1. Given ΔABC, with lengths shown in the diagram below, find the length of the line segment [CD].

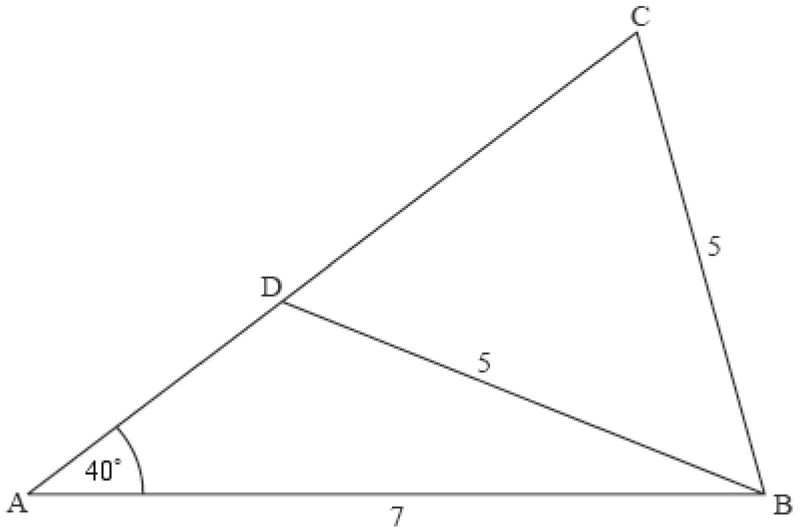


diagram not to scale

(Total 5 marks)

METHOD 1

$\frac{\sin C}{7} = \frac{\sin 40}{5}$ M1(A1)

$B\hat{C}D = 64.14...°$ A1

$CD = 2 \times 5\cos 64.14...$ M1

Note: Also allow use of sine or cosine rule.

$CD = 4.36$ A1

METHOD 2

let $AC = x$

cosine rule

$5^2 = 7^2 + x^2 - 2 \times 7 \times x \cos 40$ M1A1

$x^2 - 10.7...x + 24 = 0$

$x = \frac{10.7... \pm \sqrt{(10.7...)^2 - 4 \times 24}}{2}$ (M1)

$x = 7.54;\ 3.18$ (A1)

CD is the difference in these two values = 4.36 A1

Note: Other methods may be seen.

[5]

2. The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right), 0 \le t \le 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(3)

(b) Find the values of t for which $h(t) \ge 8$.

(3)
(Total 6 marks)

(a) Either finding depths graphically, using $\sin\frac{\pi t}{6} = \pm 1$

or solving $h'(t) = 0$ for t (M1)

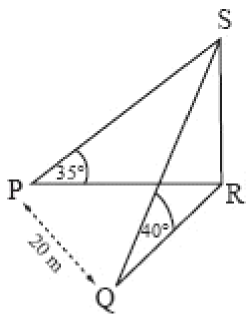
$h(t)_{max} = 12$ (m), $h(t)_{min} = 4$ (m) A1A1 N3

(b) Attempting to solve $8 + 4\sin\frac{\pi t}{6} = 8$ algebraically or graphically (M1)

$t \in [0, 6] \cup [12, 18] \cup \{24\}$ A1A1 N3

[6]

3.



The above three dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively 35° and 40°, and PQ = 20 m.

Determine the height of the flagpole.

(Total 8 marks)

PR = $h\tan 55°$, QR = $h\tan 50°$ where RS = h M1A1A1
Use the cosine rule in triangle PQR. (M1)
$20^2 = h^2\tan^2 55° + h^2\tan^2 50° - 2h\tan 55°\, h\tan 50° \cos 45°$ A1

$h^2 = \frac{400}{\tan^2 55° + \tan^2 50° - 2\tan 55° \tan 50° \cos 45°}$ (A1)

$= 379.9...$ (A1)
$h = 19.5$ (m) A1

[8]

4. The radius of the circle with centre C is 7 cm and the radius of the circle with centre D is 5 cm. If the length of the chord [AB] is 9 cm, find the area of the shaded region enclosed by the two arcs AB.

