Name:
Result:

1. *(6 points)*

Consider complex numbers $z = 3 + i$ and $w = 2\text{cis}\frac{\pi}{5}$.

(a) Write z in polar form and w in Cartesian form. [2]

Consider another complex number $c = 1 - 2i$.

(b) Calculate $z \cdot c$, express your answer in Cartesian form. [1]

(c) Describe the transformation that takes place when c is multiplied by z. [2]

(d) Describe the transformation that takes place when when w is added to c. [1]

2. *(6 points)*

Three AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as:

$$V_1 = 20\sin(2t + 20^\circ), \qquad V_2 = 30\sin(2t + 40^\circ), \qquad V_3 = 40\sin(2t + 60^\circ)$$

The combined voltage can be expressed in the form:

$$V_{\text{total}} = A\sin(bt + \theta^\circ)$$

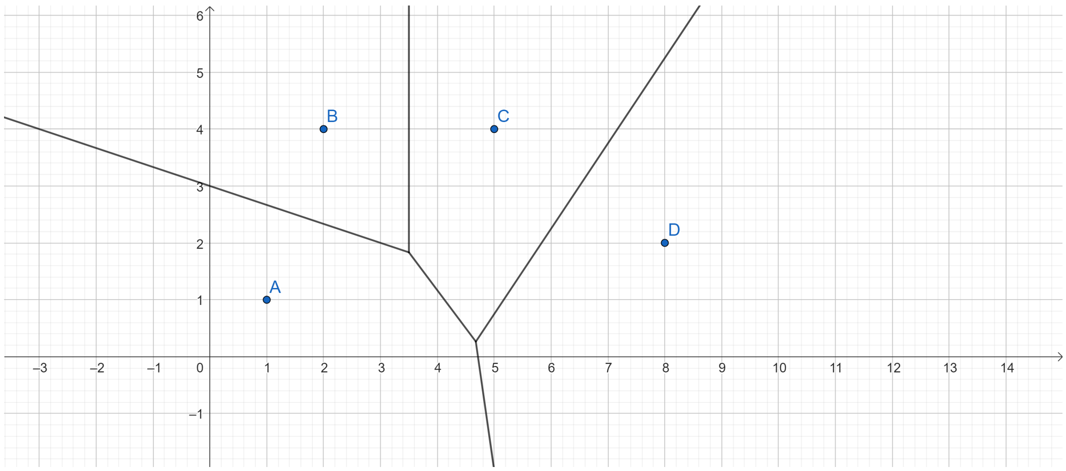
(a) State the value of b [1]

(b) Determine the values of A and θ. [4]

(c) State the maximum combined voltage. [1]

3. *(6 points)*

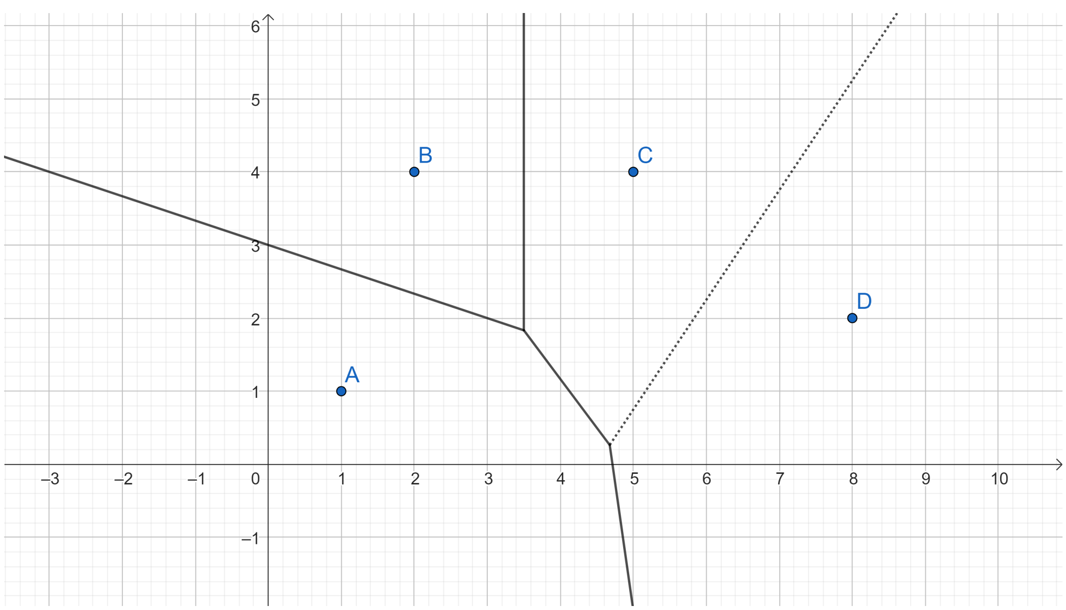
Consider the following Voronoi diagram for sites A, B, C and D.



