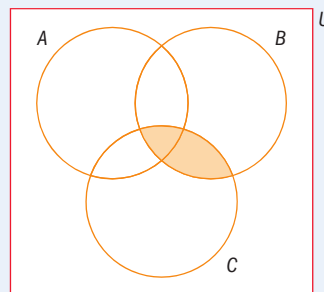


The dark region is the intersection

a $(A \cup B) \cap C$



First shade the region in the brackets $(B \cap C)$:

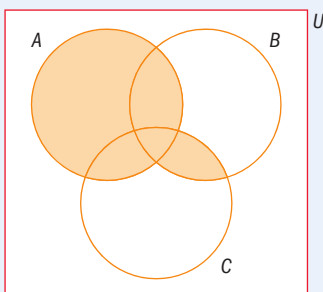


Then shade the other region A .

The union is all the shaded areas

Note that the statement $A \cup B \cap C$ has no mathematical meaning. The brackets are required to remove the ambiguity.

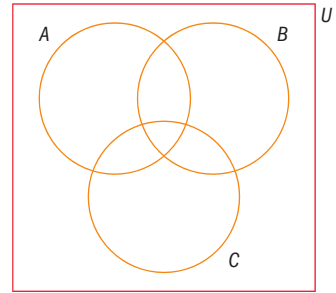
b $A \cup (B \cap C)$



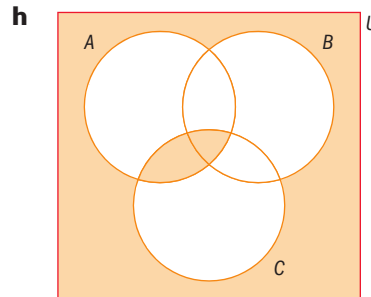
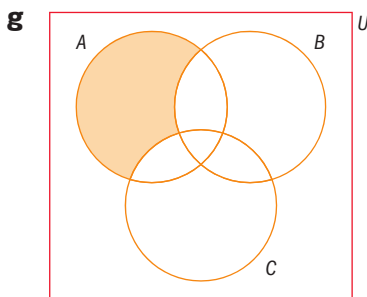
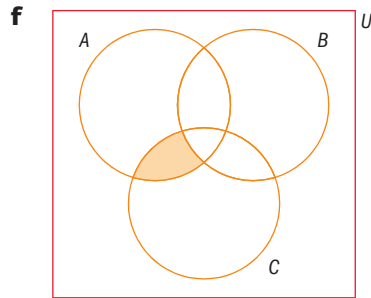
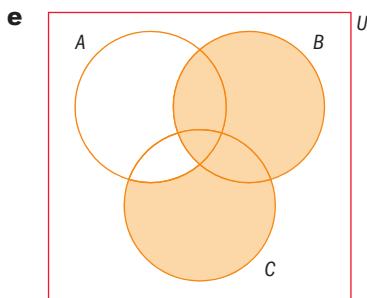
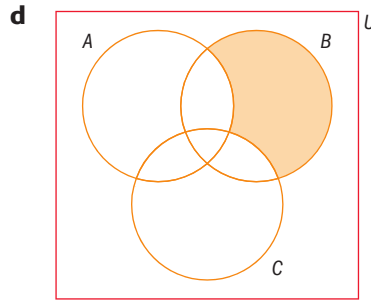
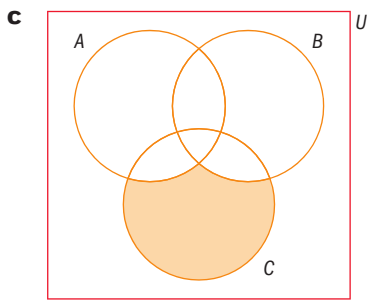
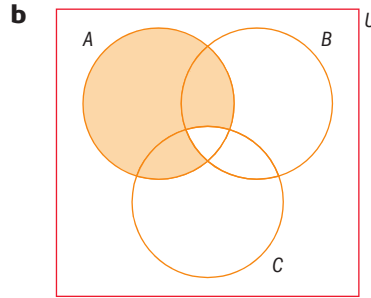
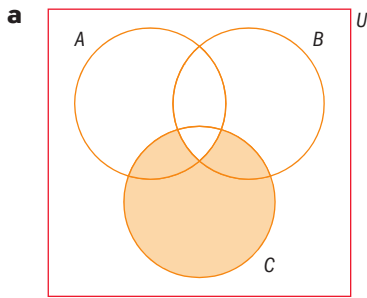
Exercise 8F

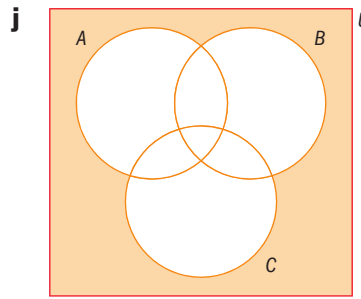
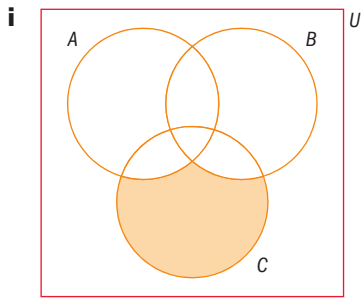
1 Shade the region on a three-set Venn diagram that shows each set:

