Variations - models [16 marks]

1. [Maximum mark: 5]

A factory produces engraved gold disks. The cost C of the disks is directly proportional to the cube of the radius r of the disk.

A disk with a radius of 0.8 cm costs 375 US dollars (USD).

(a) Find an equation which links C and r. [3]

(b) Find, to the nearest USD, the cost of disk that has a radius of 1.1 cm. [2]

2. [Maximum mark: 5]

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d, in metres is directly proportional to the square of the speed of the car, v, in kilometres per hour $(\mathrm{km}\ \mathrm{h}^{-1})$.

When a car is travelling at a speed of $50\ \mathrm{km}\ \mathrm{h}^{-1}$ it will travel $12.3\,\mathrm{m}$ after the brakes are fully applied before it completely stops.

(a) Determine an equation for d in terms of v. [2]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled $33\,\mathrm{m}$, after fully applying its brakes, before it completely stopped.
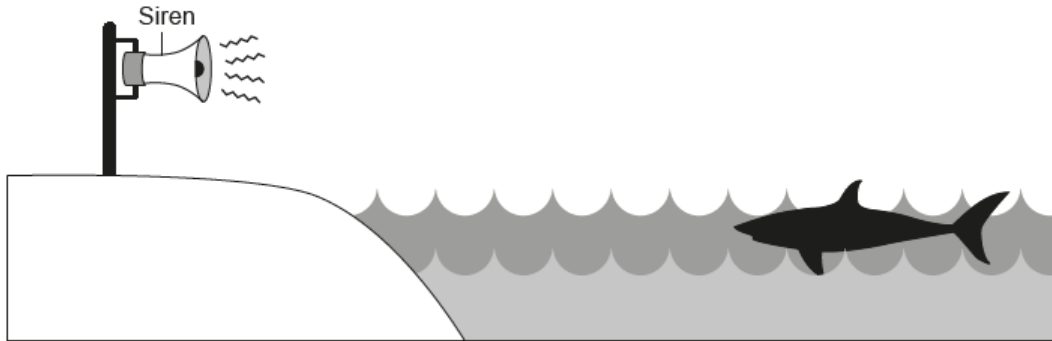
(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied. [2]

(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]

3. [Maximum mark: 6]

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre ($\mathrm{W\,m^{-2}}$).

(a)   Show that $I = \frac{9}{d^2}$.   [2]

(b)   Sketch the curve of I on the axes below showing clearly the point $(1.5,\ 4)$.
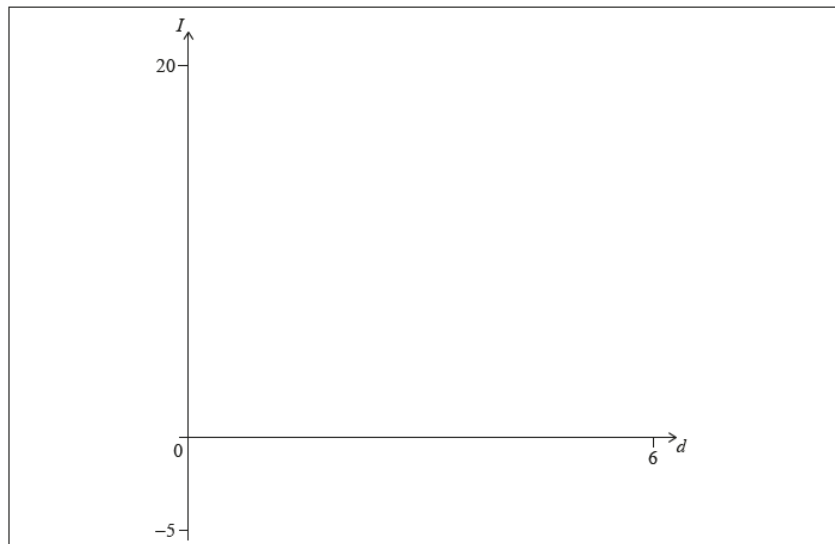
[2]

(c)   Whilst swimming, Scarlett can hear the siren only if the sound
intensity at her location is greater than $1.5 \times 10^{-6}\,\mathrm{W\,m^{-2}}$.

Find the values of d where Scarlett cannot hear the siren.   [2]