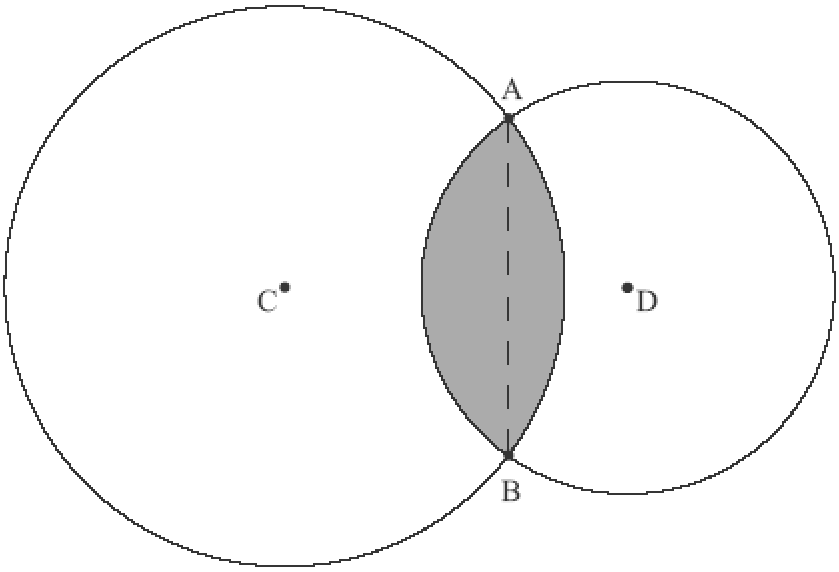


diagram not to scale

(Total 7 marks)

$\alpha = 2 \arcsin\left(\frac{4.5}{7}\right) (\Rightarrow \alpha = 1.396... = 80.010° ...)$ M1(A1)

$\beta = 2 \arcsin\left(\frac{4.5}{5}\right) (\Rightarrow \beta = 2.239... = 128.31°...)$ (A1)

Note: Allow use of cosine rule.

area $P = \frac{1}{2} \times 7^2 \times (\alpha - \sin\alpha) = 10.08...$ M1(A1)

area $Q = \frac{1}{2} \times 5^2 \times (\beta - \sin\beta) = 18.18...$ (A1)

Note: The M1 is for an attempt at area of sector minus area of triangle.

Note: The use of degrees correctly converted is acceptable.

area = 28.3(cm^2) A1

[7]

5. The points P and Q lie on a circle, with centre O and radius 8 cm, such that $P\hat{O}Q = 59°$.

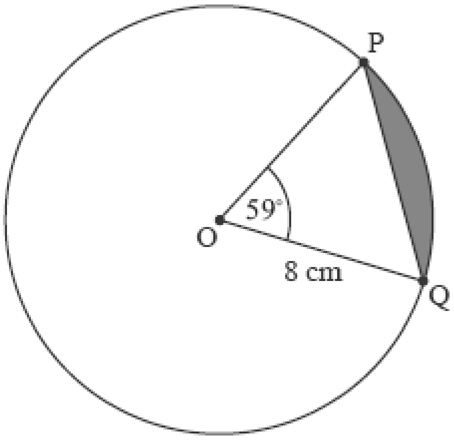


diagram not to scale

Find the area of the shaded segment of the circle contained between the arc PQ and the chord [PQ].

(Total 5 marks)

area of triangle POQ = $\frac{1}{2}8^2 \sin 59°$ M1

= 27.43 (A1)

