1. Given ΔABC, with lengths shown in the diagram below, find the length of the line segment [CD].

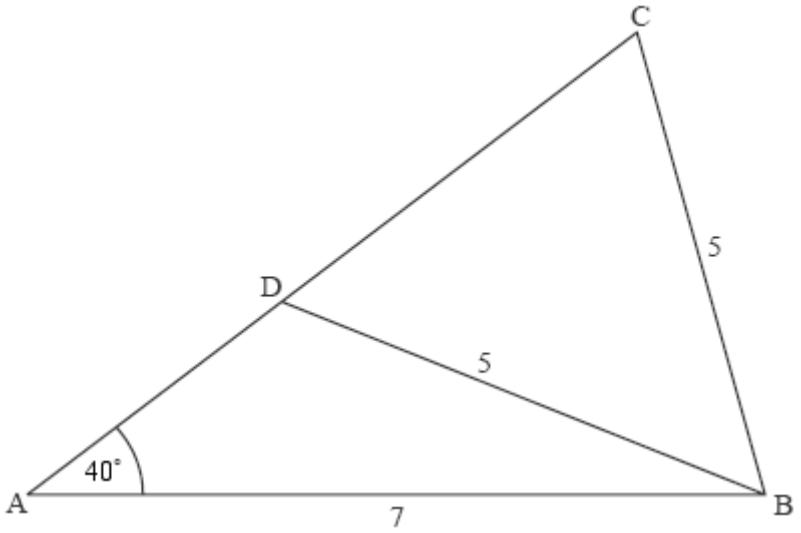


diagram not to scale

(Total 5 marks)

2. The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right), 0 \le t \le 24.$$

(a) Find the maximum depth and the minimum depth of the water.

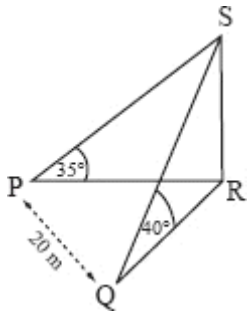
(3)

(b) Find the values of t for which $h(t) \ge 8$.

(3)

(Total 6 marks)

3.



The above three dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively 35° and 40°, and PQ = 20 m.

Determine the height of the flagpole.

(Total 8 marks)

4. The radius of the circle with centre C is 7 cm and the radius of the circle with centre D is 5 cm. If the length of the chord [AB] is 9 cm, find the area of the shaded region enclosed by the two arcs AB.

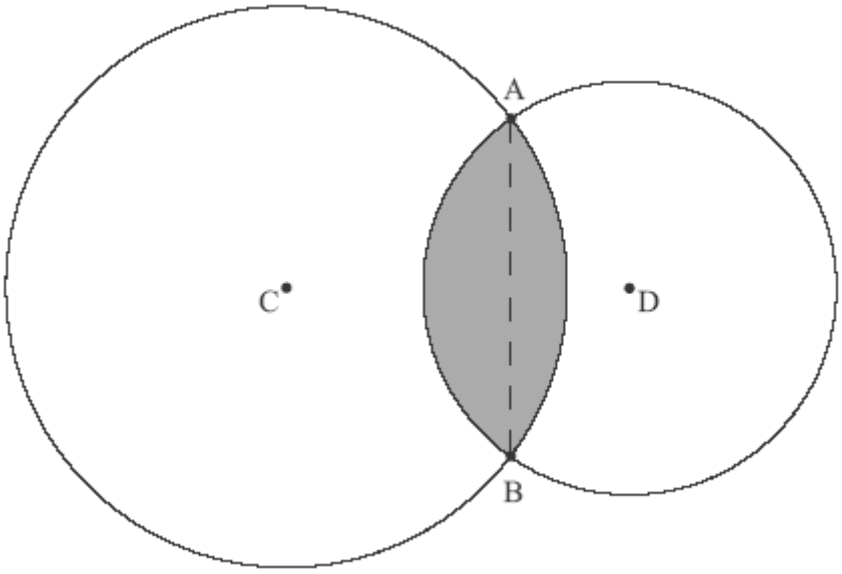


diagram not to scale

(Total 7 marks)

