Name:
Result:

1. Solve the following equations and inequalities: [5 *points*]

a) $2x^2 + 9 = 3x$

b) $x + 17 = 9\sqrt{x + 3}$

c) $12 - 2x^2 \leqslant 5x$

2. a) Sketch the graph of $y = \frac{1}{2}x^2 + 3x + 4$. Clearly indicate axes intercepts and the coordinates of the vertex. [2 *points*]

b) On the same set of axes sketch the graph of $y = -\frac{1}{2}x + 1$. Clearly indicate axes intercepts and the coordinates of the points of intersection of the two graphs. [2 *points*]

c) Find the possible values of m, for which a line with gradient m and y-intercept 1 is tangent to the parabola $y = \frac{1}{2}x^2 + 3x + 4$. [2 *points*]

NO GDC
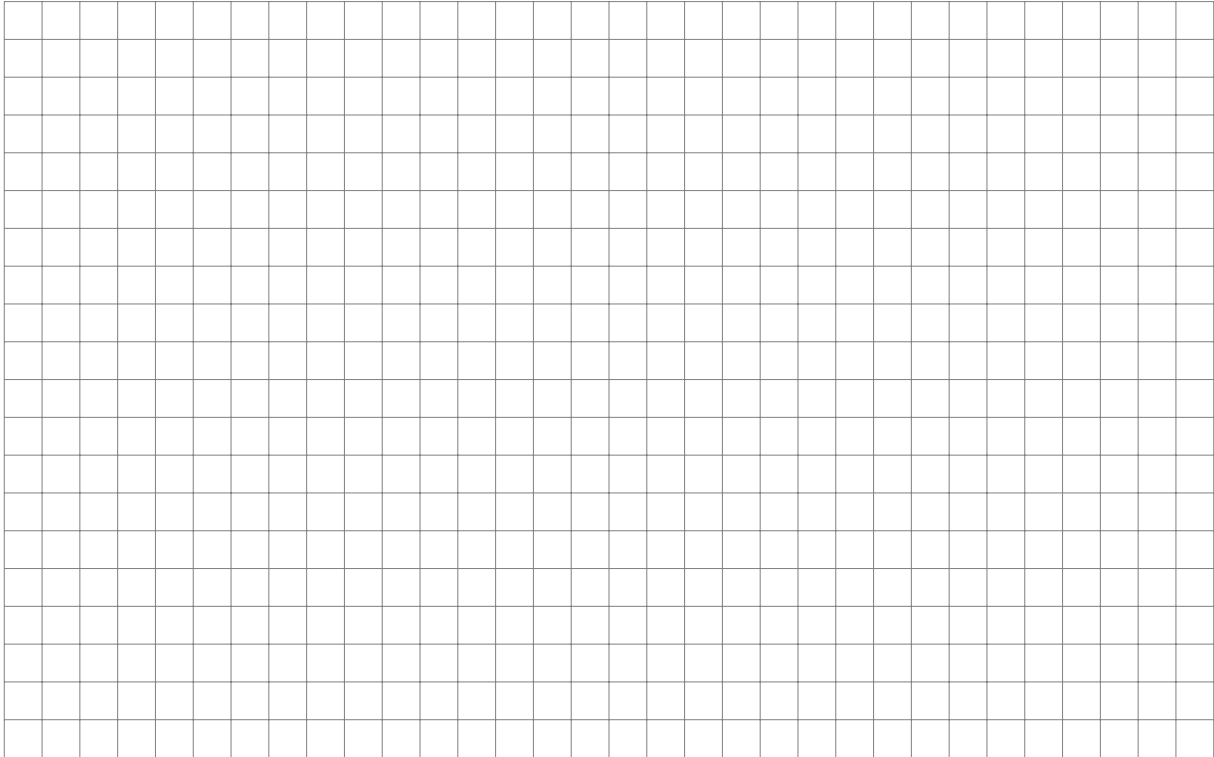
3. Certain items are sold at 80 PLN per unit. At this price 200 units are sold. Each decrease in price by 2 PLN results in a rise in sales by 10 units. The cost of producing the item (and all associated costs) are 20 PLN per item. Calculate the price at which the items should be sold to maximize the profit. Show all calculations.   [3 *points*]

4. Consider the equation:
$$x^2 + (m - 4)x - (2m + 1) = 0$$

a) Show that the equation has two distinct real solutions for all values of $m \in \mathbb{R}$. [*3 points*]

b) Let α and β be the two real solutions. Find the value of m for which the value of $\alpha^2 + \beta^2$ is minimal. [*3 points*]