area of sector = $\pi 8^2 \frac{59}{360}$ M1

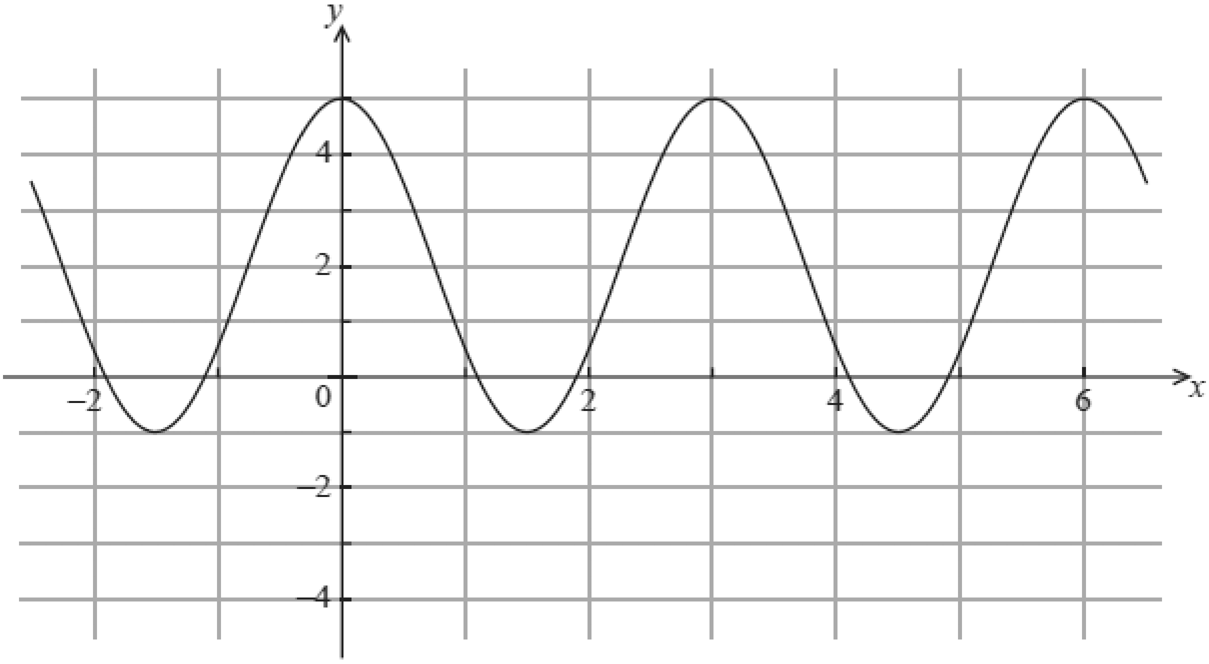
= 32.95 (A1)

area between arc and chord = 32.95 – 27.43

= 5.52 (cm^2) A1

[5]

6. The graph below shows $y = a\cos(bx) + c$.



Find the value of a, the value of b and the value of c.

(Total 4 marks)

$a = 3$ A1

$c = 2$ A1

$\text{period} = \frac{2\pi}{b} = 3$ (M1)

$b = \frac{2\pi}{3}$ $(= 2.09)$ A1

[4]

7. The vertices of an equilateral triangle, with perimeter P and area A, lie on a circle with radius r. Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^{+}$.

(Total 6 marks)

let the length of one side of the triangle be x
consider the triangle consisting of a side of the triangle and two radii

EITHER

$x^2 = r^2 + r^2 - 2r^2 \cos 120°$ M1
$= 3r^2$

OR

$x = 2r \cos 30°$ M1

THEN

$x = r\sqrt{3}$ A1

so perimeter $= 3\sqrt{3}\,r$ A1
now consider the area of the triangle

area $= 3 \times \frac{1}{2} r^2 \sin 120°$ M1

$= 3 \times \frac{\sqrt{3}}{4} r^2$ A1

$\frac{P}{A} = \frac{3\sqrt{3}r}{\frac{3\sqrt{3}}{4} r^2}$

$= \frac{4}{r}$ A1

Note: Accept alternative methods

[6]

8. In the right circular cone below, O is the centre of the base which has radius 6 cm. The points B and C are on the circumference of the base of the cone. The height AO of the cone is 8 cm and the angle $\hat{BOC}$ is 60°.

