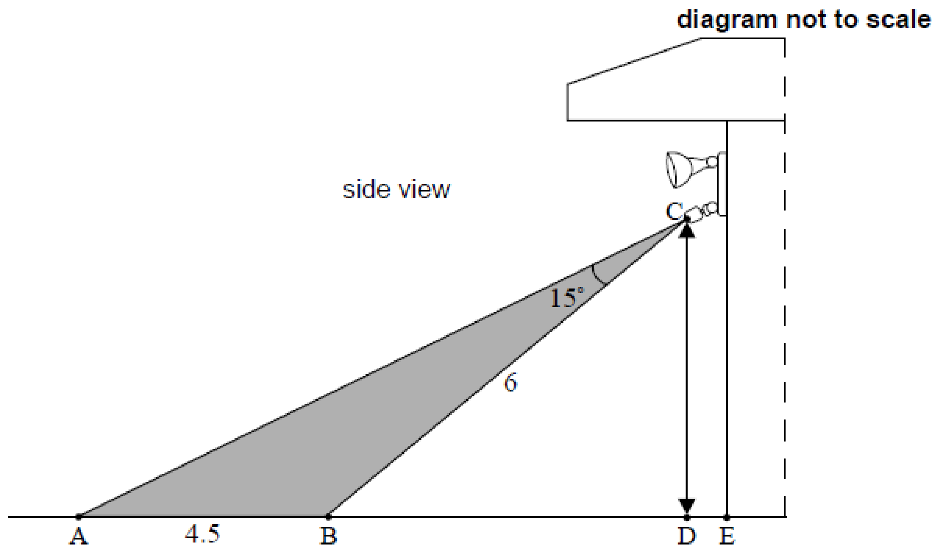
Geometry [152 marks]

1. [Maximum mark: 8] SPM.1.SL.TZ0.14

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $A\hat{C}B$ is 15°.



(a) Find $C\hat{A}B$. [3]

Markscheme

$$\frac{\sin C\hat{A}B}{6} = \frac{\sin 15°}{4.5}$$ ***(M1)(A1)***

$C\hat{A}B = 20.2°$ (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

Markscheme

$$C\hat{B}D = 20.2 + 15 = 35.2°$$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$$\tan 35.18741\ldots° = \frac{1.8}{BX}$$ ***(M1)***

Note: Award ***A1*** for their correct angle $\hat{\mathrm{CBD}}$. Award ***M1*** for correctly substituted trigonometric formula.

$\mathrm{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

= 2.45 (m) (2.44714…) ***A1***

[5 marks]

2. [Maximum mark: 9] EXN.1.SL.TZ0.11

A farmer owns a triangular field ABC. The length of side $[\mathrm{AB}]$ is $85\,\mathrm{m}$ and side $[\mathrm{AC}]$ is $110\,\mathrm{m}$. The angle between these two sides is 55°.

(a) Find the area of the field. [3]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Area $= \frac{1}{2} \times 110 \times 85 \times \sin 55^{\circ}$ **(M1)(A1)**

$= 3830\ (3829.53\ldots)\ \mathrm{m}^2$ **A1**

Note: units must be given for the final **A1** to be awarded.

[3 marks]

(b) The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\mathrm{BC}]$.

Find BD. Fully justify any assumptions you make. [6]

Markscheme

$\mathrm{BC}^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^{\circ}$ **(M1)A1**

$\mathrm{BC} = 92.7\ (92.7314\ldots)\ (\mathrm{m})$ **A1**

METHOD 1

Because the height and area of each triangle are equal they must have the same length base **R1**

D must be placed half-way along BC **A1**

$\mathrm{BD} = \frac{92.731\ldots}{2} \approx 46.4\ (\mathrm{m})$ **A1**

Note: the final two marks are dependent on the **R1** being awarded.

METHOD 2

Let $\mathrm{C\widehat{B}A} = \theta^{\circ}$

$\frac{\sin\theta}{110} = \frac{\sin 55^{\circ}}{92.731\ldots}$ **M1**

$\Rightarrow \theta = 76.3^{\circ}\ (76.3354\ldots)$

Use of area formula

$\frac{1}{2} \times 85 \times \mathrm{BD} \times \sin(76.33\ldots^{\circ}) = \frac{3829.53\ldots}{2}$ **A1**

$\mathrm{BD} = 46.4\ (46.365\ldots)\ (\mathrm{m})$ **A1**

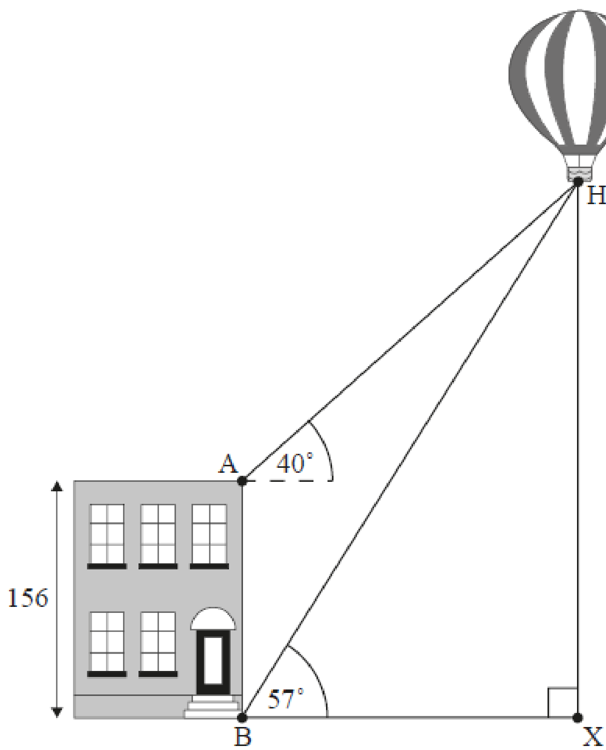
[6 marks]

3. [Maximum mark: 6] 23M.1.SL.TZ1.2

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is 40°, and the angle of elevation from point B (at the base of the building) to H is 57°. Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $\mathrm{A\hat{H}B}$. [2]

Markscheme

attempt to calculate $\mathrm{A\hat{H}B}$ using 33 **OR** use of alternate angles ***(M1)***

e.g., $180-(33+130)$ **OR** $90-(33+40)$ **OR** $57-40$

$17(^\circ)$ ***A1***

[2 marks]

(b) Calculate the distance from point B to point H. [3]

Markscheme

attempt to use sine rule ***(M1)***

$\frac{BH}{\sin(130°)} = \frac{156}{\sin(17°)}$ ***(A1)***

$(BH =)\ 409\,(m)\ \ (408.736\ldots)$ ***A1***

Note: If radians are used, answer is $151\ (150.922\ldots)$; award at most ***(M1)(A1)A0***.

[3 marks]

The hot-air balloon remains at a constant height as it moves further away from the building.