- | | | | |
|------------|----------------------|-----------|----------------------|
| a i | $(A \cup B) \cup C$ | ii | $A \cup (B \cup C)$ |
| b i | $(A \cap B) \cap C$ | ii | $A \cap (B \cap C)$ |
| c i | $(A \cup C) \cap B$ | ii | $A \cup (C \cap B)$ |
| d i | $C \cap (A \cup B)$ | ii | $B \cup (C \cap A)$ |
| e i | $(A \cup B) \cup C'$ | ii | $A \cup (B \cup C')$ |
| f i | $(A \cap B') \cap C$ | ii | $A \cap (B' \cap C)$ |
| g i | $(A \cup C) \cap B'$ | ii | $A \cup (C \cap B')$ |



2 Use set notation to name the shaded region in each Venn diagram.

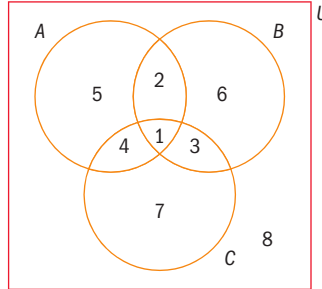




3 In this Venn diagram, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

List the elements of:

- a** $A \cap B \cap C$
- b** $A' \cap B \cap C$
- c** $A \cap B' \cap C$
- d** $A \cap B \cap C'$
- e** $A' \cap B' \cap C$
- f** $A' \cap B \cap C'$
- g** $A \cap B' \cap C'$
- h** $A' \cap B' \cap C'$



4 For the Venn diagram in question 3, list the elements of:

- a** $A \cap (B \cup C)$
- b** $A' \cap (B \cup C)$
- c** $(A \cup B') \cap C$
- d** $(A \cup B) \cap C'$
- e** $(A' \cup B') \cap C$
- f** $(A' \cup B) \cap C'$
- g** $B \cap (A' \cup C')$
- h** $B' \cap (A' \cup C)$

8.4 Problem-solving using Venn diagrams

Here is the problem from the first investigation in this chapter:

Investigation – a contradiction?

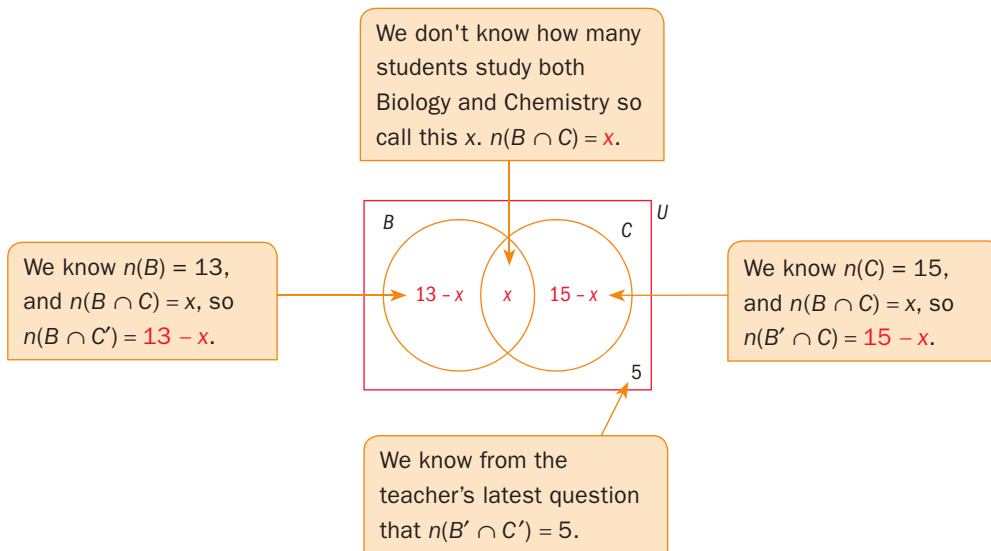
A teacher asks her class how many of them study Chemistry. She finds that there are 15. She then asks how many study Biology and finds that there are 13. Later, she remembers that there are 26 students in the class. But $15 + 13 = 28$. Has she miscounted?

We can represent this problem on a Venn diagram.

Let B be the set of students studying Biology, and C be the set of students studying Chemistry. Then $n(B) = 13$, $n(C) = 15$ and $n(U) = 26$.

The teacher asks another question and finds out that 5 of the students study neither Biology nor Chemistry, so $n(B' \cap C') = 5$.

