Tangent & normal lines [45 marks]

1. [Maximum mark: 6] 18M.1.SL.TZ2.T_14

Consider the function $f\left(x\right) = \frac{x^4}{4}$.

(a) Find $f'(x)$ [1]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

x^3 ***(A1) (C1)***

Note: Award ***(A0)*** for $\frac{4x^3}{4}$ and not simplified to x^3.

[1 mark]

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$. [2]

Markscheme

$\left(-\frac{1}{2}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $-\frac{1}{2}$ into their derivative.

$-\frac{1}{8}$ (-0.125) ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (a).

[2 marks]

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$. [3]

Markscheme

$x^3 = 8$ ***(A1)(M1)***

Note: Award ***(A1)*** for 8 seen maybe seen as part of an equation $y = 8x + c$, ***(M1)*** for equating their derivative to 8.

$(x =)$ 2 ***(A1) (C3)***

Note: Do not accept (2, 4).

[3 marks]

2. [Maximum mark: 7] 19M.2.SL.TZ2.T_5

Consider the function $f\left(x\right) = \frac{1}{3}x^3 + \frac{3}{4}x^2 - x - 1$.

(d) Find $f'\left(x\right)$. [3]

Markscheme

$x^2 + \frac{3}{2}x - 1$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award at most ***(A1)(A1)(A0)*** if there are extra terms.

[3 marks]

(e) Find the gradient of the graph of $y = f\left(x\right)$ at $x = 2$. [2]

Markscheme

$2^2 + \frac{3}{2} \times 2 - 1$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 2 in their derivative of the function.

6 ***(A1)*(ft)*(G2)***

Note: Follow through from part (d).

[2 marks]

(f) Find the equation of the tangent line to the graph of $y = f\left(x\right)$ at $x = 2$. Give the equation in the form $ax + by + d = 0$ where, a, b, and $d \in \mathbb{Z}$. [2]

Markscheme

$\frac{8}{3} = 6\left(2\right) + c$ ***(M1)***

Note: Award ***(M1)*** for 2, their part (a) and their part (e) substituted into equation of a straight line.

$c = -\frac{28}{3}$

OR

$\left(y - \frac{8}{3}\right) = 6\left(x - 2\right)$ ***(M1)***

Note: Award ***(M1)*** for 2, their part (a) and their part (e) substituted into equation of a straight line.

OR

$y = 6x - \frac{28}{3}\ (y = 6x - 9.33333\ldots)$ ***(M1)***

Note: Award ***(M1)*** for their answer to (e) and intercept $-\frac{28}{3}$ substituted in the gradient-intercept line equation.

$-18x + 3y + 28 = 0$ (accept integer multiples) ***(A1)*****(ft)*****(G2)***

Note: Follow through from parts (a) and (e).

[2 marks]

3. [Maximum mark: 6] 18N.1.SL.TZ0.T_11

Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{dy}{dx}$. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$15x^2 - 3$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for $15x^2$, ***(A1)*** for -3. Award at most ***(A1)(A0)*** if additional terms are seen.

[2 marks]

The curve has a tangent at the point P(−1, −2).

(b) Find the gradient of this tangent at point P. [2]

Markscheme

$15(-1)^2 - 3$ ***(M1)***

Note: Award ***(M1)*** for substituting −1 into their $\frac{dy}{dx}$.

$= 12$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$. [2]

Markscheme

$(y - (-2)) = 12\,(x - (-1))$ ***(M1)***

OR

$-2 = 12(-1) + c$ ***(M1)***

Note: Award ***(M1)*** for point **and** their gradient substituted into the equation of a line.

$y = 12x + 10$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (b).

[2 marks]

4. [Maximum mark: 7] EXN.1.SL.TZ0.7

Consider the curve $y = x^2 - 4x + 2$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 4$ **A1**

[1 mark]

(b) Show that the normal to the curve at the point where $x = 1$ is $2y - x + 3 = 0$. [6]

Markscheme

Gradient at $x = 1$ is -2 **M1**

Gradient of normal is $\frac{1}{2}$ **A1**

When $x = 1$ $y = 1 - 4 + 2 = -1$ **(M1)A1**

EITHER

$y + 1 = \frac{1}{2}(x - 1)$ **M1**

$2y + 2 = x - 1$ or $y + 1 = \frac{1}{2}x - \frac{1}{2}$ **A1**

OR

$-1 = \frac{1}{2} \times 1 + c$ **M1**

$y = \frac{1}{2}x - \frac{3}{2}$ **A1**

THEN

$2y - x + 3 = 0$ **AG**

[6 marks]

5. [Maximum mark: 7] 22M.1.SL.TZ1.9

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3,\ x \neq 0$.

