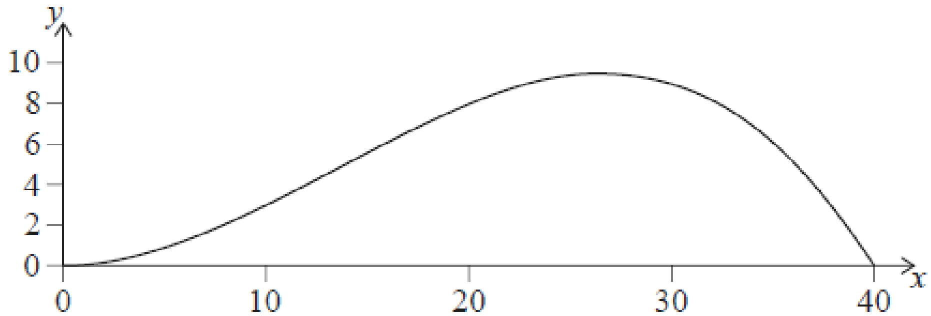
Areas (calculus) [57 marks]

1. [Maximum mark: 8] 23M.1.SL.TZ1.9

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is
$y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40.$

(b.i) Write down an integral to find the exact cross-sectional area. [2]

(b.ii) Calculate the value of the cross-sectional area to two decimal places. [2]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

2. [Maximum mark: 9] 23M.1.SL.TZ2.13

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.

The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

(a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals. [2]

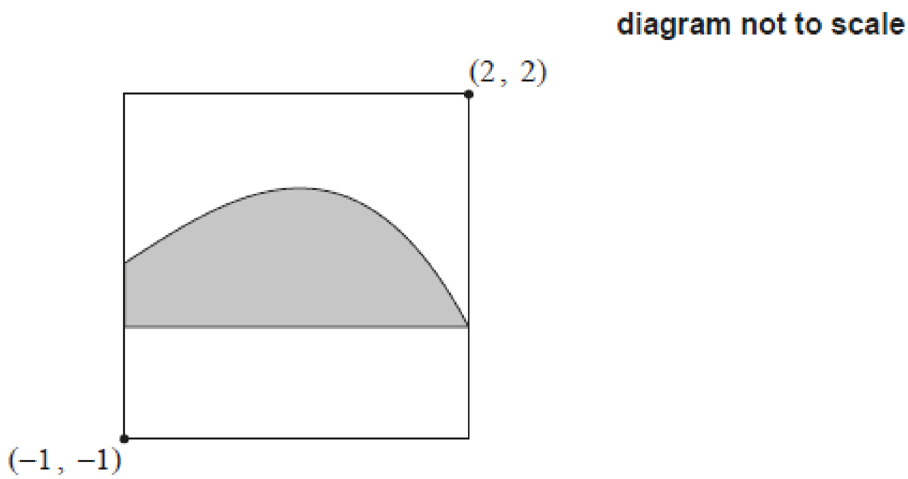
It is known that $f\prime(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]

(c) **Hence** find the actual area of the **entire** cross-section. [3]

3. [Maximum mark: 7] 22M.1.SL.TZ1.6

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, \ -1)$ and the top right corner has coordinates $(2, \ 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	−1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \frac{-x^3-3x^2+4x+12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]

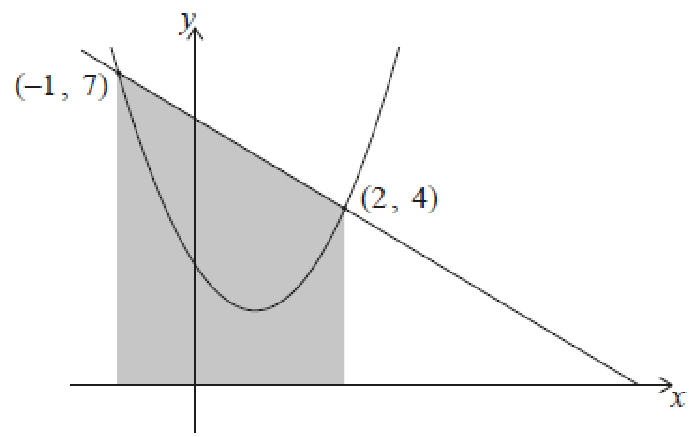
4. [Maximum mark: 7] 22M.1.SL.TZ2.6

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2,\ 4)$ and $(-1,\ 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