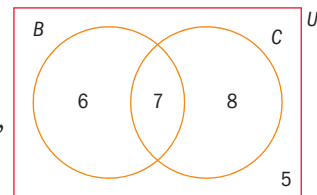
We can put what we know, and what we don't know, on a Venn diagram:



We also know that $n(U) = 26$. From the Venn diagram we can write

$$\begin{aligned} (13 - x) + x + (15 - x) + 5 &= 26 \\ 33 - x &= 26 \\ x &= 7 \end{aligned}$$

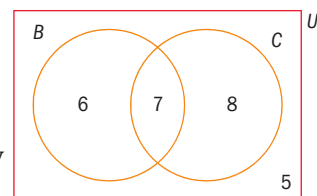
So now we can substitute for x on the Venn diagram, and answer questions like 'How many students study Chemistry but not Biology?'



Exercise 8G

Use the Venn diagram to answer these questions:

- 1 How many students study Biology **only**? (That is, 'Biology, but not Chemistry'.)
- 2 How many students study **exactly** one science? (That is, 'Biology or Chemistry, but not **both**'.)
- 3 How many students study **at least** one science? (That is, 'Biology or Chemistry, or **both**'.)
- 4 How many students study one science? (That is, 'Biology or Chemistry, or **both**'.)
- 5 How many students do not study Biology?
- 6 How many students do not study Chemistry?
- 7 How many Chemists study Biology?
- 8 How many Biologists do not study Chemistry?
- 9 How many science students do not study both Biology and Chemistry?



If you study two sciences, then you necessarily must study one!

Example 9

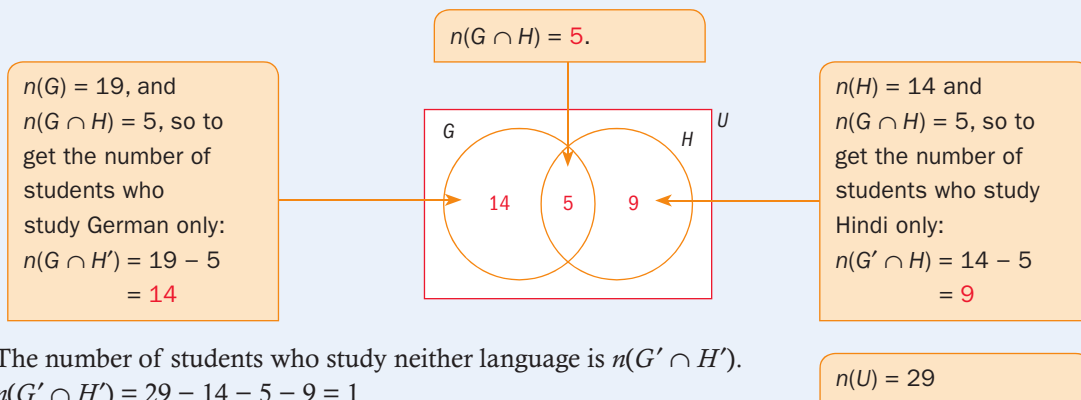
In a class of 29 students, 19 study German, 14 study Hindi and 5 study both languages. Work out the number of students that study neither language.

Answer

Let G be the set of students who study German, and H the set of those studying Hindi. From the information in the question

$$n(U) = 29 \quad n(G) = 19 \quad n(H) = 14 \quad n(G \cap H) = 5$$

Draw a Venn diagram:



The number of students who study neither language is $n(G' \cap H')$.

$$n(G' \cap H') = 29 - 14 - 5 - 9 = 1$$

1 student studies neither language.

Exercise 8H

- There are 25 students in a class. 17 study French, 12 study Malay, and 10 study both languages. Show this information on a Venn diagram. Find the number of students who:
 - study French only
 - study Malay or French or both
 - study neither subject
 - do not study both subjects.
- In a class 20 people take Geography, 17 take History, 10 take both subjects, and 1 person takes neither subject. Draw a Venn diagram to show this information. Find the number of students who:
 - are in the class
 - do not study History
 - study Geography but not History
 - study Geography or History but not both.
- Of the 32 students in a class, 18 play the violin, 16 play the piano, and 7 play neither. Find the number of students who:
 - play the violin but not the piano
 - do not play the violin
 - play the piano but not the violin
 - play the piano or the violin, but not both.

- 4 There are 30 students in a mathematics class. 20 of the students have studied probability, 14 have studied set theory, and two people have studied neither.
Find the number of students who:
- have studied both topics
 - have studied exactly one of these subjects
 - have studied set theory, but not probability.
- 5 There are 25 girls in a PE group. 13 have taken aerobics before and 17 have taken gymnastics. One girl has done neither before.
Find the number of girls who:
- have taken both activities
 - have taken gymnastics but not aerobics
 - have taken at least one of these activities.

You can use the same ideas to draw Venn diagrams with more sets; see the following example.