(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

Markscheme

the angle of depression from the hot air balloon) gets smaller ***A1***

(as the horizontal distance increases)

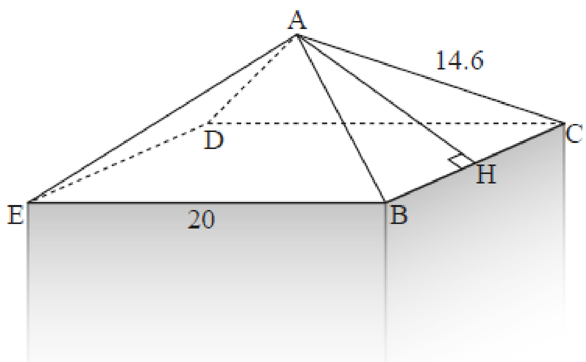
[1 mark]

4. [Maximum mark: 7] 23M.1.SL.TZ1.11

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly $20\ \mathrm{cm}$. The length from the apex of the pyramid, A, to any corner of the base of the pyramid is $14.6\ \mathrm{cm}$, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



(a) Write down the upper bound and lower bound for the possible lengths of edge AC. [2]

Markscheme

$14.55\,(\mathrm{cm})$ to $14.65\,(\mathrm{cm})$ ***A1A1***

Note: Award ***A1*** for each value. Accept $14.55 \leq \mathrm{AC} < 14.65$.

[2 marks]

Point H is the midpoint of BC.

(b) Determine the upper bound and lower bound for AH, the slant height of the pyramid. [3]

Markscheme

attempt to use Pythagorean theorem **OR** trig ratio to find slant height ***(M1)***

a correct expression for either the **upper** or **lower** bound ***(A1)***

$\sqrt{14.55^2 - 10^2}$ **OR** $\sqrt{14.65^2 - 10^2}$ **OR**

$\sin\left(46.5844\ldots^{\circ}\right)=\frac{\mathrm{AH}}{14.55}$ **OR** $\sin\left(46.9533\ldots^{\circ}\right)=\frac{\mathrm{AH}}{14.65}$

(lower bound =) $10.6\ (\mathrm{cm})\ (10.5689\ldots)$ **AND**

(upper bound =) $10.7\ (\mathrm{cm})\ (10.7061\ldots)$ ***A1***

[3 marks]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than 22°.

(c) Show that this post is safe for children. Justify your answer. [2]

Markscheme

METHOD 1

attempt to find the maximum angle measure of the post using trigonometry ***(M1)***

e.g. $\cos\theta=\frac{10}{10.7061\ldots}$ **OR** $\frac{\sin\theta}{3.82393\ldots}=\frac{\sin(90^{\circ})}{10.7061\ldots}$

Note: Accept an inequality.

$(\theta=)\ 20.9\ (^{\circ})\ \ (20.9265\ldots(^{\circ}))$ ***A1***

and hence the post is safe ***AG***

Note: Use of radians gives an answer of $0.365\ (0.365237\ldots)$; award at most ***(M1)A0*** since this value cannot be directly compared to 22°.

Award at most ***(M1)A0*** for an angle calculated using their lower bound from part (b).

METHOD 2

attempt to find the longest slant height for angle to be a maximum of 22° ***(M1)***

e.g. $\cos\left(22^{\circ}\right)=\frac{10}{x}$

$(x=10.7853\ldots)$

$10.7061\ldots<10.7853\ldots$ ***A1***

and hence the post is safe ***AG***

Note: A comparison to their upper bound from part (b) is required for ***A1*** to be awarded. Use of radians gives an unreasonable answer of $-10.0003\ldots$; award at most ***(M1)A0***.

[2 marks]

5. [Maximum mark: 5] 23M.1.SL.TZ1.13

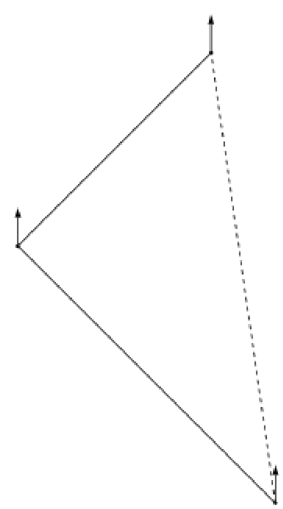
A boat travels $8\ \mathrm{km}$ on a bearing of 315° and then a further $6\ \mathrm{km}$ on a bearing of 045°.

Find the bearing on which the boat should travel to return directly to the starting point. [5]

Markscheme

METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



Note: Values do not need to be seen on the diagram to award the ***A1***.

recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

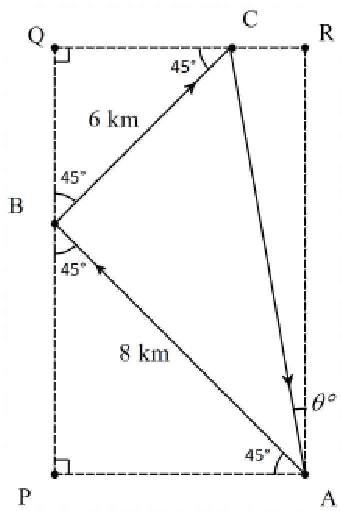
e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

$=172^\circ\ (171.869\ldots^\circ)$ ***A1***

METHOD 2

diagram showing (approximately) correct directions (and order) for the 315° and 045°

(these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$\mathrm{AP} = \mathrm{BP} = 8\frac{\sqrt{2}}{2} = 5.6568\ldots$$

$$\mathrm{CQ} = \mathrm{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426\ldots$$

Note: This may be done using a vector approach.

using $\tan\theta^\circ = \frac{\mathrm{AP}-\mathrm{CQ}}{\mathrm{PB}+\mathrm{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180^\circ - \arctan\left(\frac{8\frac{\sqrt{2}}{2}+6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2}-6\frac{\sqrt{2}}{2}}\right)$$

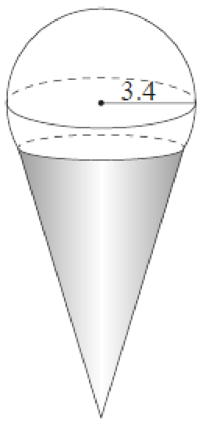
$$= 172^\circ \quad (171.869\ldots^\circ)$$ ***A1***

[5 marks]

6. [Maximum mark: 5] 23M.1.SL.TZ2.8

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of 3.4 cm. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

Markscheme

EITHER

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5} \times \frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

THEN

$132\text{ cm}^3\ \left(131.708\ldots\text{ cm}^3\right)$ ***A1***

[3 marks]

The cone has a slant height of 11 cm and a radius of 3 cm.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

