Variations - models [16 marks]

1. [Maximum mark: 5] EXN.1.SL.TZ0.2

A factory produces engraved gold disks. The cost C of the disks is directly proportional to the cube of the radius r of the disk.

A disk with a radius of 0.8 cm costs 375 US dollars (USD).

(a) Find an equation which links C and r. [3]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$C = kr^3$ **(M1)**

$375 = k \times 0.8^3 \Rightarrow k = 732\ (732.421\ldots)$ **(M1)**

$C = 732r^3$ **A1**

[3 marks]

(b) Find, to the nearest USD, the cost of disk that has a radius of 1.1 cm. [2]

Markscheme

$C = 732.42\ldots \times 1.1^3$ **(M1)**

$C = \$975\ (974.853\ldots)$ **A1**

Note: accept $\$974$ from use of $C = 732r^3$.

[2 marks]

2. [Maximum mark: 5] 23M.1.SL.TZ1.6

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d, in metres is directly proportional to the square of the speed of the car, v, in kilometres per hour (km h^{-1}).

When a car is travelling at a speed of 50 km h^{-1} it will travel 12.3 m after the brakes are fully applied before it completely stops.

(a) Determine an equation for d in terms of v. [2]

Markscheme

attempt to set up a direct variation equation that includes a constant, k, or the calculation of a constant using 12.3 and 50 ***(M1)***

e.g., $d = kv^2$ **OR** $12.3 = k \times 50^2$

$(k =)\ 0.00492\ \left(\frac{1}{203.252\ldots}\right)$

$d = 0.00492v^2$ **OR** $d = \frac{v^2}{203}$ ***A1***

[2 marks]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled 33 m, after fully applying its brakes, before it completely stopped.

(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied.

[2]

Markscheme

substituting 33 for d in their part (a) ***(A1)***

$33 = 0.00492 \times v^2$ **OR** $33 = \frac{v^2}{203.252...}$

$(v =)\ 81.9\ (\text{km h}^{-1})\ \ (81.8982...\ (\text{km h}^{-1}))$ ***A1***

[2 marks]

(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]

Markscheme

Award ***R1*** for a reasonable variable that exists after the brakes are applied such as:

- road material
- weather conditions
- condition/type of brakes
- weight/type of vehicle
- gradient/incline of road
- traction
- wind resistance
- friction

R1

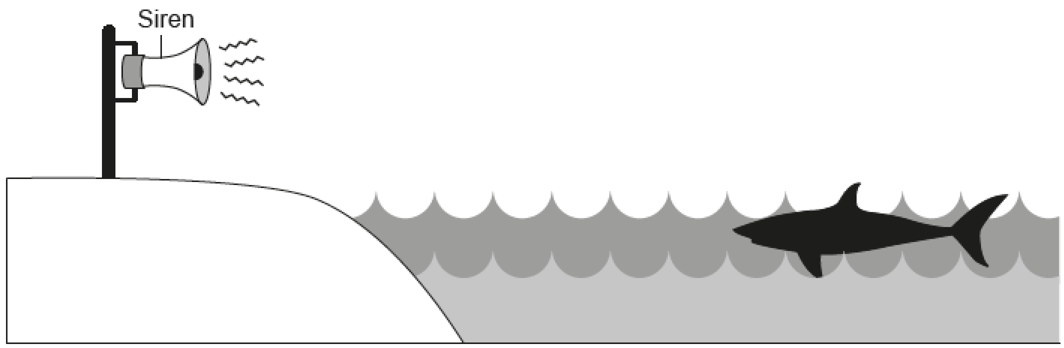
Note: Do not accept a variable that refers to the timing of the brakes being applied such as:

- slow reaction time
- inexperienced driver

[1 mark]

3. [Maximum mark: 6] 21M.1.SL.TZ1.11

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.



The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre ($\mathrm{W\,m^{-2}}$).

