Logs - models [36 marks]

1. [Maximum mark: 4] SPM.1.SL.TZ0.8

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.

$$L = 10\log_{10}\left(S \times 10^{12}\right), S \geq 0.$$

(a) An orchestra has a sound intensity of 6.4×10^{-3} W m^{-2}. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of 112 dB. Find the sound intensity, S. [2]

2. [Maximum mark: 6] EXN.1.SL.TZ0.5

The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

(a) Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}. [2]

(b.i) Write an expression for C in terms of pH. [2]

(b.ii) Find the hydrogen ion concentration in a solution with pH 4.2. Give your answer in the form $a \times 10k$ where $1 \leq a < 10$ and k is an integer. [2]

3. [Maximum mark: 7] 23M.1.SL.TZ1.8

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

(b) Write down G as a function of p. [1]

(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0,\ 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

4. [Maximum mark: 6] 22N.1.SL.TZ0.10

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

5. [Maximum mark: 8] 22M.1.SL.TZ1.11

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

(c) Given $0 < M < 8$, find the range for N. [2]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. [2]

6. [Maximum mark: 5] 22M.1.SL.TZ2.4

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee. [2]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. [3]