Markscheme

$\pi \times 3 \times 11$ ***(A1)***

$103.672\ldots\left(\text{cm}^2\right)$ **OR** $33\pi\left(\text{cm}^2\right)$

$104\left(\text{cm}^2\right)$ ***A1***

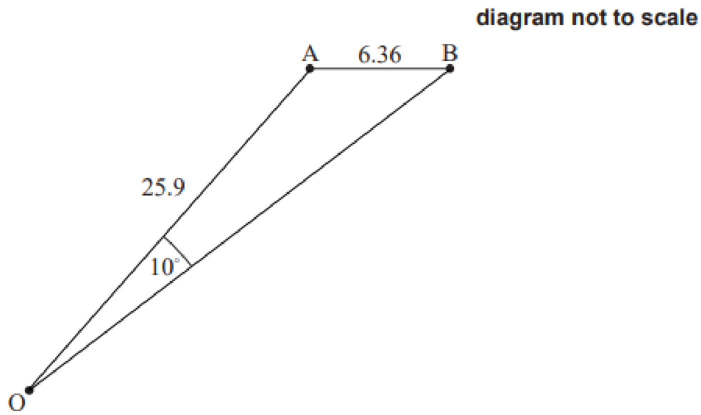
[2 marks]

7. [Maximum mark: 17] 23M.2.SL.TZ2.1

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is $6.36\,\mathrm{m}$.

Odette is playing football in the park and is standing at point O, such that $\mathrm{A\hat{O}B} = 10^\circ$, $\mathrm{OA} = 25.9\,\mathrm{m}$ and $\mathrm{O\hat{A}B}$ is obtuse.



(a) Calculate the size of $\mathrm{A\hat{B}O}$. [3]

Markscheme

attempt to use sine rule ***(M1)***

$\frac{\sin \mathrm{A\hat{B}O}}{25.9} = \frac{\sin 10^\circ}{6.36}$ ***(A1)***

$45.0^\circ\ (45.0036\ldots^\circ)$ ***A1***

Note: Accept an answer of 45° for full marks.

[3 marks]

(b) Calculate the area of triangle AOB. [4]

Markscheme

$\left(\mathrm{O\hat{A}B} =\right) 124.996\ldots^\circ$ ***(A1)***

attempt to use area of triangle formula ***(M1)***

$\frac{1}{2} \times 25.9 \times 6.36 \times \sin\left(124.996\ldots^\circ\right)$ ***(A1)***

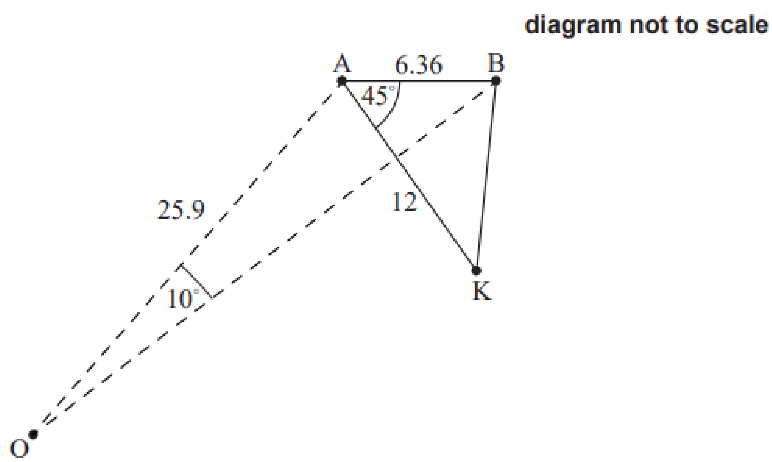
$67.5\,\mathrm{m}^2\ (67.4700\ldots\,\mathrm{m}^2)$ ***A1***

Note: Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(A1)(M1)(A1)A0*** for an unsupported answer of 67.5. Accept $67.4670\ldots\,\mathrm{m}^2$ from use of 3 sf values.

Full follow through marks can be awarded for this part even if their $\mathrm{O\hat{A}B}$ is not obtuse, provided that all working is shown.

[4 marks]

Odette's friend, Khemil, is standing at point K such that he is $12\,\mathrm{m}$ from A and $\mathrm{K\hat{A}B} = 45^\circ$.



(c) Calculate Khemil's distance from B. [3]

Markscheme

attempt to use cosine rule ***(M1)***

$\left(\mathrm{BK} =\right)\sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ ***(A1)***

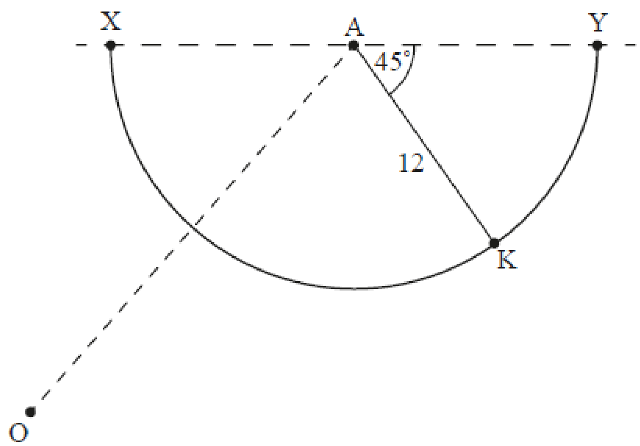
$8.75\,(\mathrm{m})\ (8.74738\ldots\,(\mathrm{m}))$ ***A1***

Note: Award ***(M1)(A1)(A0)*** for radian answer of $10.25\,(\mathrm{m})\ (10.2109\ldots\,(\mathrm{m}))$ with or without working shown.

[3 marks]

XY is a semicircular path in the park with centre A, such that $\mathrm{K\hat{A}Y} = 45^{\circ}$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $\mathrm{KX} = 22.2\,\mathrm{m}$, $\mathrm{K\hat{O}X} = 53.8^{\circ}$ and $\mathrm{O\hat{K}X} = 51.1^{\circ}$.

(d) Find whether Odette or Khemil is closer to the football. [4]

Markscheme

METHOD 1

attempt to use sine rule with measurements from triangle OKX ***(M1)***

$\frac{\mathrm{OX}}{\sin 51.1^{\circ}} = \frac{22.2}{\sin 53.8^{\circ}}$ ***(A1)***

$(\mathrm{OX} =)\ 21.4\,(\mathrm{m})\ (21.4099\ldots)\,(\mathrm{m})$ ***A1***

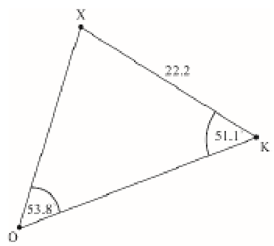
$(21.4\,(\mathrm{m}) < 22.2\,(\mathrm{m}))$

Odette is closer to the football / Khemil is further from the football ***A1***

Note: For the final ***A1*** to be awarded $21.4\ (21.4099\ldots)$ must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths ***(A1)***



51.1° is smallest angle in triangle OXK ***R1***

opposite side (OX) is smallest length ***R1***

therefore Odette is closest ***A1***

[4 marks]

Khemil runs along the semicircular path to pick up the football.