Example 10

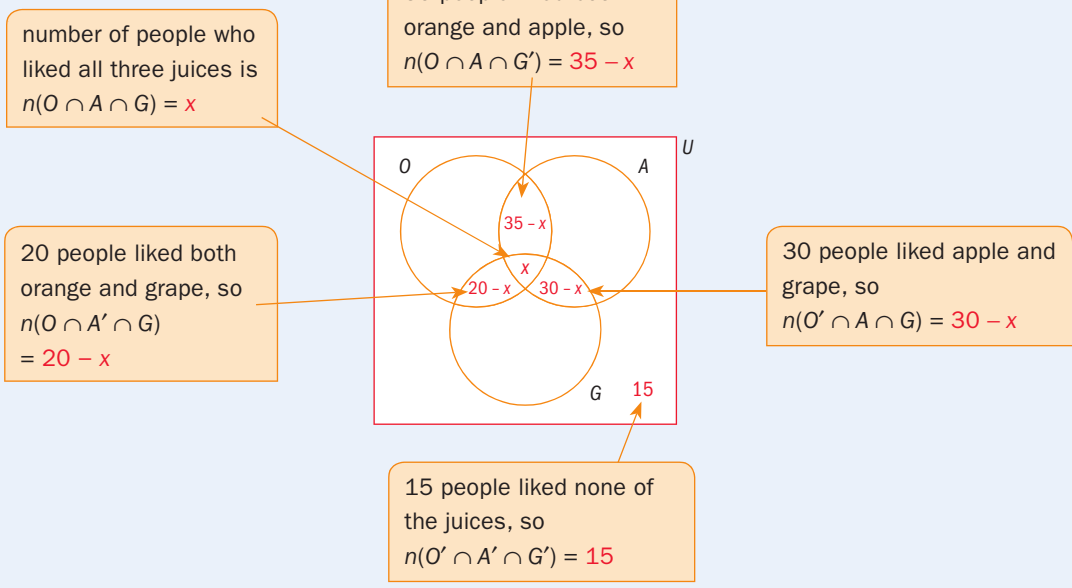
145 people answered a survey to find out which flavor of fruit juice, orange, apple or grape, they preferred.

The replies showed:

15 liked none of the three	35 liked orange and apple
55 liked grape	20 liked orange and grape
80 liked apple	30 liked apple and grape
75 liked orange	

Find the number of people who liked all three types of juice.

Answer



▶ Continued on next page

75 people liked orange, so $n(O \cap A' \cap G')$
 $= 75 - ((35 - x) + x + (20 - x)) = 20 + x$

80 people liked apple, so $n(O' \cap A \cap G')$
 $= 80 - ((35 - x) + x + 30 - x) = 15 + x$

55 people liked grape, so $n(O' \cap A' \cap G)$
 $= 55 - ((20 - x) + x + (30 - x)) = 5 + x$

To find x , use $n(U) = 145$
 $145 = n(O) + (15 + x) + (30 - x) + (5 + x) + 15$
 We know that $n(O) = 75$, so
 $145 = 75 + (15 + x) + (30 - x) + (5 + x) + 15$
 $145 = 140 + x$
 $x = 5$

There are a number of ways of combining the various parts of the Venn diagram to make U . They will all give the same answer.

In the IB examination you won't be asked to draw a Venn diagram with more than three sets.

Exercise 8I

Use the information from Example 10 to answer these questions.

- 1 Find the number in the survey above who
 - a liked exactly two of the three flavors of juice
 - b did not like orange juice
 - c liked one flavor of juice only
 - d did not like either orange or apple juice
 - e did not like orange juice and did not like apple juice
 - f liked at least two of the three flavors of juice
 - g liked fewer than two of the three flavors of juice.

Find the number of orange juice drinkers who

- h liked apple juice
- i did not like grape juice
- j liked no other flavors of juice
- k liked exactly one other flavor of juice.

- 2** In a group of 105 students, 70 students passed Mathematics, 60 students passed History and 45 students passed Geography; 30 students passed Mathematics and History, 35 students passed History and Geography, 25 passed Mathematics and Geography, and 15 passed all three subjects.

Draw a Venn diagram to illustrate this information.

Find the number of students who

- a** passed at least one subject
 - b** passed exactly two subjects
 - c** passed Geography and failed Mathematics
 - d** passed all three subjects given that they passed two
 - e** failed Mathematics given that they passed History.
- 3** In a youth camp, each participant must take part in at least one of the following activities: chess, backgammon or dominoes. Of the total of 55 in the camp, 25 participants participated in chess, 24 in backgammon, and 30 in dominoes; 15 in both chess and backgammon, 10 in both backgammon and dominoes, 5 in both chess and dominoes, and 2 in all three events.



Draw a Venn diagram to show this information.

How many of the participants are not taking part in at least one activity?

Find the number of participants who

- a** take part in one activity only
 - b** take part in exactly two activities
 - c** do not take part in at least two activities
 - d** take part in chess, given that they take part in dominoes
 - e** take part in backgammon, given that they do not take part in dominoes.
- 4** Fatty's Delight sells chicken, duck, and barbecued pork rice. Of the 160 customers one day, 57 had chicken rice, 60 had duck rice and 48 had barbecued pork rice. 30 customers ordered chicken and duck rice, 25 ordered duck and barbecued pork rice, 35 ordered chicken and barbecued pork rice, and 20 ordered all three types.