(a) Show that $I = \frac{9}{d^2}$. [2]

Markscheme

$I = \frac{k}{d^2}$ ***(M1)***

$4 = \frac{k}{1.5^2}$ ***M1***

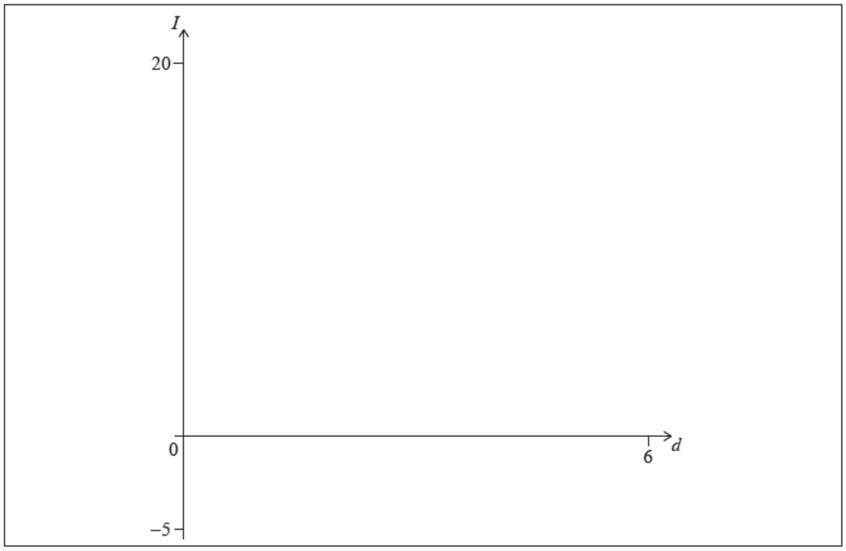
$I = \frac{9}{d^2}$ ***AG***

Note: The ***AG*** line must be seen for the second ***M1*** to be awarded.

Award no marks for substituting 1.5 and 4 into $I = \frac{9}{d^2}$ (i.e., working backwards).

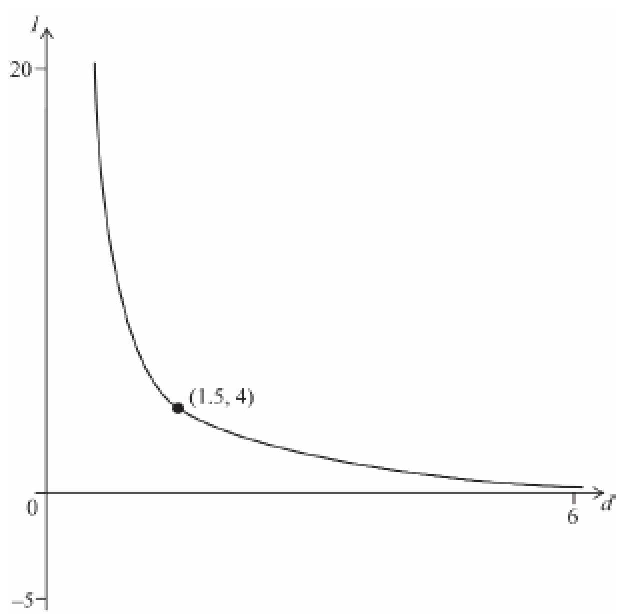
[2 marks]

(b) Sketch the curve of I on the axes below showing clearly the point $(1.5,\ 4)$.



[2]

Markscheme



A1A1

Note: Award ***A1*** for correct general shape (concave up) with no I-intercept, passing through the marked point $(1.5,\ 4)$; the point must be labelled with either the coordinates or the values 1.5 and 4 on the x and y axes.

Award ***A1*** for the curve showing asymptotic behavior (i.e. I tends to 0, as d tends to infinity), extending to at least $d = 6$; the curve must not cross nor veer away from the horizontal asymptote.

[2 marks]

(c) Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6}\,\mathrm{W\,m^{-2}}$.

Find the values of d where Scarlett cannot hear the siren. [2]

Markscheme

$1.5 \times 10^{-6} \geq \frac{9}{d^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct inequality.

$d \geq 2450\,(\mathrm{m})\ \ (2449.48\ldots)$ ***A1***

Note: Award ***A0*** for $d = 2450$.

[2 marks]