

Name:

Mathematics preIB Examination

January 10, 2023

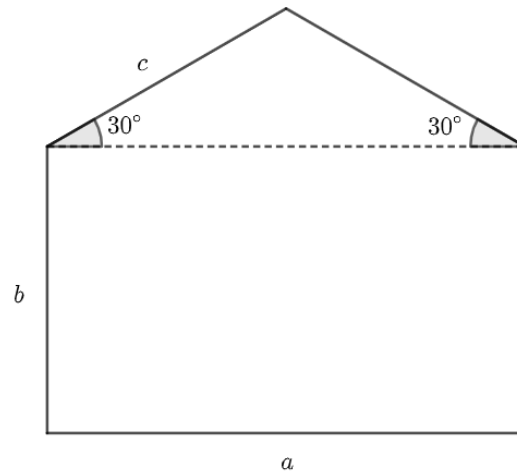
1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **required** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**48 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 10]

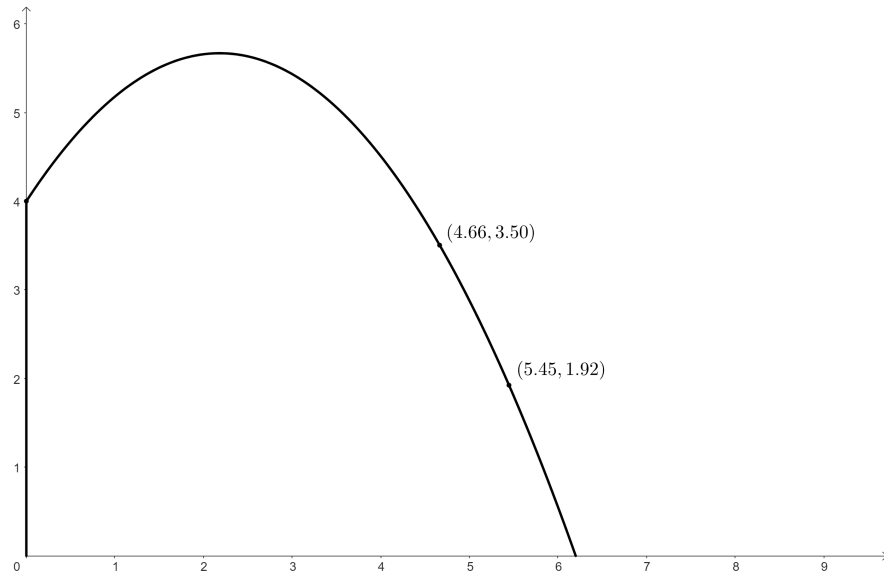
Consider a window in a shape of a rectangle with an isosceles triangle on top as shown on the diagram below:



- Express c in terms of a . [2]
- Hence express the perimeter of the window in terms a and b . [1]
- Assume that the height, b , is 2 metres and the perimeter of the window is $2\sqrt{3} + 7$ metres, find the value of a . [2]
- Let the perimeter be 10 metres. Find the value of a , which maximizes the area of the window. [5]

2. [Maximum mark: 10]

A tunnel has a cross section which consists of vertical line segment of length 4 and part of a parabola, as shown on the diagram below:



All distances are measured in metres.

- a) Use the points on the graph of the parabola to find its equation in the form $y = ax^2 + bx + c$. [3]
- b) Find the maximum height of the tunnel. [2]
- c) Find the maximum width of the tunnel. [2]
- d) Determine if a container which is 7 metres long, 5 metres wide and 3 metres high can fit through the tunnel. Justify your answer. [3]

3. [Maximum mark: 10]

According to the producer the temperature inside a freezer is given by the formula:

$$T(t) = -19 - 0.7 \sin\left(\frac{\pi}{10}(t - 6)\right)$$

where T is measured in $^{\circ}C$ and t is measured in minutes since the thermostat is switched on.

a) State the period of T . [2]

b) Find the maximum and minimum temperature in the freezer. [2]

c) Calculate how long, according to the producer, the temperature in the fridge will exceed $-18.5^{\circ}C$ during the first 30 minutes since switching on the thermostat. [3]

Tomasz made several measurements of the temperature inside the freezer. The results are displayed below:

t	0	1	3	4	5	8	11	15	16	20	21	25
T	-18.3	-18.3	-18.4	-18.6	-18.8	-19.4	-19.7	-19.3	-19.1	-18.3	-18.2	-18.8

d) Use technology and the above measurements to find a *sine* function $T(t)$. [2]

e) Use your result from part (d) to estimate the temperature in the fridge 10 minutes after switching the thermostat on. [1]

4. [Maximum mark: 10]

Tomasz starts at point A and walks 3.2 km on a bearing of 035 to reach point B . He then walks another 1.4 km on a bearing of 153 to reach points C .

a) Find the direct distance from C to A . [3]

b) Find the bearing of A from C . [3]

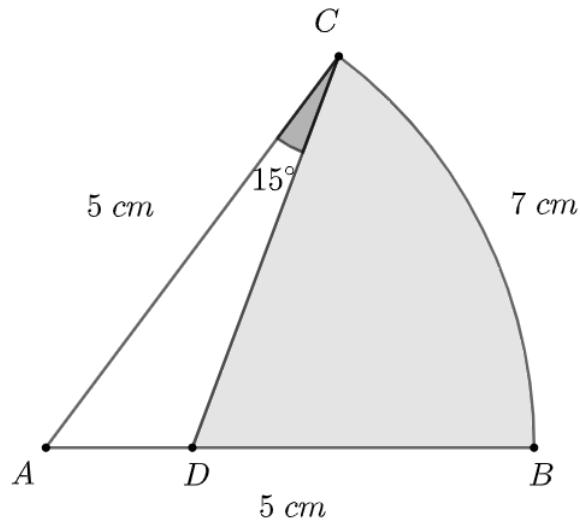
c) Find the area of the triangle ABC . [2]

A 125-metre tower is located at point C .

d) Find the angle of elevation from point A to the top the tower. [2]

5. [Maximum mark: 8]

Consider the following diagram:



ABC is a circular sector with $AB = AC = 5\text{ cm}$ and the length of the arc $BC = 7\text{ cm}$. Point D is on AB such that $\angle ACD = 15^\circ$ as shown.

- a) Find size of $\angle BAC$. [2]
- b) Find length of AD . [3]
- c) Find the area of the shaded region. [3]