(e) Calculate the distance that Khemil runs. [3]

Markscheme

attempt to use length of arc formula ***(M1)***

$\frac{135}{360} \times 2\pi \times 12$ ***(A1)***

$28.3\,(\mathrm{m})\ (9\pi,\ 28.2743\ldots)\ (\mathrm{m})$ ***A1***

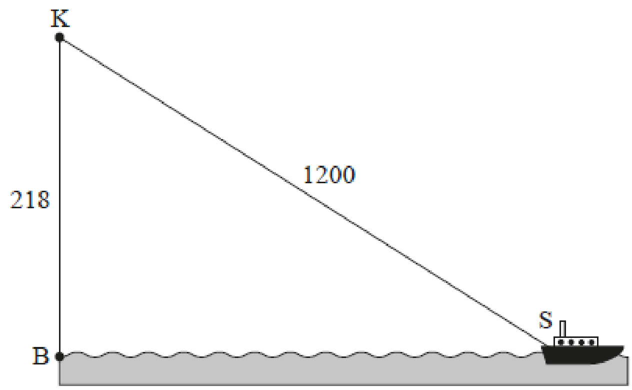
[3 marks]

8. [Maximum mark: 6] 22N.1.SL.TZ0.1

Kacheena stands at point K, the top of a $218\,\mathrm{m}$ vertical cliff. The base of the cliff is located at point B. A ship is located at point S, $1200\,\mathrm{m}$ from Kacheena.

This information is shown in the following diagram.

diagram not to scale



(a) Find the angle of elevation from the ship to Kacheena. [2]

Markscheme

$\sin\left(\mathrm{B\hat{S}K}\right) = \frac{218}{1200}$ **OR** $\frac{\sin\left(\mathrm{B\hat{S}K}\right)}{218} = \frac{\sin(90^\circ)}{1200}$ ***(M1)***

Note: Award ***M1*** for a correct trig formula. Accept other variables representing $\mathrm{B\hat{S}K}$.

$\left(\mathrm{B\hat{S}K} =\right) 10.5^\circ \quad (10.4668\ldots)$ ***A1***

Note: Award ***A1*** for the radian answer, $0.182681\ldots$. Award ***M1A0*** if the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

[2 marks]

(b) Find the horizontal distance from the base of the cliff to the ship. [2]

Markscheme

$\text{SB}^2 + 218^2 = 1200^2$ **OR** $\cos(10.4468\ldots) = \frac{\text{SB}}{1200}$ **OR** $\tan(10.4468\ldots) = \frac{218}{\text{SB}}$ **OR** $\frac{\text{BS}}{\sin(79.5331\ldots°)} = \frac{1200}{\sin(90°)}$ ***(M1)***

$1800\,(\text{m})\ \left(\sqrt{1392476},\ 1180.03\ldots\right)$ ***A1***

[2 marks]

(c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

Markscheme

1.18×10^3 ***A1A1***

Note: Award ***A1*** for 1.18
Award ***A1*** for 10^3
Accept their rounded answer to part (b).
Award ***A0A0*** for answers of the type: 11.8×10^2

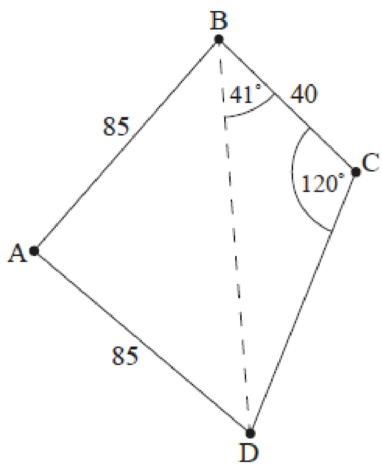
[2 marks]

9. [Maximum mark: 17] 22N.2.SL.TZ0.2

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB}=85\,\mathrm{m},\ \mathrm{AD}=85\,\mathrm{m},\ \mathrm{BC}=40\,\mathrm{m},\ \mathrm{C\hat{B}D}=41^{\circ},\ \mathrm{B\hat{C}D}=120^{\circ}$$

diagram not to scale



(a.i) Write down the value of angle BDC. [1]

Markscheme

19° ***A1***

[1 mark]

(a.ii) Hence use triangle BDC to find the length of path BD. [3]

Markscheme

$\frac{\mathrm{BD}}{\sin 120^{\circ}}=\frac{40}{\sin 19^{\circ}}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted sine rule for BCD, ***A1*** for their correct substitution.

$(\mathrm{BD}=)\ 106\,\mathrm{m}\quad(106.401\ldots)$ ***A1***

[3 marks]

(b) Calculate the size of angle $\mathrm{B\widehat{A}D}$, correct to five significant figures. [3]

Markscheme

METHOD 1 (cosine rule)

$\cos \mathrm{BAD} = \frac{85^2+85^2-106.401\ldots^2}{2\times85\times85}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted cosine rule, ***A1*** for their correct substitution.

77.495 ***A1***

Note: Accept an answer of 77.149 from use of 3 sf answer from part (a). The final answer must be correct to five significant figures.

METHOD 2 (right angled trig/isosceles triangles)

$\sin\left(\frac{\mathrm{BAD}}{2}\right) = \frac{53.2008\ldots}{85}$ ***(A1)(M1)***

Note: Award ***A1*** for $53.2008\ldots$ seen. Award ***M1*** for correctly substituted trig ratio. Follow through from part (a).

$77.495\ldots$ ***A1***

Note: Use of 3 sf answer from part (a), results in 77.149.

[3 marks]

The size of angle $\mathrm{B\widehat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\widehat{A}D} = 77^\circ$ for the rest of this question.

(c) Find the area bounded by the path BD, and fences AB and AD. [3]

Markscheme

EITHER

$(\text{Area} =)\ \frac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula, ***A1*** for correct substitution. Award at most ***(M1)(A1)A0*** if an angle other than 77° is used.

OR

$(\text{Area} =)\ \frac{1}{2} \times (2 \times 85 \times \sin(38.5^\circ)) \times (85 \times \cos(38.5^\circ))$ ***(M1)(A1)***

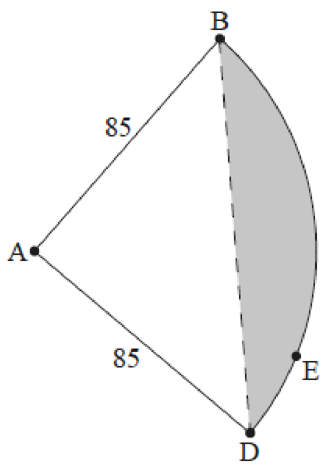
Note: Award ***M1*** for substituted area formula $A = \frac{1}{2}bh$, ***A1*** for correct substitution.