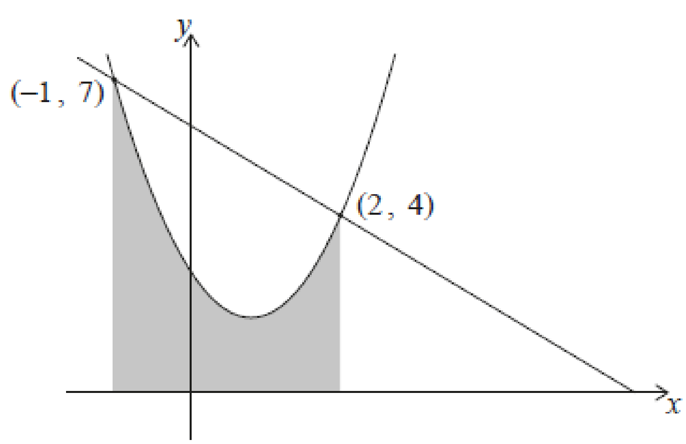


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



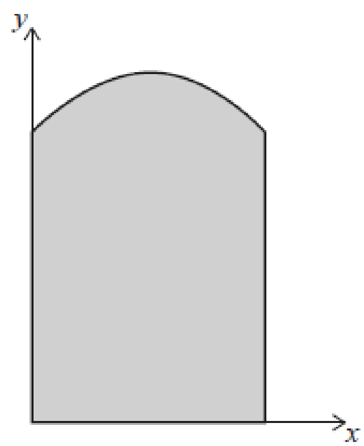
(b.i) Write down an integral for the area of the shaded region in **diagram 2**. [2]

(b.ii) Calculate the area of this region. [1]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

5. [Maximum mark: 8] 21N.1.SL.TZ0.13

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x-axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0,\ 10)$ and $(8,\ 10)$ and its vertex is $(4,\ 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \le x \le 8$.

(a.i) Write down the value of c. [1]

(a.ii) Hence form two equations in terms of a and b. [2]

(a.iii) Hence find the equation of the quadratic curve. [2]

(b) Find the area of the shaded region in Irina's design. [3]

6. [Maximum mark: 5] 21N.1.SL.TZ0.6

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

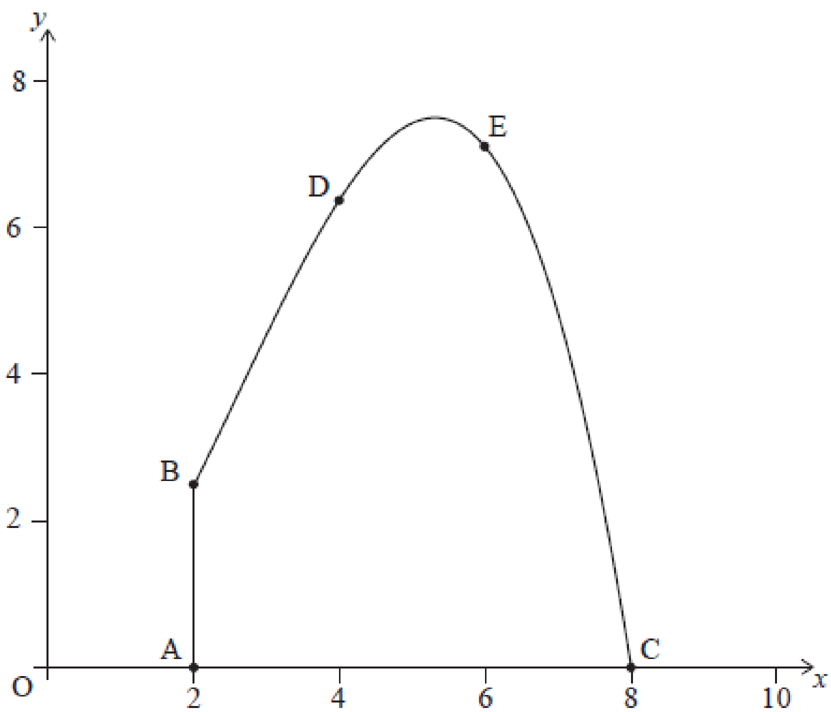
(b) The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

Find the percentage error of the estimate found in part (a). [2]

7. [Maximum mark: 13] 21M.2.SL.TZ1.5

The cross-sectional view of a tunnel is shown on the axes below. The line $[\mathrm{AB}]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by

$y = -0.1x^3 + 0.8x^2,\ 2 \le x \le 8$, relative to an origin O.



Point A has coordinates $(2,\ 0)$, point B has coordinates $(2,\ 2.4)$, and point C has coordinates $(8,\ 0)$.

(a.i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(a.ii) Hence find the maximum height of the tunnel. [4]

When $x = 4$ the height of the tunnel is $6.4\,\mathrm{m}$ and when $x = 6$ the height of the tunnel is $7.2\,\mathrm{m}$. These points are shown as D and E on the diagram, respectively.

(b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]

(c.i) Write down the integral which can be used to find the cross-sectional area of the tunnel. [2]

(c.ii) Hence find the cross-sectional area of the tunnel. [2]