5. The points P and Q lie on a circle, with centre O and radius 8 cm, such that $\hat{POQ} = 59°$.

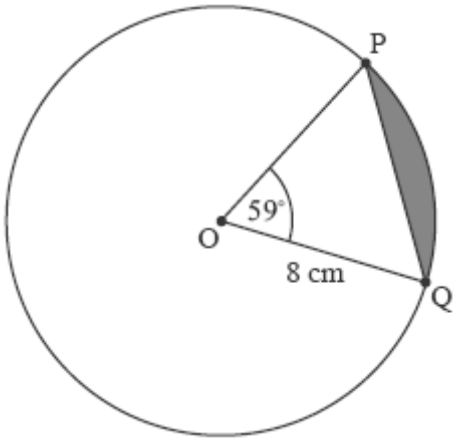
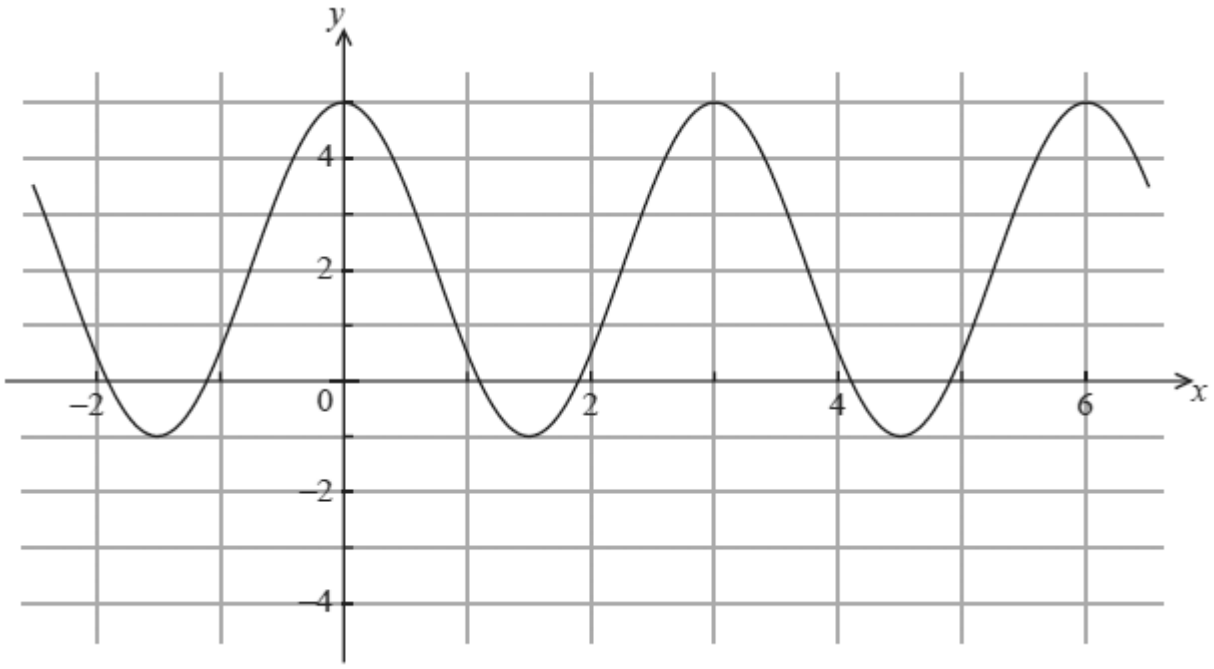


diagram not to scale

Find the area of the shaded segment of the circle contained between the arc PQ and the chord [PQ].

(Total 5 marks)

6. The graph below shows $y = a\cos(bx) + c$.



Find the value of a, the value of b and the value of c.

(Total 4 marks)

7. The vertices of an equilateral triangle, with perimeter P and area A, lie on a circle with radius r.

Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^+$.

(Total 6 marks)

8. In the right circular cone below, O is the centre of the base which has radius 6 cm.
The points B and C are on the circumference of the base of the cone. The height AO of the cone is 8 cm and the angle $B\hat{O}C$ is 60°.

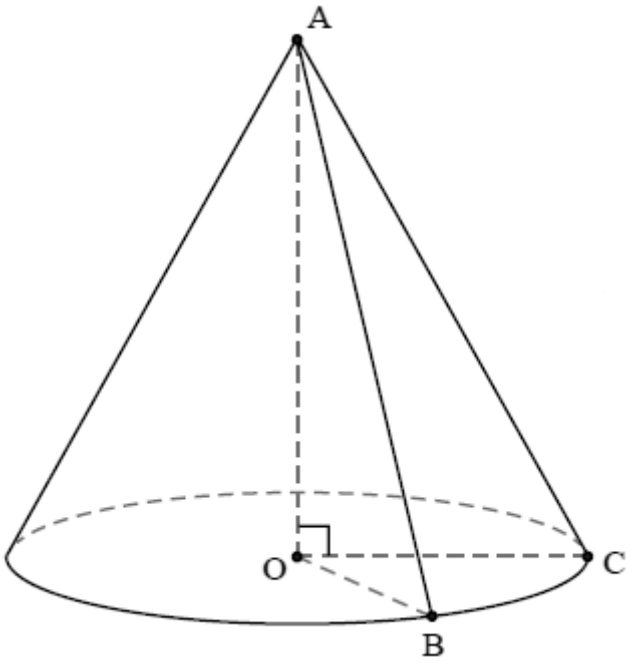


diagram not to scale

Calculate the size of the angle $B\hat{A}C$.

(Total 6 marks)

9. Consider the triangle ABC where $B\hat{A}C = 70°$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that $\frac{BD}{DC} = 2$.
Determine the length of AD.