Draw a Venn diagram to show these data.

Find the number of customers who

- a** ordered more than one type of rice
- b** did not order a rice dish from Fatty's Delight
- c** did not order chicken rice
- d** ordered duck rice and one other rice dish.

- 5** In a community center in Buona Vista there are 170 youths. Of these, 65 take up climbing, 65 bouldering and 50 swimming; 15 take up climbing and bouldering, 10 bouldering and swimming, and 5 swimming and climbing. 17 youths take up other activities.

Let x be the number of youths who take up all three activities.

Show the above information in a Venn diagram.

Show clearly the number in each separate region in terms of x .

Form an equation satisfied by x , and hence find its value.

Find the number of youths who

- a** take up one activity only
 - b** take up at least two activities
 - c** take part in fewer than two activities
 - d** take up bouldering given that they have already taken up climbing
 - e** take up one other activity given that they have already taken up swimming.
- 6** 65 elderly men failed a medical test because of defects in at least one of these organs: the heart, lungs or kidneys. 29 had heart disease, 28 lung disease and 31 kidney disease. 8 of them had both lung and heart diseases, 11 had lung and kidney diseases, while 12 had kidney and heart diseases.

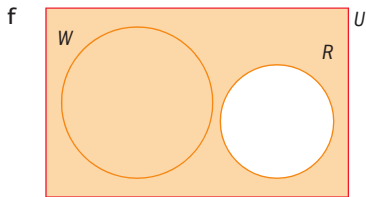
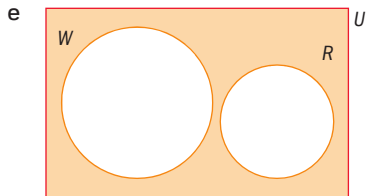
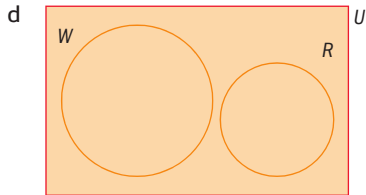
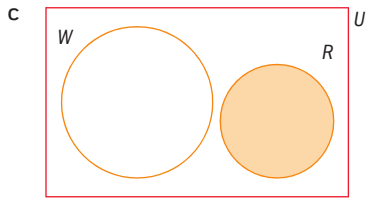
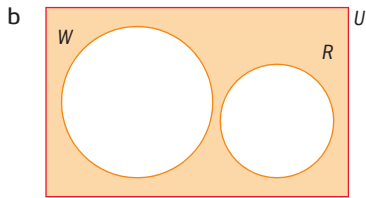
Draw a Venn diagram to show this information. You will need to introduce a variable.

Find the number of men who

- a** suffer from all three diseases
 - b** suffer from at least two diseases
 - c** suffer from lung disease and exactly one other disease
 - d** suffer from heart disease and lung disease but not kidney disease
 - e** suffer from lung disease only.
- 7** Each of the 116 students in the Fourth Year of a school studies at least one of the subjects History, English and Art.
- Of the 50 students who study Art,
- 15 also study History and English,
 - 12 study neither History nor English, and
 - 17 study English but not History.
- Of the 66 students who do not study Art,
- 39 study both History and English,
 - x study History only, and
 - $2x$ study English only.

Draw a Venn diagram showing the number of students in each subset. Hence find

- a** the value of x
- b** the total number of students studying English.

