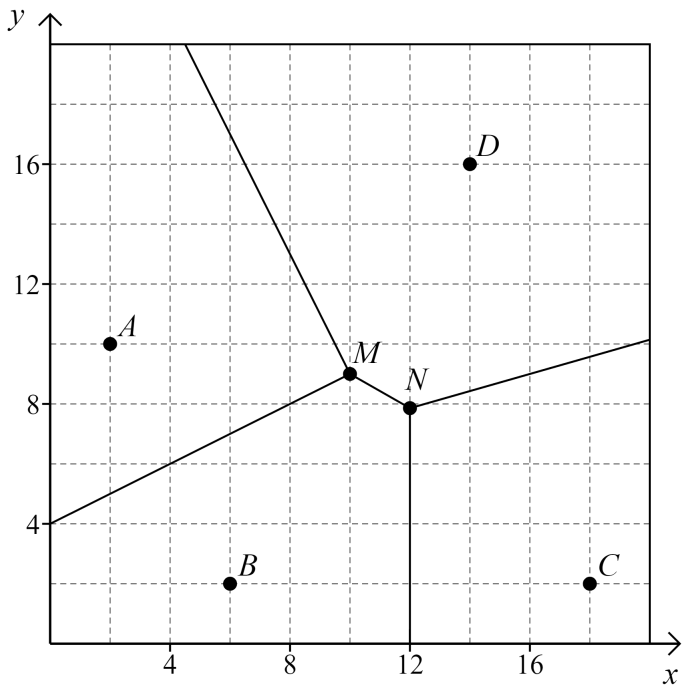
1. [Maximum points: 25]

A square pond has sides of length 20 m. The depth of the pond at points A, B, C and D is measured. Each point has integer coordinates. This is shown in the Voronoi diagram below.



(a) Write down the equation of the perpendicular bisector of line segment [2]

(i) AB

(ii) BC

(b) Find the equation of the perpendicular bisector of the following line segments. Write your answers in the form $y = mx + c$ where $m, c \in \mathbb{Q}$. [6]

(i) AD

(ii) CD

(c) Determine the coordinates of point M. [3]

(d) Show that the coordinates of point N are equal to $\left(12, \frac{55}{7}\right)$. [1]

(e) To two decimal places calculate the area of the cell containing point [8]

(i) A

(ii) B

(iii) C

(f) Hence show the area of the cell containing point D is $141.39\,m^2$ to two decimal places. [1]

The table below shows the depth of the water at each point.

Point	A	B	C	D
Depth	0.8 m	1.1 m	0.5 m	1.6 m

(g) Estimate [4]

(i) the volume of water in the pond

(ii) the average depth of the pond

2. [Maximum points: 15]

The table below shows the methods of payment in a convenience store over one year. The percentage given represents the percentage of customers using each method.

Method	Percentage
Cash	43
Smart Device	19
Credit / Debit Card	34
Vouchers	4

The total number of customers who paid by cash was 169,506.

(a) Calculate [4]

(i) the total number of customers

(ii) the number of customers who paid with a smart device

The table below shows the payment method of a random sample of customers from the next year.

Method	Frequency
Cash	125
Smart Device	62
Credit / Debit Card	96
Vouchers	10

A χ^2 goodness of fit test at the 10% significance level is used to determine whether the distribution has changed.

(b) Write down [2]

(i) the null hypothesis

(ii) the alternate hypothesis

(c) State the degrees of freedom for the test. [1]

(d) Find the total number of customers sampled. [1]

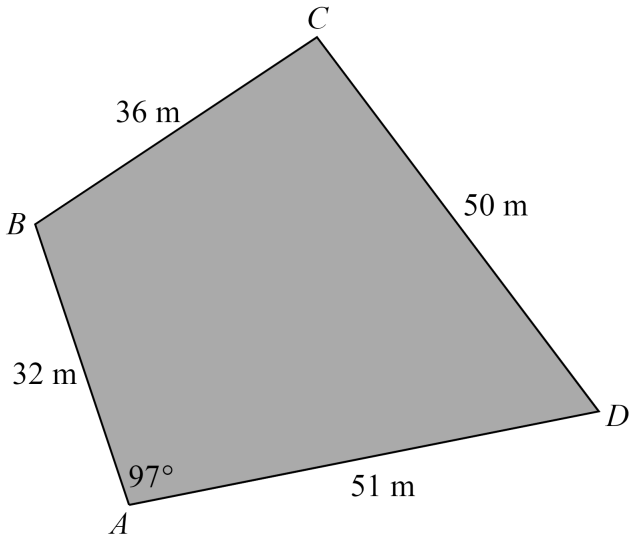
(e) Calculate the expected frequencies. [3]

(f) Find the *p*-value for the test. [2]

(g) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum points: 15]

A field is in the shape of a quadrilateral. Its measurements are shown in the diagram below.



(a) Find the value of the following to three decimal places [4]

(i) length BD

(ii) $\angle BCD$

(b) Hence calculate the area of the field. [3]

(c) Find the value of $\angle ADC$ to three decimal places. [2]

The field is to be divided into two seperate fields using a straight fence from point A to point M on one of the other edges.

(d) Using your answer to part (c) explain why point M must lie between C and D. [3]

(e) Find the distance of point M from point C. [3]

4. [Maximum points: 14]

A Japanese katana sword is made by repeatedly stretching and then folding a sheet of steel in half. The first two folds are shown below. After being stretched and folded the overall thickness of the metal remains the same.

One Fold *Two Folds*

After the first fold there are two layers and after the second fold there are four layers.

(a) Write down the number of layers after [2]

(i) 3 folds

(ii) 4 folds

(b) Sketch a diagram, similar to the ones above, showing the layers after three folds. [3]

Let L_n represent the number of layers after n folds.

(c) Find an expression for L_n. [2]

(d) Use your expression to determine the number of layers after 10 folds. [2]

A typical sword can contain more than one million layers of steel.

(e) Find the minimum number of folds needed to achieve this. [3]

The diagrams above include spaces between each layer in order to make each layer clearly visible. In practice there is no space between each layer.

(f) If the overall thickness of the steel is 2 cm determine the thickness of one layer after the number of folds in part (e). Write your answer in cm in the form $a \times 10^k$ where $a \in \mathbb{R}$, $k \in \mathbb{Z}$ and $1 \leq a < 10$. [2]