(a) State the equation of the edge between B and C. [1]

(b) Find the equation of the edge between C and D, give your answer in the form $Ax + By + C = 0$, where $A, B, C \in \mathbb{Z}$. [2]

(c) A new site E is to be added at $(8, 4)$. Complete the Voronoi diagram with the new site added. [2]



(d) State the coordinates of the point, which is equidistant to sites C, D and E. [1]

4. *(7 points)*

Consider points $A(1,0,1), B(0,3,1)$ and $C(2,5,3)$.

(a) Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence find the area of the triangle ABC. [3]

(b) Point D is on the line segment AB such that AB is perpendicular to CD. Use your answer to part (a) to find the length of CD. [2]

(c) Find the angle that the triangle makes with the plane $z = 1$. [2]

5. *(17 points)*

A box contains 10 balls: 2 red, 3 blue and 5 green. A game consists of picking 3 balls (without replacement), for each blue ball picked the player wins 10 tokens. If however the player picks a red ball, all his winnings are lost.

(a) Find the probability that the player picks 3 blue balls. Express your answer as a fraction in simplest terms. [2]

(b) Show that the probability that the player picks 2 blue and 1 green ball is. $\frac{1}{8}$. [2]

The probability that the player picks 1 blue and 2 green balls is $\frac{1}{4}$. Let X be the winnings of the player.

(c) Complete the distribution table for X: [3]

x	0	10	20	30
$P(X = x)$				

(d) Calculate $E(X)$ [2]

(e) Hence state how much the game should cost for it to be fair. [1]

(f) If Tomasz plays the game 10 times, find the probability that: [4]

(i) he wins tokens on more than half of these games.

(ii) he wins tokens exactly 7 times, given that he won tokens on more than half of his games.

(g) Tomasz gets bored, if he doesn't win. He will continue playing until a third game in which he doesn't win occurs. Find the probability that he will play exactly 10 games. [3]

6. *(10 points)*

In this question, i denotes a unit vector due east, j denotes a unit vector due north.
The positions of two yachts *Amelia* and *Borubar* are given by the equations:

$$r_A = (2+t)i + (3+2t)j \qquad r_B = (-1+4t)i + 3tj$$

t is measured in hours since 7 am and distances are measured in kilometres.

(a) Find the speed of *Borubar*. [1]

(b) Find the distance between the two yachts at 9 am. [2]

(c) Find the shortest distance between the two yachts and the time at which it occurs. Give your answer to the nearest minute. [3]

At 10 am a speedboat sets off from *Amelia* and travels in a straight line. It will be reach *Borubar* at exactly 10:06 am.

(e) Find the velocity vector of the speedboat. [2]

(f) Find the bearing at which it travels. [2]

7. *(16 points)*

The number of cars passing through a certain toll gate on a motorway follows Poisson distribution with 7 cars passing every 5 minutes.

(a) Find the probability that more than 48 cars will pass through the gate between 8:00 and 8:30. [2]

(b) Find the probability that there will be more than 8 cars passing in at least four of the six 5-minute intervals between 8:00 and 8:30 (8:00 - 8:05, 8:05 - 8:10 etc.) [3]

Each car passing through the gate pays a toll of 20 PLN. The toll machine can process at most 4 payments per minute. Let Y be the total toll paid by all passing cars during 1 minute.

(c) Explain why Y does not follow Poisson distribution. [1]

(d) Calculate $P(Y = 20)$. [3]

(e) Show that $E(Y) > 21$ [4]

The number of cars passing through a different toll gate on another motorway also follows Poisson distribution. On average 9 cars pass every 10 minutes. Let Z be the total number of cars passing through the two gates in 20 minutes.

(f) Calculate the probability that Z is at most 40. State any assumptions that you've made in your calculations. [3]