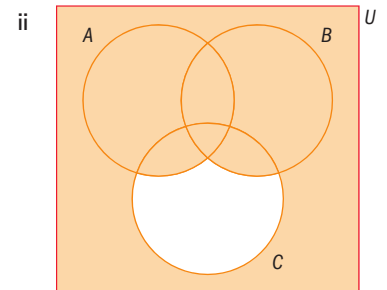
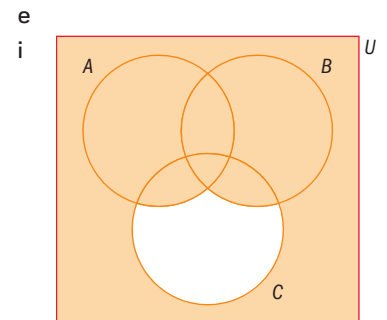
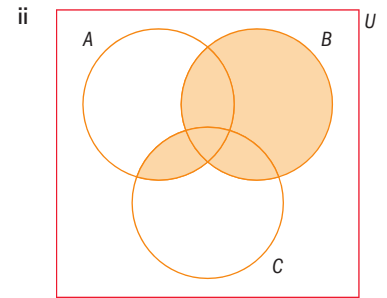
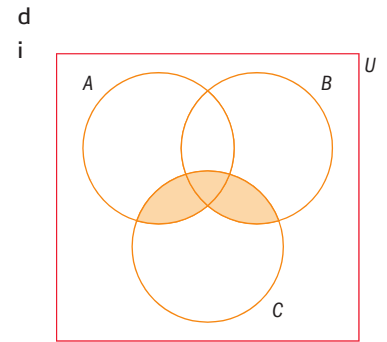
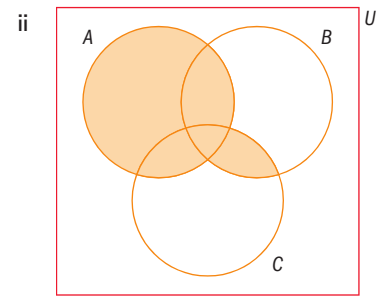
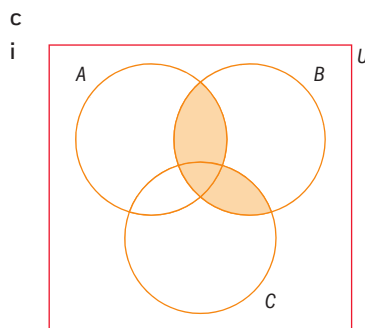
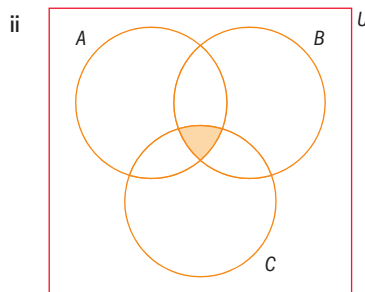
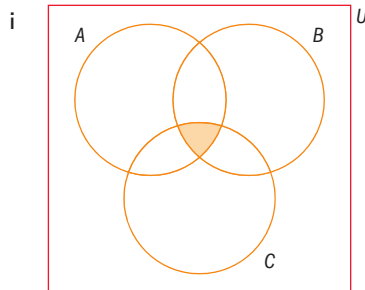
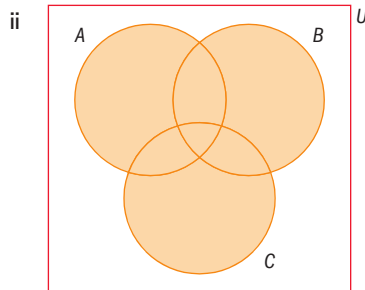
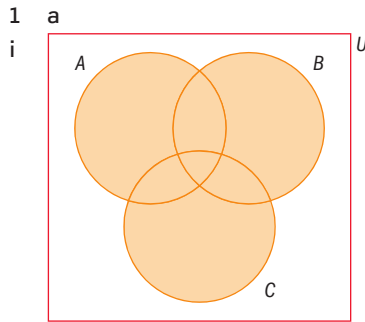


- 4 a {1, 2, 3, 4} b {1, 4}
 c \emptyset d \emptyset e {4}
 f {0, 1, 2, 3, 4, 5}
 g {0, 1, 2, 3, 4, 5}
 h {2, 4, 5, 6, 7} i {1, 2, 3, 4, 5}
 j {0, 1, 2, 3, 4, 5, 6, 7}
 k False l False
 m True n False o False

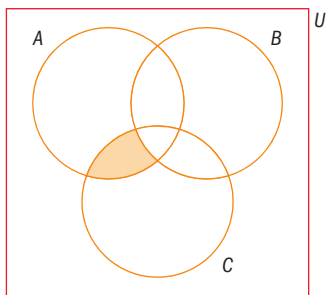
Exercise 8E

- 1 a False b True
 c False d False
 e False f False
 g False h False
 2 a b, c, d, e, f, g, h, k
 b b, d, e, f c c, g, h, k
 d c, d, e, k e b, f, g, h
 3 a q, t, x, w b p, r
 c p, q, r, t, x, w d q, x, w
 e p, q, r, x, w

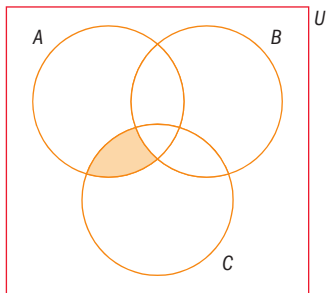
Exercise 8F



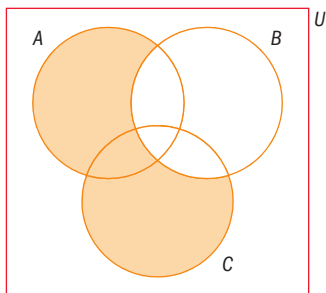
f
i



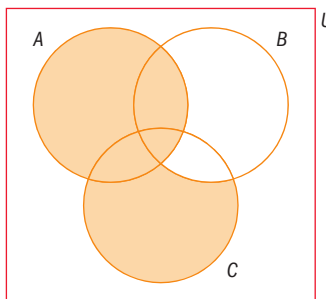
ii



g
i



ii



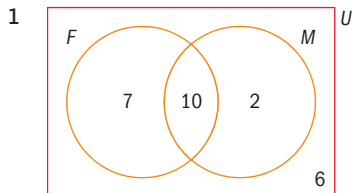
- 2 a $(A' \cup B') \cap C$
 b $A \cap (B' \cup C')$
 c $(A' \cap B') \cap C$
 d $A' \cap (B \cap C)$
 e $(A' \cap C) \cup B$
 f $A \cap (C' \cup B)'$
 g $A \cap (B \cup C)'$
 h $(A \cap C) \cup (A \cup (B \cup C))'$
 i $(A \cup B)' \cap C$
 j $A' \cap (B \cup C)'$