(Total 6 marks)

10. Triangle ABC has AB = 5cm, BC = 6 cm and area 10 cm^2.

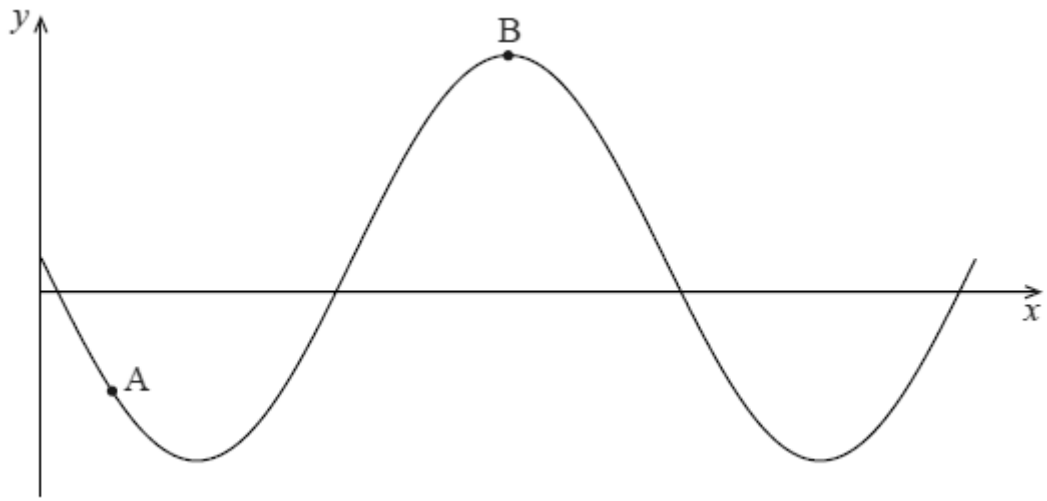
(a) Find $\sin \hat{B}$.

(2)

(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

(4)
(Total 6 marks)

11. The diagram below shows a curve with equation $y = 1 + k \sin x$, defined for $0 \leq x \leq 3\pi$.



The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and B(a, b) is the maximum point.

(a) Show that $k = -6$.

(2)

(b) Hence, find the values of a and b.

(3)

(Total 5 marks)

12. The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius R and the inner circle has radius r.

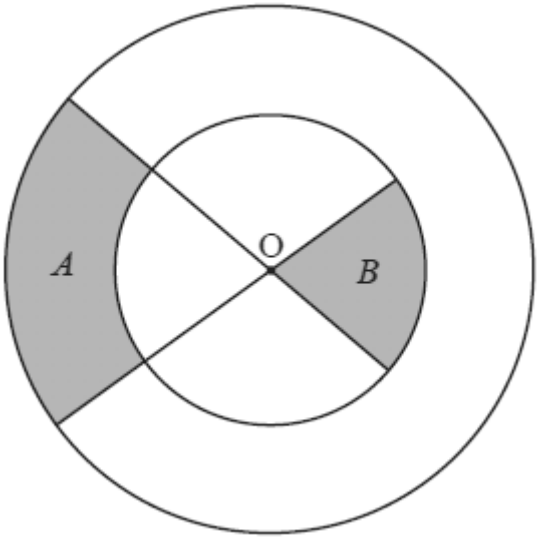


diagram not to scale

Consider the shaded regions with areas A and B. Given that $A : B = 2 : 1$, find the **exact** value of the ratio $R : r$.

(Total 5 marks)

13. A triangle has sides of length $(n^2 + n + 1)$, $(2n + 1)$ and $(n^2 - 1)$ where $n > 1$.

(a) Explain why the side $(n^2 + n + 1)$ must be the longest side of the triangle.

(3)

(b) Show that the largest angle, θ, of the triangle is 120°.

(5)

(Total 8 marks)

14. Consider triangle ABC with $\text{B}\hat{\text{A}}\text{C} = 37.8°$, AB = 8.75 and BC = 6.

Find AC.

(Total 7 marks)

15. In a triangle ABC, $\hat{A} = 35°$, BC = 4 cm and AC = 6.5 cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

(Total 7 marks)

16. The lengths of the sides of a triangle ABC are $x - 2$, x and $x + 2$. The largest angle is 120°.

(a) Find the value of x.

(6)

(b) Show that the area of the triangle is $\frac{15\sqrt{3}}{4}$.

(3)

(c) Find $\sin A + \sin B + \sin C$ giving your answer in the form $\frac{p\sqrt{q}}{r}$ where $p, q, r \in \mathbb{Z}$.

(4)

(Total 13 marks)

17. A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is 60°.

(a) Calculate the length of the third side of the field.

(3)

(b) Find the area of the field in the form $p\sqrt{3}$, where p is an integer.

(3)

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length x metres.

(c) (i) Show that the area o the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

(ii) Hence, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(8)

(d) Prove that $\frac{\text{BD}}{\text{DC}} = \frac{5}{8}$.

(6)
(Total 20 marks)