$3520\ \text{m}^2 \quad (3519.91\ldots)$ ***A1***

[3 marks]

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

Markscheme

$85\ \text{m}$ ***A1***

[1 mark]

(e) Find the perimeter of the proposed park, ABED. [3]

Markscheme

$85 + 85 + \frac{77}{360} \times 2\pi \times 85$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted into $\frac{\theta}{360} \times 2\pi \times r$, ***M1*** for addition of AB and AD.

$284 \text{ m} \ \ (284.231\ldots)$ ***A1***

[3 marks]

(f) Find the area of the shaded region in the proposed park. [3]

Markscheme

$\frac{77}{360} \times \pi \times (85)^2 - 3519.91\ldots$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted area of sector formula, ***M1*** for subtraction of their area from part (c).

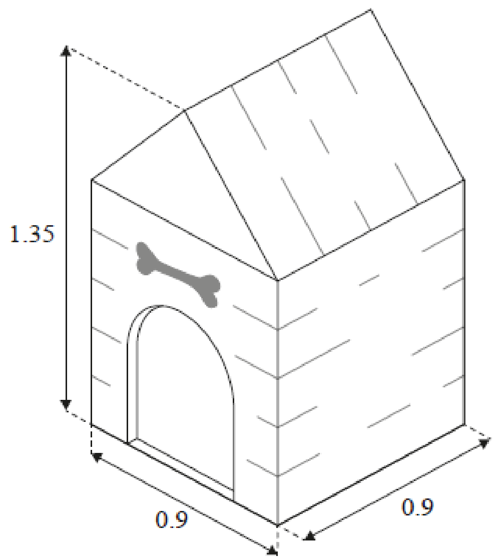
$1330 \text{ m}^2 \ \ (1334.93\ldots)$ ***A1***

[3 marks]

10. [Maximum mark: 5] 22M.1.SL.TZ1.1

The front view of a doghouse is made up of a square with an isosceles triangle on top.

The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted. [5]

Markscheme

height of triangle at roof $= 1.35 - 0.9 = 0.45$ ***(A1)***

Note: Award ***A1*** for 0.45 (height of triangle) seen on the diagram.

$\text{slant height} = \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \frac{0.45}{\text{slant height}}$ ***(M1)***

$= \sqrt{0.405}$ $\left(0.636396\ldots,\ 0.45\sqrt{2}\right)$ ***A1***

Note: If using $\sin(45^\circ) = \frac{0.45}{\text{slant height}}$ then ***(A1)*** for angle of 45°, ***(M1)*** for a correct trig statement.

area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756\ldots)$ ***M1***

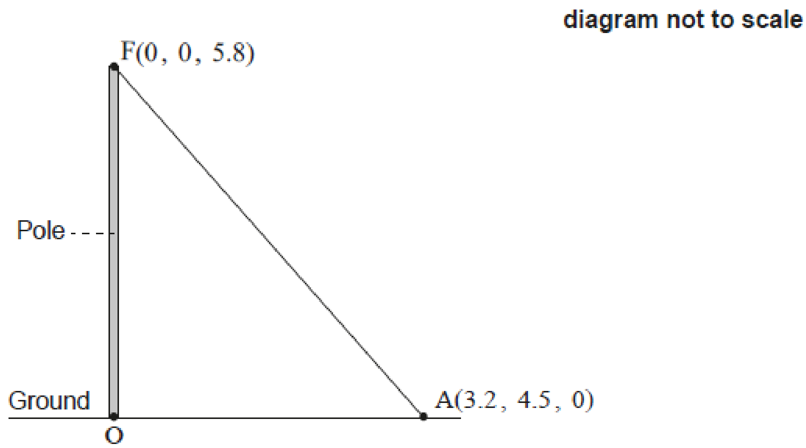
area painted $= \left(2 \times \sqrt{0.405} \times 0.9 = 2 \times 0.572756\ldots\right)$

1.15m^2 $\left(1.14551\ldots\text{ m}^2,\ 0.81\sqrt{2}\text{m}^2\right)$ ***A1***

[5 marks]

11. [Maximum mark: 4] 22M.1.SL.TZ1.2

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0,\ 0,\ 5.8)$. All units are in metres.



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2,\ 4.5,\ 0)$. The rope forms a straight line from A to F.

(a) Find the length of the rope connecting A to F. [2]

Markscheme

$\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01 \quad (8.00812\ldots)\ \mathrm{m}$ ***A1***

[2 marks]

(b) Find $\mathrm{F\hat{A}O}$, the angle the rope makes with the ground. [2]

Markscheme

$\mathrm{F\hat{A}O} = \sin^{-1}\left(\frac{5.8}{8.00812\ldots}\right)$ **OR** $\cos^{-1}\left(\frac{5.52177\ldots}{8.00812\ldots}\right)$ **OR** $\tan^{-1}\left(\frac{5.8}{5.52177\ldots}\right)$ ***(M1)***

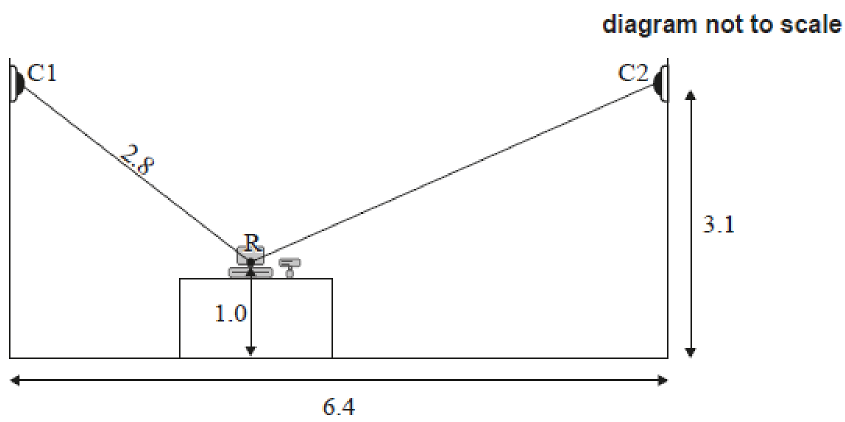
$46.4^\circ \quad (46.4077\ldots^\circ)$ ***A1***

[2 marks]

12. [Maximum mark: 8] 22M.1.SL.TZ2.3

The owner of a convenience store installs two security cameras, represented by points $\mathrm{C1}$ and $\mathrm{C2}$. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of $3.1\,\mathrm{m}$, and the horizontal distance between the cameras is $6.4\,\mathrm{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\mathrm{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\mathrm{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

Markscheme

$\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =) 48.6^\circ \quad (48.5903\ldots^\circ)$ ***A1***

[2 marks]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

Markscheme

METHOD 1

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903\ldots)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