- 3 a 1 b 3 c 4
 d 2 e 7 f 6
 g 5 h 8
 4 a 1, 2, 4 b 3, 6, 7
 c 1, 4, 7 d 2, 5, 6
 e 3, 4, 7 f 2, 6, 8
 g 2, 3, 6 h 4, 7, 8

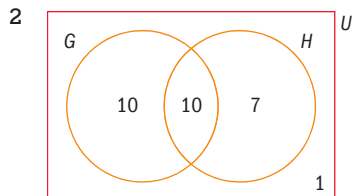
Exercise 8G

- 1 6 2 14 3 21
 4 21 5 13 6 11
 7 7 8 6 9 14

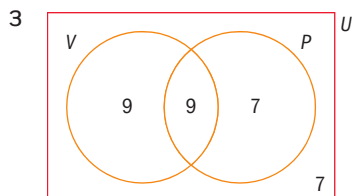
Exercise 8H



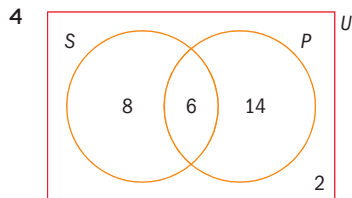
- a 7 b 19 c 6 d 15



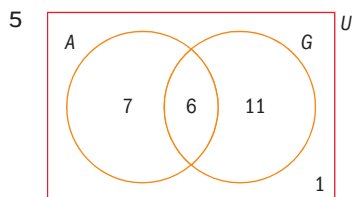
- a 28 b 11 c 10 d 17



- a 9 b 14 c 7 d 16



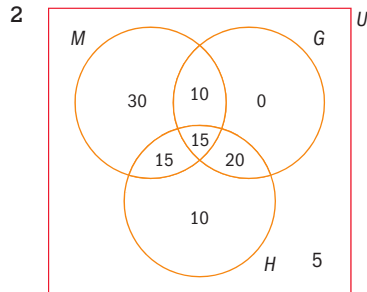
- a 6 b 22 c 8



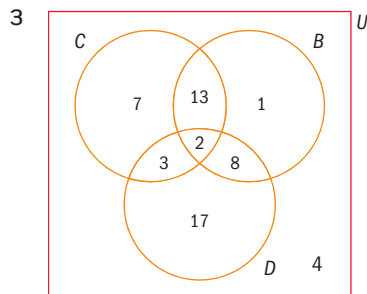
- a 6 b 11 c 24

Exercise 8I

- 1 a 70 b 70 c 55
 d 25 e 25 f 75
 g 70 h 35 i 55
 j 25 k 45

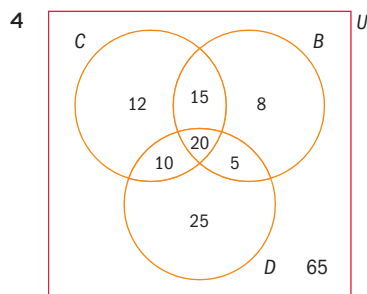


- a 100 b 45 c 20
 d 15 e 30

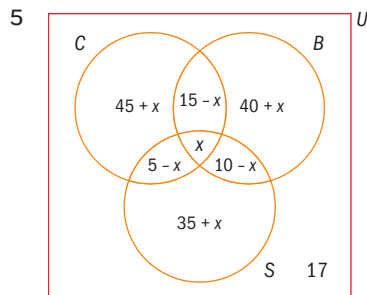


4 are not fulfilling their responsibilities.

