Logs - models [36 marks]

1. [Maximum mark: 4] SPM.1.SL.TZ0.8

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.

$$L = 10\log_{10}\left(S \times 10^{12}\right), S \geq 0.$$

(a) An orchestra has a sound intensity of 6.4×10^{-3} W m^{-2}. Calculate the intensity level, L of the orchestra. [2]

Markscheme

$10\log_{10}\left(6.4 \times 10^{-3} \times 10^{12}\right)$ ***(M1)***

= 98.1(dB) (98.06179…) ***A1***

[2 marks]

(b) A rock concert has an intensity level of 112 dB. Find the sound intensity, S. [2]

Markscheme

$112 = 10\log_{10}\left(S \times 10^{12}\right)$ ***(M1)***

0.158 (W m^{-2}) (0.158489… (W m^{-2})) ***A1***

[2 marks]

2. [Maximum mark: 6] EXN.1.SL.TZ0.5

The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

(a) Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$pH = -\log_{10}(5.2 \times 10^{-8}) = 7.29\ (7.28399\ldots)$ **(M1)A1**

[2 marks]

(b.i) Write an expression for C in terms of pH. [2]

Markscheme

$C = 10^{-pH}$ **(M1)A1**

Note: Award **M1** for an exponential equation with 10 as the base.

[2 marks]

(b.ii) Find the hydrogen ion concentration in a solution with $pH\ 4.2$. Give your answer in the form $a \times 10k$ where $1 \leq a < 10$ and k is an integer. [2]

Markscheme

$C = 10^{-4.2} = 6.30957\ldots \times 10^{-5}$ **(M1)**

6.31×10^{-5} **A1**

[2 marks]

3. [Maximum mark: 7] 23M.1.SL.TZ1.8

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

Markscheme

attempt to substitute 5000 for G ***(M1)***

$0.301p = \log_{10} 5000$

$(p =)\ 12.3$ (bits) $(12.2889\ldots)$ ***A1***

[2 marks]

(b) Write down G as a function of p. [1]

Markscheme

$(G =) 10^{0.301p}$ **OR** 2^p ***A1***

[1 mark]

(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$. [3]

Markscheme

attempt to substitute 28 for p in given equation or $G(p)$ ***(M1)***

$0.301 \times 28 = \log_{10} G$ **OR** $(G =)\ 10^{0.301 \times 28}$

$(G =)\ 2.68 \times 10^8\ (2.67916\ldots \times 10^8)$ ***A1A1***

Note: Award ***A1*** for 2.68, ***A1*** for 10^8. Award ***M1A1A0*** for a correct final answer not written in scientific notation or written incorrectly in scientific notation (e.g., $268\,000\,000$ or 26.8×10^7 or $2.68\text{E}08$).

[3 marks]

There is a point on the graph of the function $G(p)$ with coordinates $(0,\ 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

Markscheme

If a password has an **entropy of** 0 (bits), then the password can be **guessed in one try** / then the **password is known** ***R1***

Note: Reference must be made to both entropy and number of guesses/password known for ***R1*** to be awarded.

Do not accept "no password" as this contradicts the context.

[1 mark]

4. [Maximum mark: 6] 22N.1.SL.TZ0.10

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Markscheme

$m = 1 - 2.5 \log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ \ (4.19960\ldots)$ ***A1***

[2 marks]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

Markscheme

attempt to solve $7 = 1 - 2.5 \log_{10}(b)$ ***(M1)***

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5 \log_{10}(b)$.

$b = 0.00398 \ \ (0.00398107\ldots)$ ***A1***

[2 marks]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

Markscheme

$\frac{0.0525}{0.00398107}$ ***(M1)***

$= 13.2 \ (13.1874\ldots)$ ***A1***

[2 marks]

5. [Maximum mark: 8] 22M.1.SL.TZ1.11

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

Markscheme

$\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

Markscheme

EITHER

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ \left(= 10^5\right)$ ***A1***

[2 marks]

(c) Given $0 < M < 8$, find the range for N. [2]

Markscheme

$0.001 < N < 100000 \ \left(10^{-3} < N < 10^5\right)$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

[2 marks]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year.

[2]

Markscheme

$N = \frac{10^5}{10^{7.2}} \quad (= 0.0063095\ldots)$ ***(M1)***

length of time $= \frac{1}{0.0063095\ldots} = 10^{2.2}$

$= 158$ years ***A1***

[2 marks]

6. [Maximum mark: 5] 22M.1.SL.TZ2.4

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee. [2]

Markscheme

$(\text{pH} =) \ -\log_{10}\left(1.3 \times 10^{-5}\right)$ ***(M1)***

$4.89 \ \ (4.88605\ldots)$ ***A1***

[2 marks]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. [3]

Markscheme

EITHER

calculating pH

$(\text{pH} =) \ -\log_{10}\left(10 \times 1.3 \times 10^{-5}\right)$ ***(M1)***

$3.89 \ \ (3.88605\ldots)$ ***A1***

($3.89 < 4.89$, therefore) the unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0A0A0***.

OR

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of x increases, the value of y decreases. ***M1***

Since the C-value (x-value) of the unknown liquid is larger than that of the coffee, the pH level (y-value) is lower. ***R1***

The unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0R0A0***.

[3 marks]