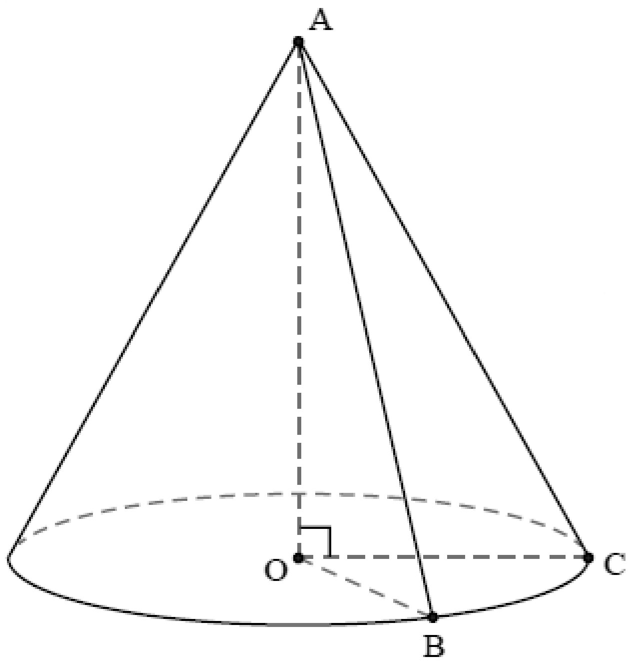


diagram not to scale

Calculate the size of the angle $\hat{BAC}$.

(Total 6 marks)

AC = AB = 10 (cm) A1
triangle OBC is equilateral (M1)
BC = 6 (cm) A1

EITHER

$\hat{BAC} = 2\arcsin\frac{3}{10}$ M1A1

$\hat{BAC} = 34.9°$ (accept 0.609 radians) A1

OR

$\cos\hat{BAC} = \frac{10^2 + 10^2 - 6^2}{2 \times 10 \times 10} = \frac{164}{200}$ M1A1

$\hat{BAC} = 34.9°$ (accept 0.609 radians) A1

Note: Other valid methods may be seen.

[6]

9. Consider the triangle ABC where $B\hat{A}C = 70°$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that $\frac{BD}{DC} = 2$.

Determine the length of AD.

(Total 6 marks)

10. Triangle ABC has AB = 5cm, BC = 6 cm and area 10 cm^2.

(a) Find $\sin \hat{B}$.

(2)

(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

(4)

(Total 6 marks)

(a) $\text{area} = \frac{1}{2} \times \text{BC} \times \text{AB} \times \sin B$ (M1)

$\left(10 = \frac{1}{2} \times 5 \times 6 \times \sin B\right)$

$\sin \hat{B} = \frac{2}{3}$ A1

(b) $\cos B = \pm\frac{\sqrt{5}}{3}$ $(= \pm 0.7453\ldots)$ or $B = 41.8\ldots$ and $138.1\ldots$ (A1)

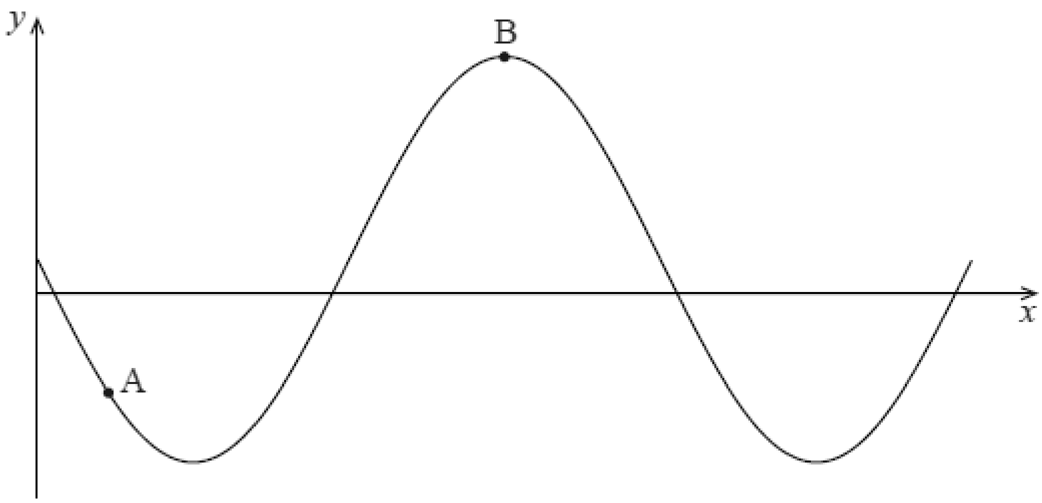
$\text{AC}^2 = \text{BC}^2 + \text{AB}^2 - 2 \times \text{BC} \times \text{AB} \times \cos B$ (M1)