- a 25 b 24 c 29
 d 5 e 14

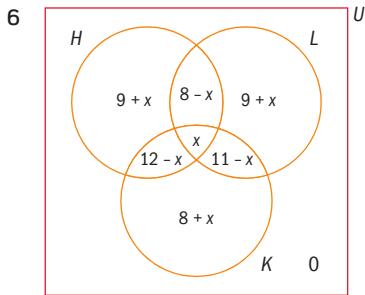


- a 50 b 65
 c 103 d 15

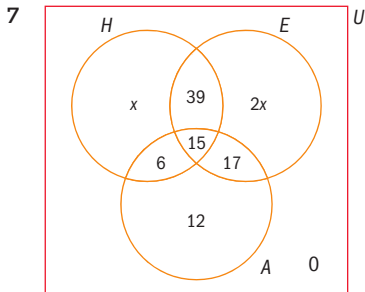


$x = 3$

- a 129 b 24 c 146
 d 15 e 9



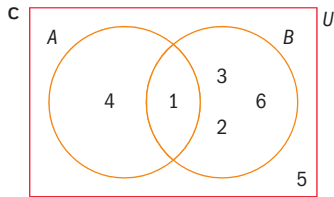
- a 8 b 15 c 3
d 0 e 17



- a $x=9$ b 89

Exercise 8J

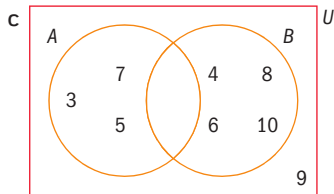
- 1 a {1, 4}
b {1, 2, 3, 6}



- d $\frac{2}{6}$ e $\frac{4}{6}$ f $\frac{4}{6}$

- g $\frac{1}{6}$ h $\frac{5}{6}$

- 2 a {3, 5, 7} b {4, 6, 8, 10}

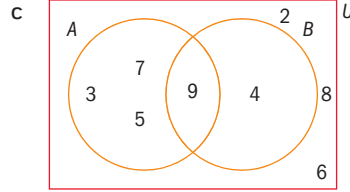


- d $\frac{3}{8}$ e $\frac{4}{8}$ f $\frac{5}{8}$

- g $\frac{4}{8}$ h 0 i $\frac{7}{8}$

- l $\frac{1}{8}$ m 1

- 3 a {3, 5, 7, 9}
b {4, 9}



- d $\frac{4}{8}$ e $\frac{2}{8}$ f $\frac{1}{8}$ g $\frac{5}{8}$

- 4 a {HH, HT, TH, TT}

- b $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$

- 5 a {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

- b $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

- 6 a $\frac{1}{16}$ b $\frac{1}{16}$ c $\frac{4}{16}$ d $\frac{4}{16}$

- e $1 - \left(\frac{1}{16} + \frac{4}{16} + \frac{4}{16} + \frac{1}{16} \right) = \frac{6}{16}$

- f {HHHH, HHHT, HHHT, HTHH, HHTT, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, THTT, TTHT, TTTH, TTTT}

Exercise 8K

- 1 a $\frac{23}{40}$ b $\frac{5}{40}$ c $\frac{5}{40}$

- d $\frac{15}{20}$ e $\frac{8}{23}$ f $\frac{8}{23}$

- 2 a $\frac{14}{30}$ b $\frac{8}{30}$ c $\frac{6}{10}$

- d $\frac{8}{20}$ e $\frac{4}{16}$ f 0

- 3 a $\frac{8}{17}$ b $\frac{2}{17}$ c $\frac{8}{17}$

- d $\frac{7}{9}$ e 0 f 1

- 4 a $\frac{12}{34}$ b $\frac{16}{34}$ c $\frac{28}{34}$

- d $\frac{12}{22}$ e $\frac{6}{18}$ f $\frac{10}{22}$

- 5 a $\frac{13}{24}$ b $\frac{4}{24}$ c $\frac{8}{24}$

- d $\frac{17}{24}$ e $\frac{7}{24}$ f $\frac{12}{24}$

- g $\frac{9}{24}$

- 6 a $\frac{5}{22}$ b $\frac{18}{22}$

- c $\frac{10}{15}$ d $\frac{3}{8}$

- 7 a $\frac{12}{28}$ b $\frac{4}{13}$ c $\frac{4}{16}$

- d $\frac{3}{28}$ e $\frac{12}{21}$

- 8 a $\frac{12}{27}$ b $\frac{12}{20}$ c $\frac{7}{19}$

- d $\frac{2}{7}$ e $\frac{12}{17}$

Exercise 8L

- 1 $A \cap B = \{1\}$

- 2 $A \cap B = \emptyset$, so A and B are mutually exclusive

- 3 $A \cap B = \{2\}$

- 4 $A \cap B = \emptyset$, so A and B are mutually exclusive

- 5 $A \cap B = \{9\}$

- 6 $A \cap B = \emptyset$, so A and B are mutually exclusive

- 7 $A \cap B = \{6\}$

- 8 $A \cap B = \emptyset$, so A and B are mutually exclusive

Exercise 8M

- 1 Not independent

- 2 Independent

- 3 Not independent

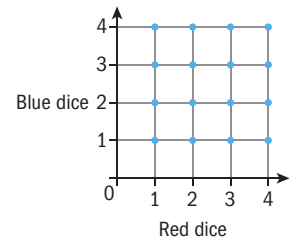
- 4 Independent events

- 5 Not independent events

- 6 Not independent events

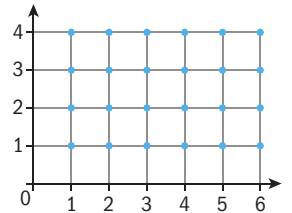
Exercise 8N

- 1



- a $\frac{3}{8}$ b $\frac{3}{8}$ c $\frac{1}{4}$ d $\frac{9}{16}$

- 2



- a $\frac{6}{24}$ b $\frac{13}{24}$ c $\frac{6}{24}$

- d $\frac{11}{24}$ e $\frac{4}{24}$