Note: Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

$1.85\,(\text{m}) \quad (1.85202\ldots)$ ***(A1)***

Note: Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$

$4.55\,\text{m} \quad (4.54797\ldots)$ ***(A1)***

Note: Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$

$5.01\,\text{m} \quad (5.00939\ldots\,\text{m})$ ***A1***

METHOD 2

attempt to use cosine rule ***(M1)***

$\left(c^2 =\right) 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

Note: Award ***A1*** for $48.5903\ldots^\circ$ substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\,\text{m} \quad (5.00939\ldots\,\text{m})$ ***A1***

[4 marks]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

Markscheme

camera 1 is closer to the cash register (than camera 2 and both cameras are at the same height on the wall) ***R1***

the larger angle of depression is from camera 1 ***A1***

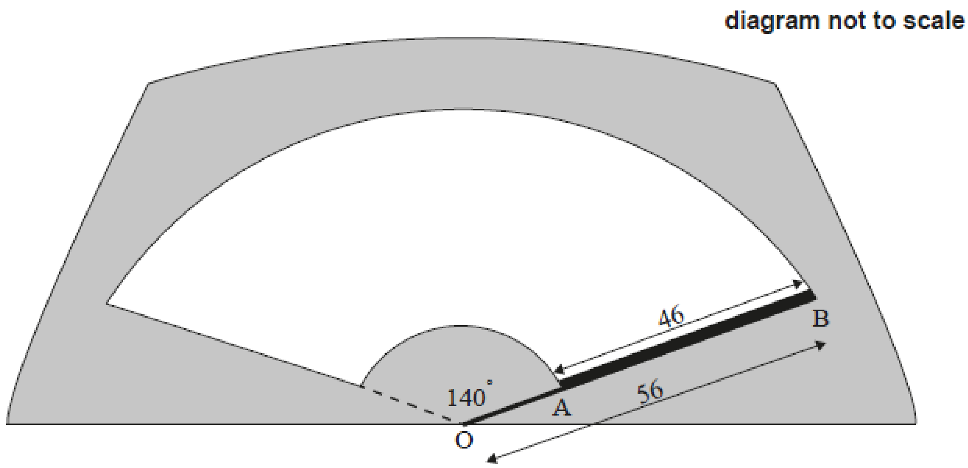
Note: Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "$1.85 < 4.55$" or "$2.8 < 5.01$" as evidence for the ***R1***.

[2 marks]

13. [Maximum mark: 5] 22M.1.SL.TZ2.1

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of 140°. The windscreen is cleared by a rubber blade of length $46\,\mathrm{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\,\mathrm{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.



(a) Calculate the length of the arc made by B, the end of the rubber blade. [2]

Markscheme

attempt to substitute into length of arc formula ***(M1)***

$\frac{140^\circ}{360^\circ} \times 2\pi \times 56$

$137\,\mathrm{cm} \quad \left(136.833\ldots,\ \frac{392\pi}{9}\,\mathrm{cm}\right)$ ***A1***

[2 marks]

(b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

Markscheme

subtracting two substituted area of sectors formulae ***(M1)***

$\left(\frac{140^\circ}{360^\circ} \times \pi \times 56^2\right) - \left(\frac{140^\circ}{360^\circ} \times \pi \times 10^2\right)$ **OR** $\frac{140^\circ}{360^\circ} \times \pi \times \left(56^2 - 10^2\right)$ ***(A1)***

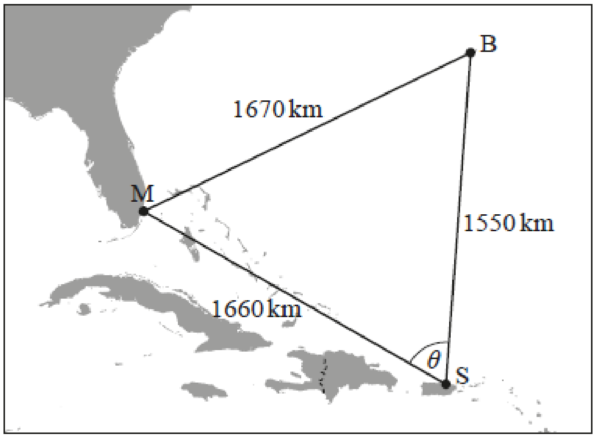
$3710\,\mathrm{cm}^2 \quad \left(3709.17\ldots\,\mathrm{cm}^2\right)$ ***A1***

[3 marks]

14. [Maximum mark: 5] 21N.1.SL.TZ0.2

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

(a) Calculate the value of θ, the measure of angle $\mathrm{M\hat{S}B}$. [3]

Markscheme

attempt at substituting the cosine rule formula ***(M1)***

$\cos\theta = \frac{1660^2+1550^2-1670^2}{2(1660)(1550)}$ ***(A1)***

$(\theta =)\ 62.6^\circ \quad (62.5873\ldots)$ (accept 1.09 rad $(1.09235\ldots)$) ***A1***

[3 marks]

(b) Find the area of the Bermuda Triangle. [2]

Markscheme

correctly substituted area of triangle formula ***(M1)***

$A = \frac{1}{2}(1660)(1550)\sin(62.5873\ldots)$

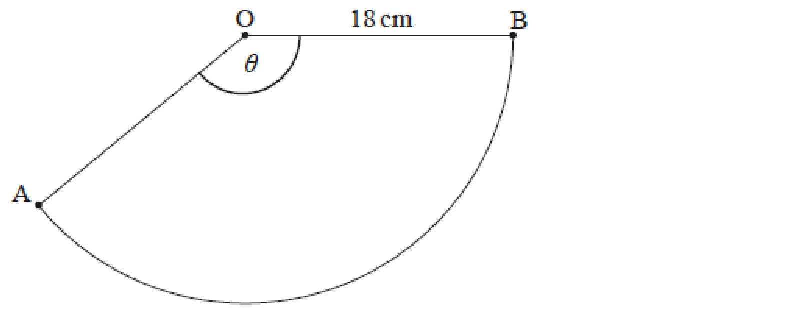
$(A =)\ 1140\,000\ \ (1.14 \times 10^6,\ 1142\,043.327\ldots)\ \mathrm{km}^2$ ***A1***

Note: Accept $1150\,000\ \ (1.15 \times 10^6,\ 1146\,279.893\ldots)\ \mathrm{km}^2$ from use of 63°. Other angles and their corresponding sides may be used.

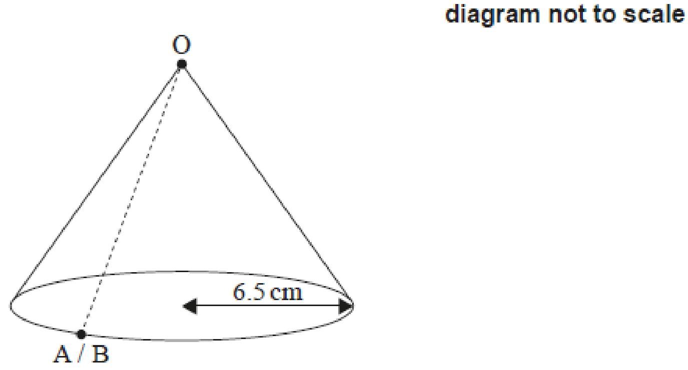
[2 marks]

15. [Maximum mark: 5] 21N.1.SL.TZ0.8

Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of $18\ \mathrm{cm}$ and $\mathrm{AOB} = \theta$ as shown in the diagram.