$\text{AC} = \sqrt{5^2 + 6^2 - 2 \times 5 \times 6 \times 0.7453\ldots}$ or $\sqrt{5^2 + 6^2 + 2 \times 5 \times 6 \times 0.7453\ldots}$

AC = 4.03 or 10.28 A1A1

[6]

11. The diagram below shows a curve with equation $y = 1 + k \sin x$, defined for $0 \le x \le 3\pi$.



The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and B(a, b) is the maximum point.

(a) Show that $k = -6$.

(2)

(b) Hence, find the values of a and b.

(3)
(Total 5 marks)

(a) $-2 = 1 + k\sin\left(\frac{\pi}{6}\right)$ M1

$-3 = \frac{1}{2}k$ A1

$k = -6$ AG N0

(b) **METHOD 1**

maximum $\Rightarrow \sin x = -1$ M1

$a = \frac{3\pi}{2}$ A1

$b = 1 - 6(-1)$
$= 7$ A1 N2

12. The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius R and the inner circle has radius r.

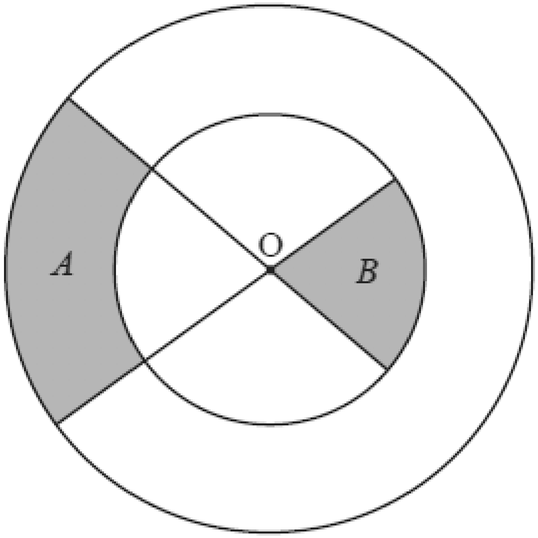


diagram not to scale

Consider the shaded regions with areas A and B. Given that $A : B = 2 : 1$, find the **exact** value of the ratio $R : r$.

(Total 5 marks)

$A = \frac{\theta}{2}(R^2 - r^2)$	A1	
$B = \frac{\theta}{2}r^2$	A1	
from $A: B = 2:1$, we have $R^2 - r^2 = 2r^2$	M1	
$R = \sqrt{3}r$	(A1)	
hence exact value of the ratio $R : r$ is $\sqrt{3}$:1	A1	N0

[5]

13. A triangle has sides of length $(n^2 + n + 1)$, $(2n + 1)$ and $(n^2 - 1)$ where $n > 1$.

(a) Explain why the side $(n^2 + n + 1)$ must be the longest side of the triangle.

(3)

(b) Show that the largest angle, θ, of the triangle is 120°.

(5)
(Total 8 marks)

(a) a reasonable attempt to show either that $n^2 + n + 1 > 2n + 1$ or $n^2 + n + 1 > n^2 - 1$ M1

complete solution to each inequality A1A1

(b) $\cos\theta = \frac{(2n+1)^2 + (n^2-1)^2 - (n^2+n+1)^2}{2(2n+1)(n^2-1)}$ M1A1

$= \frac{-2n^3 - n^2 + 2n + 1}{2(2n+1)(n^2-1)}$ M1

$= -\frac{(n-1)(n+1)(2n+1)}{2(2n+1)(n^2-1)}$ A1

$= -\frac{1}{2}$ A1

$\theta = 120°$ AG

[8]

14. Consider triangle ABC with $B\hat{A}C = 37.8°$, AB = 8.75 and BC = 6.

Find AC.

