Quadratic modelling - exam questions [43 marks]

1. [Maximum mark: 8] 23M.1.SL.TZ2.10

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5,\ t \geq 0,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

2. [Maximum mark: 5] 22M.1.SL.TZ1.3

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

(c) State an appropriate domain for t in this model. [2]

3. [Maximum mark: 17] SPM.2.SL.TZ0.5

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, $d \text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of 6 m s^{-1}, Model A can be represented by the equation $6p + q = 2$.

(a.i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1}. [2]

(a.ii) Find the values of p and q. [2]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

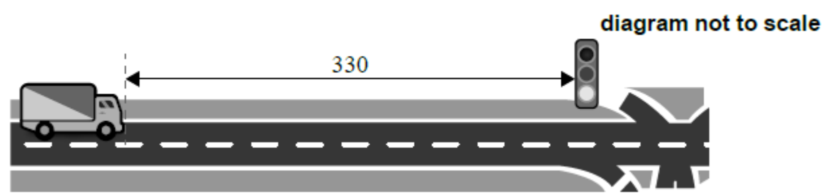
(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \mathrm{m\,s^{-1}}$. [2]

The actual braking distance at $20\ \mathrm{m\,s^{-1}}$ is $320\ \mathrm{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

(g) It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ \mathrm{m\,s^{-1}}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\ \mathrm{m}$.



Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]

4. [Maximum mark: 13] EXM.2.SL.TZ0.3

Urvashi wants to model the height of a moving object. She collects the following data showing the height, h metres, of the object at time t seconds.

t (seconds)	2	5	7
h (metres)	34	38	24

She believes the height can be modeled by a quadratic function, $h\left(t\right)=at^2+bt+c$, where $a,\ b,\ c\in\mathbb{R}$.

(a) Show that $4a+2b+c=34$. [1]

(b) Write down two more equations for a, b and c. [3]

(c) Solve this system of three equations to find the value of a, b and c. [4]

Hence find

(d.i) when the height of the object is zero. [3]

(d.ii) the maximum height of the object. [2]