To make the hat, sides $[\mathrm{OA}]$ and $[\mathrm{OB}]$ are joined together. The hat has a base radius of $6.5\ \mathrm{cm}$.



(a.i) Write down the perimeter of the base of the hat in terms of π. [1]

Markscheme

$13\pi\ \mathrm{cm}$ ***A1***

Note: Answer must be in terms of π.

[1 mark]

(a.ii) Find the value of θ. [2]

Markscheme

METHOD 1

$\frac{\theta}{360} \times 2\pi(18) = 13\pi$ **OR** $\frac{\theta}{360} \times 2\pi(18) = 40.8407\ldots$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into length of an arc formula.

$(\theta =)\ 130^{\circ}$ ***A1***

METHOD 2

$\frac{\theta}{360} \times \pi \times 18^2 = \pi \times 6.5 \times 18$ ***(M1)***

$(\theta =)\ 130^{\circ}$ ***A1***

[2 marks]

(b) Find the surface area of the outside of the hat. [2]

Markscheme

EITHER

$\frac{130}{360} \times \pi(18)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into area of a sector formula.

OR

$\pi(6.5)(18)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into curved area of a cone formula.

THEN

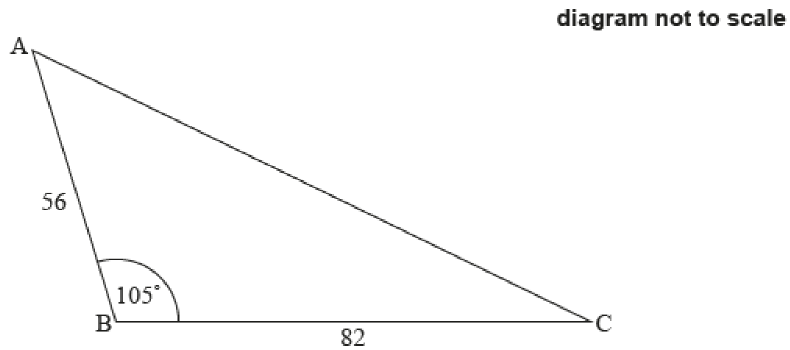
(Area$=$) $368\,\text{cm}^2$ $(367.566\ldots,\ 117\pi)$ ***A1***

Note: Allow ***FT*** from their part (a)(ii) even if their angle is not obtuse.

[2 marks]

16. [Maximum mark: 5] 21M.1.SL.TZ1.9

A triangular field ABC is such that $\mathrm{AB} = 56\,\mathrm{m}$ and $\mathrm{BC} = 82\,\mathrm{m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.



Calculate the maximum possible area of the field. [5]

Markscheme

attempt to find any relevant maximum value ***(M1)***

largest sides are 56.5 and 82.5 ***(A1)***

smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

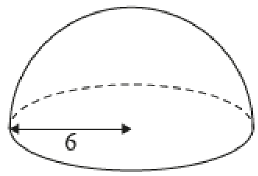
$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280\left(\mathrm{m}^2\right) \quad (2275.37\ldots)$ ***A1***

[5 marks]

17. [Maximum mark: 6] 21M.1.SL.TZ1.3

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is $6\,\text{mm}$.



(a) Calculate the **total** surface area of one piece of candy. [4]

Markscheme

$\frac{1}{2} \times 4 \times \pi \times 6^2 + \pi \times 6^2$ **OR** $3 \times \pi \times 6^2$ ***(M1)(A1)(M1)***

Note: Award ***M1*** for use of surface area of a sphere formula (or curved surface area of a hemisphere), ***A1*** for substituting correct values into hemisphere formula, ***M1*** for adding the area of the circle.

$= 339\ \text{mm}^2\ \ (108\pi,\ 339.292\ldots)$ ***A1***

[4 marks]

(b) The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\,\text{mm}^2$.

Calculate the weight of chocolate required to coat one piece of candy. [2]

Markscheme

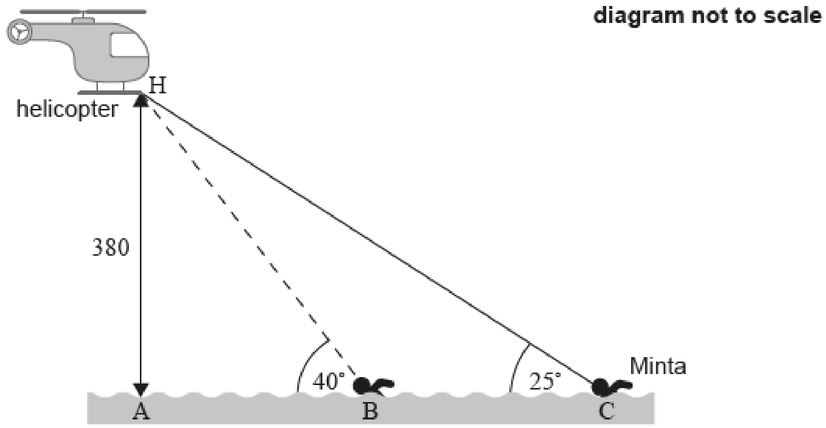
$\frac{339.292\ldots}{240}$ ***(M1)***

$= 1.41\ (\text{g})\ \left(\frac{9\pi}{20},\ 0.45\pi,\ 1.41371\ldots\right)$ ***A1***

[2 marks]

18. [Maximum mark: 7] 21M.1.SL.TZ2.4

The diagram below shows a helicopter hovering at point H, $380\,\mathrm{m}$ vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Write down the size of the angle of depression from H to C. [1]

Markscheme

25° ***A1***

[1 mark]

(b) Find the distance from A to C. [2]

Markscheme

$\mathrm{AC} = \frac{380}{\tan 25^{\circ}}$ **OR** $\mathrm{AC} = \sqrt{\left(\frac{380}{\sin 25^{\circ}}\right)^2 - 380^2}$ **OR** $\frac{380}{\sin 25^{\circ}} = \frac{\mathrm{AC}}{\sin 65^{\circ}}$ ***(M1)***

$\mathrm{AC} = 815\,\mathrm{m}\ (814.912\ldots)$ ***A1***

[2 marks]

(c) Find the distance from B to C. [3]

Markscheme

METHOD 1

attempt to find AB ***(M1)***

$\mathrm{AB} = \frac{380}{\tan 40^{\circ}}$

$= 453\,\mathrm{m}\ (452.866\ldots)$ ***(A1)***

$\mathrm{BC} = 814.912\ldots - 452.866\ldots$

$= 362\,\mathrm{m}\ (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$\mathrm{HB} = \frac{380}{\sin 40^{\circ}}$

$591\,\mathrm{m}\ (= 591.175\ldots)$ ***(A1)***

$\mathrm{BC} = \frac{591.175\ldots \times \sin 15^{\circ}}{\sin 25^{\circ}}$

$= 362\,\mathrm{m}\ (362.046\ldots)$ ***A1***

[3 marks]

(d) Find Minta's speed, in metres per hour. [1]

Markscheme

$362.046\ldots \times 4$

$= 1450\,\mathrm{m\,h^{-1}}\ (1448.18\ldots)$ ***A1***

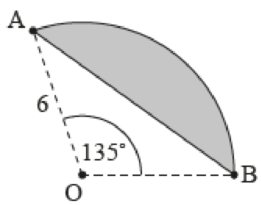
[1 mark]

19. [Maximum mark: 7] 21M.1.SL.TZ2.9

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius $6\,\mathrm{m}$, such that $\mathrm{AOB} = 135^\circ$. The straight border of the lawn is defined by chord $[\mathrm{AB}]$.