(Total 7 marks)

$\frac{\sin B}{6.5} = \frac{\sin 35°}{4}$ M1

$\hat{B} = 68.8°$ or $111°$ A1A1

$\hat{C} = 76.2°$ or $33.8°$ (accept 34°) A1

$\frac{\text{AB}}{\sin\text{C}} = \frac{\text{BC}}{\sin A}$

$\frac{\text{AB}}{\sin 76.2°} = \frac{4}{\sin 35°}$ (M1)

AB = 6.77 cm

$\frac{\text{AB}}{\sin 33.8°} = \frac{4}{\sin 35°}$ A1

AB = 3.88cm (accept 3.90) A1

[7]

15. In a triangle ABC, $\hat{A} = 35°$, BC = 4 cm and AC = 6.5 cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

(Total 7 marks)

$\frac{\sin B}{6.5} = \frac{\sin 35°}{4}$ M1

$\hat{B} = 68.8°$ or $111°$ A1A1

$\hat{C} = 76.2°$ or $33.8°$ (accept 34°) A1

$\frac{\text{AB}}{\sin \text{C}} = \frac{\text{BC}}{\sin A}$

$\frac{\text{AB}}{\sin 76.2°} = \frac{4}{\sin 35°}$ (M1)

AB = 6.77 cm

$\frac{\text{AB}}{\sin 33.8°} = \frac{4}{\sin 35°}$ A1

AB = 3.88cm (accept 3.90) A1

[7]

16. The lengths of the sides of a triangle ABC are $x - 2$, x and $x + 2$. The largest angle is 120°.

(a) Find the value of x.

(6)

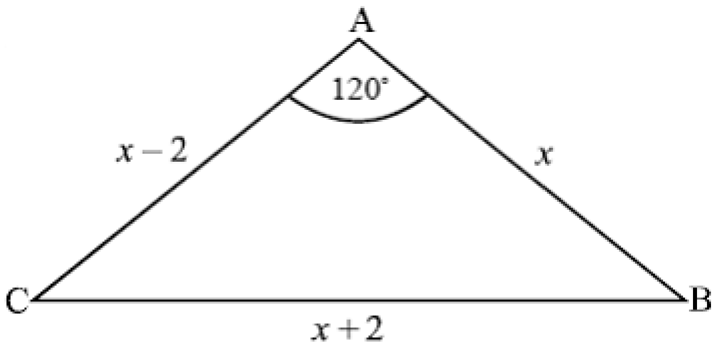
(b) Show that the area of the triangle is $\frac{15\sqrt{3}}{4}$.

(3)

(c) Find $\sin A + \sin B + \sin C$ giving your answer in the form $\frac{p\sqrt{q}}{r}$ where $p, q, r \in \mathbb{Z}$.

(4)
(Total 13 marks)

(a)



	(M1)
$(x+2)^2 = (x-2)^2 + x^2 - 2(x-2)\,x\cos 120°$	M1A1
$x^2 + 4x + 4 = x^2 - 4x + 4 + x^2 + x^2 - 2x$	(M1)
$0 = 2x^2 - 10x$	A1
$0 = x(x-5)$	
$x = 5$	A1

(b) Area $= \frac{1}{2} \times 5 \times 3 \times \sin 120°$ M1A1

$= \frac{1}{2} \times 15 \times \frac{\sqrt{3}}{2}$ A1

$= \frac{15\sqrt{3}}{4}$ AG

(c) $\sin A = \frac{\sqrt{3}}{2}$

$\frac{15\sqrt{3}}{4} = \frac{1}{2} \times 5 \times 7 \times \sin B \Rightarrow \sin B = \frac{3\sqrt{3}}{14}$ M1A1

Similarly $\sin C = \frac{5\sqrt{3}}{14}$ A1

$\sin A + \sin B + \sin C = \frac{15\sqrt{3}}{14}$ A1

[13]

17. A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is 60°.

(a) Calculate the length of the third side of the field.

