Tangent & normal lines [45 marks]

1. [Maximum mark: 6] 18M.1.SL.TZ2.T_14

Consider the function $f(x) = \frac{x^4}{4}$.

(a) Find $f'(x)$ [1]

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$. [2]

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$. [3]

2. [Maximum mark: 7] 19M.2.SL.TZ2.T_5

Consider the function $f(x) = \frac{1}{3}x^3 + \frac{3}{4}x^2 - x - 1$.

(d) Find $f'(x)$. [3]

(e) Find the gradient of the graph of $y = f(x)$ at $x = 2$. [2]

(f) Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 2$. Give the equation in the form $ax + by + d = 0$ where, a, b, and $d \in \mathbb{Z}$. [2]

3. [Maximum mark: 6] 18N.1.SL.TZ0.T_11

Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

The curve has a tangent at the point P(−1, −2).

(b) Find the gradient of this tangent at point P. [2]

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$. [2]

4. [Maximum mark: 7] EXN.1.SL.TZ0.7

Consider the curve $y = x^2 - 4x + 2$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(b) Show that the normal to the curve at the point where $x = 1$ is $2y - x + 3 = 0$. [6]

5. [Maximum mark: 7] 22M.1.SL.TZ1.9

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3,\ x \neq 0$.

(a) Find $f\prime(x)$. [3]

(b) Find the equation of the normal to the curve $y = f(x)$ at $(1,\ 2)$ in the form $ax + by + d = 0$, where $a,\ b,\ d \in \mathbb{Z}$. [4]

6. [Maximum mark: 6] 20N.1.SL.TZ0.T_13

Consider the graph of the function $f(x) = x^2 - \frac{k}{x}$.

(a) Write down $f'(x)$. [3]

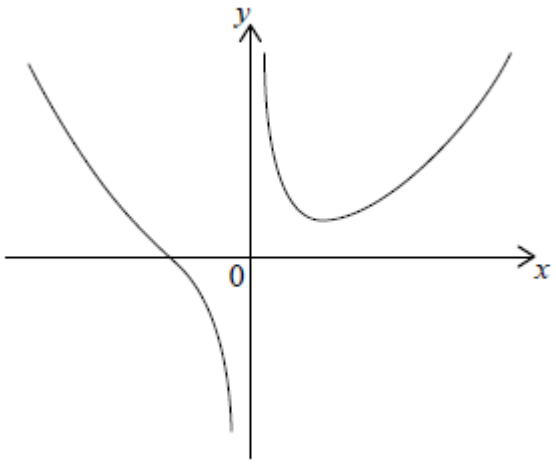
The equation of the tangent to the graph of $y = f(x)$ at $x = -2$ is $2y = 4 - 5x$.

(b) Write down the gradient of this tangent. [1]

(c) Find the value of k. [2]

7. [Maximum mark: 6] 19N.1.SL.TZ0.T_14

The diagram shows the curve $y = \frac{x^2}{2} + \frac{2a}{x},\ x \neq 0$.



The equation of the vertical asymptote of the curve is $x = k$.

(a) Write down the value of k. [1]

(b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(c) At the point where $x = 2$, the gradient of the tangent to the curve is 0.5.

Find the value of a. [2]