The lawn is shown as the shaded region in the following diagram.

diagram not to scale



(a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]

Markscheme

$135^\circ \times \frac{12\pi}{360^\circ}$ ***(M1)(A1)***

$14.1\,(\mathrm{m})\ \ (14.1371\ldots)$ ***A1***

[3 marks]

(b) Find the area of the lawn. [4]

Markscheme

evidence of splitting region into two areas ***(M1)***

$135^\circ \times \frac{\pi 6^2}{360^\circ} - \frac{6\times 6\times \sin 135^\circ}{2}$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituting into area of sector formula, ***M1*** for evidence of substituting into area of triangle formula.

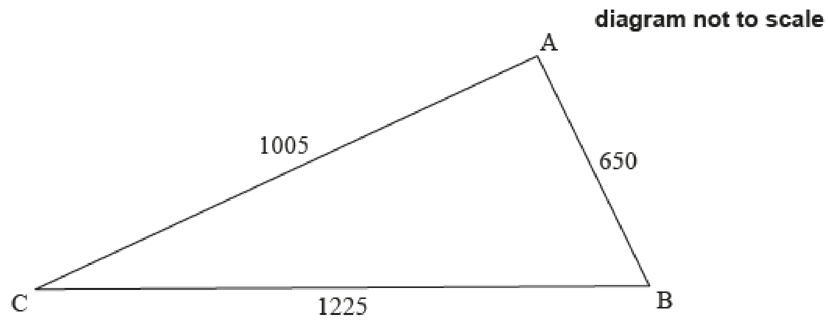
$42.4115\ldots - 12.7279\ldots$

$29.7\,\mathrm{m}^2\ \ (29.6835\ldots)$ ***A1***

[4 marks]

20. [Maximum mark: 15] 21M.2.SL.TZ2.2

A farmer owns a field in the shape of a triangle ABC such that $\mathrm{AB} = 650\,\mathrm{m},\ \mathrm{AC} = 1005\,\mathrm{m}$ and $\mathrm{BC} = 1225\,\mathrm{m}$.



(a) Find the size of $\mathrm{A\hat{C}B}$. [3]

Markscheme

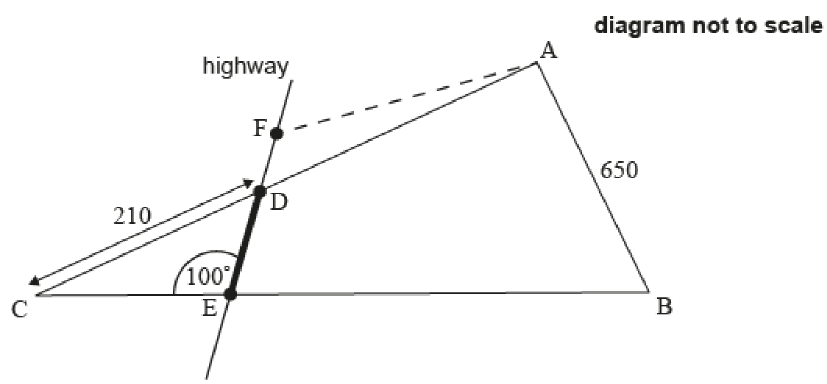
use of cosine rule *(M1)*

$\mathrm{A\hat{C}B} = \cos^{-1}\left(\frac{1005^2+1225^2-650^2}{2\times1005\times1225}\right)$ *(A1)*

$= 32^\circ \quad (31.9980\ldots)$ ***A1***

[3 marks]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $\mathrm{DC} = 210\,\mathrm{m}$ and $\mathrm{C\hat{E}D} = 100^\circ$, as shown in the diagram below.



(b) Find DE. [3]

Markscheme

use of sine rule *(M1)*

$\frac{\text{DE}}{\sin 31.9980\ldots^\circ} = \frac{210}{\sin 100^\circ}$ *(A1)*

$(\text{DE} =) 113\,\text{m}\ \ (112.9937\ldots)$ ***A1***

[3 marks]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

Markscheme

METHOD 1

$180^\circ - (100^\circ + \text{their part } (a))$ *(M1)*

$= 48.0019\ldots^\circ$ **OR** $0.837791\ldots$ *(A1)*

substituted area of triangle formula *(M1)*

$\frac{1}{2} \times 112.9937\ldots \times 210 \times \sin 48.002^\circ$ *(A1)*

$8820\,\text{m}^2\ \ (8817.18\ldots)$ ***A1***

METHOD 2

$\frac{\text{CE}}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100}$ *(M1)*

$(\text{CE} =)\ 158.472\ldots$ *(A1)*

substituted area of triangle formula *(M1)*

EITHER

$\frac{1}{2} \times 112.993\ldots \times 158.472\ldots \times \sin 100$ *(A1)*

OR

$\frac{1}{2} \times 210\ldots \times 158.472\ldots \times \sin(\text{their part } (a))$ *(A1)*

THEN

$8820\,\mathrm{m}^2\ \ (8817.18\ldots)$ ***A1***

METHOD 3

$\mathrm{CE}^2 = 210^2 + 112.993\ldots^2 - (2 \times 210 \times 112.993\ldots \times \cos(180 - 100 - \text{their part }(a)))$ ***(M1)***

$(\mathrm{CE} =)\ 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993\ldots \times 158.472\ldots \times \sin 100$ ***(A1)***

$8820\,\mathrm{m}^2\ \ (8817.18\ldots)$ ***A1***

[5 marks]

(d) Estimate DF. You may assume the highway has a width of zero. [4]

Markscheme

$1005 - 210$ **OR** 795 ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

$8817.18\ldots = \frac{1}{2} \times \mathrm{DF} \times (1005 - 210) \times \sin 48.002\ldots^\circ$ ***(A1)***

$(\mathrm{DF}=)\ 29.8\,\mathrm{m}\ \ (29.8473\ldots)$ ***A1***

[4 marks]