(a) Find $f\prime(x)$. [3]

Markscheme

$f\prime(x) = -2x^{-2} + 6x$ **OR** $f\prime(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

Note: Award ***A1*** for $6x$ seen, and ***(M1)*** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from either x^{-2} or $\frac{2}{x^2}$ seen in their final answer), ***A1*** for $-\frac{2}{x^2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

[3 marks]

(b) Find the equation of the normal to the curve $y = f(x)$ at $(1,\ 2)$ in the form $ax + by + d = 0$, where $a,\ b,\ d \in \mathbb{Z}$. [4]

Markscheme

finding gradient at $x = 1$

$\frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_{\perp} = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

Note: Award ***M1*** for correctly substituting $x = 1$ and $y = 2$ and their $m_{\perp}$.

$x + 4y - 9 = 0$ ***A1***

Note: Do not award the final ***A1*** if the answer is not in the required form. Accept integer multiples of the equation.

[4 marks]

6. [Maximum mark: 6] 20N.1.SL.TZ0.T_13

Consider the graph of the function $f(x) = x^2 - \frac{k}{x}$.

(a) Write down $f\prime(x)$. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$2x + \frac{k}{x^2}$ ***(A1)(A1)(A1) (C3)***

Note: Award ***(A1)*** for $2x$, ***(A1)*** for $+k$, and ***(A1)*** for x^{-2} or $\frac{1}{x^2}$.
Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

The equation of the tangent to the graph of $y = f(x)$ at $x = -2$ is $2y = 4 - 5x$.

(b) Write down the gradient of this tangent. [1]

Markscheme

$-2.5 \ \left(\frac{-5}{2}\right)$ ***(A1) (C1)***

[1 mark]

(c) Find the value of k. [2]

Markscheme

$-2.5 = 2 \times (-2) + \frac{k}{(-2)^2}$ ***(M1)***

Note: Award ***(M1)*** for equating their gradient from part (b) to their substituted derivative from part (a).

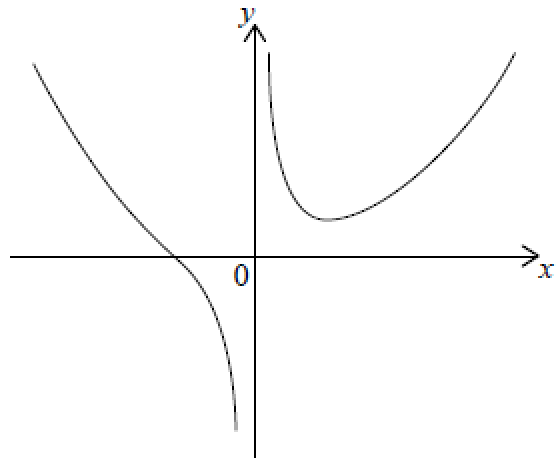
$(k =)\ 6$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

7. [Maximum mark: 6] 19N.1.SL.TZ0.T_14

The diagram shows the curve $y = \frac{x^2}{2} + \frac{2a}{x},\ x \neq 0.$



The equation of the vertical asymptote of the curve is $x = k$.

(a) Write down the value of k. [1]

Markscheme

$(k =)\ 0$ ***(A1) (C1)***

Note: Award ***(A1)*** for an answer of "$x = 0$".

[1 mark]

(b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

Markscheme

$x - \frac{2a}{x^2}$ ***(A1)(A1)(A1) (C3)***

Note: Award ***(A1)*** for x, ***(A1)*** for $-2a$, ***(A1)*** for x^{-2} or $\frac{1}{x^2}$. Award at most ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

(c) At the point where $x = 2$, the gradient of the tangent to the curve is 0.5.

Find the value of a. [2]

Markscheme

$0.5 = 2 - \frac{2a}{2^2}$ ***(M1)***

Note: Award ***(M1)*** for *their* correctly substituted derivative equated to 0.5.

$(a =)\ 3$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (b) providing their answer is **not** $a = 0$ as this value contradicts the graph.

[2 marks]