(3)

(b) Find the area of the field in the form $p\sqrt{3}$, where p is an integer.

(3)

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length x metres.

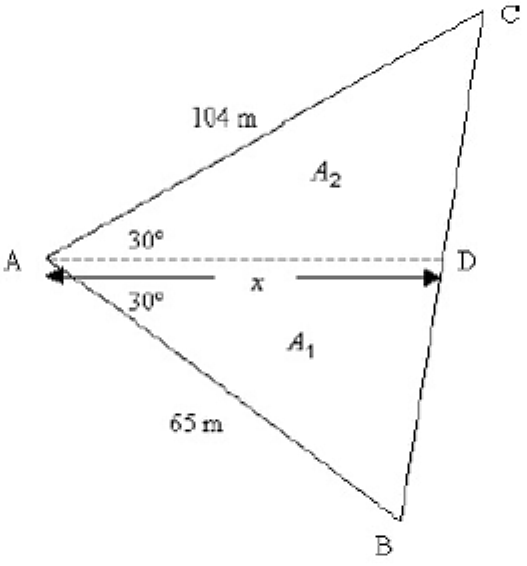
(c) (i) Show that the area o the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

(ii) Hence, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(8)

(d) Prove that $\frac{\text{BD}}{\text{DC}} = \frac{5}{8}$.

(6)
(Total 20 marks)



(a)		Using the cosine rule ($a^2 = b^2 + c^2 - 2bc\cos A$)	(M1)	
		Substituting correctly		
		$BC^2 = 65^2 + 104^2 - 2\,(65)\,(104)\cos 60°$	A1	
		$= 4225 + 10\,816 - 6760 = 8281$		
		$\Rightarrow BC = 91\text{m}$	A1	N2
(b)		Finding the area using $= \frac{1}{2} bc \sin A$	(M1)	
		Substituting correctly, area $= \frac{1}{2}(65)\,(104)\sin 60°$	A1	
		$= 1690\sqrt{3}$ (accept $p = 1690$)	A1	N2
(c)	(i)	Smaller area $A_1 = \left(\frac{1}{2}\right)(65)\,(x)\sin 30°$	(M1)A1	
		$= \frac{65x}{4}$	AG	N0
		Larger area $A_2 = \left(\frac{1}{2}\right)(104)\,(x)\sin 30°$	M1	
		$= 26x$	A1	N1

(ii) Using $A_1 + A_2 = A$ (M1)

Substituting $\frac{65x}{4} + 26x = 1690\sqrt{3}$ A1

Simplifying $\frac{169x}{4} = 1690\sqrt{3}$ A1

Solving $x = \frac{4 \times 1690\sqrt{3}}{169}$

$\Rightarrow x = 40\sqrt{3}$ (accept $q = 40$) A1 N1

(d) Using sin rule in ΔADB and ΔACD (M1)

Substituting correctly $\frac{\text{BD}}{\sin 30°} = \frac{65}{\sin \text{A}\hat{\text{D}}\text{B}} \Rightarrow \frac{\text{BD}}{65} = \frac{\sin 30°}{\sin \text{A}\hat{\text{D}}\text{B}}$ A1

and $\frac{\text{DC}}{\sin 30°} = \frac{104}{\sin \text{A}\hat{\text{D}}\text{C}} \Rightarrow \frac{\text{DC}}{104} = \frac{\sin 30°}{\sin \text{A}\hat{\text{D}}\text{C}}$ A1

Since $\text{A}\hat{\text{D}}\text{B} + \text{A}\hat{\text{D}}\text{C} = 180°$ R1

It follows that $\sin \text{A}\hat{\text{D}}\text{B} = \sin \text{A}\hat{\text{D}}\text{C}$ R1

$\frac{\text{BD}}{65} = \frac{\text{DC}}{104} \Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{65}{104}$ A1

$\Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{5}{8}$ AG N0

[20]