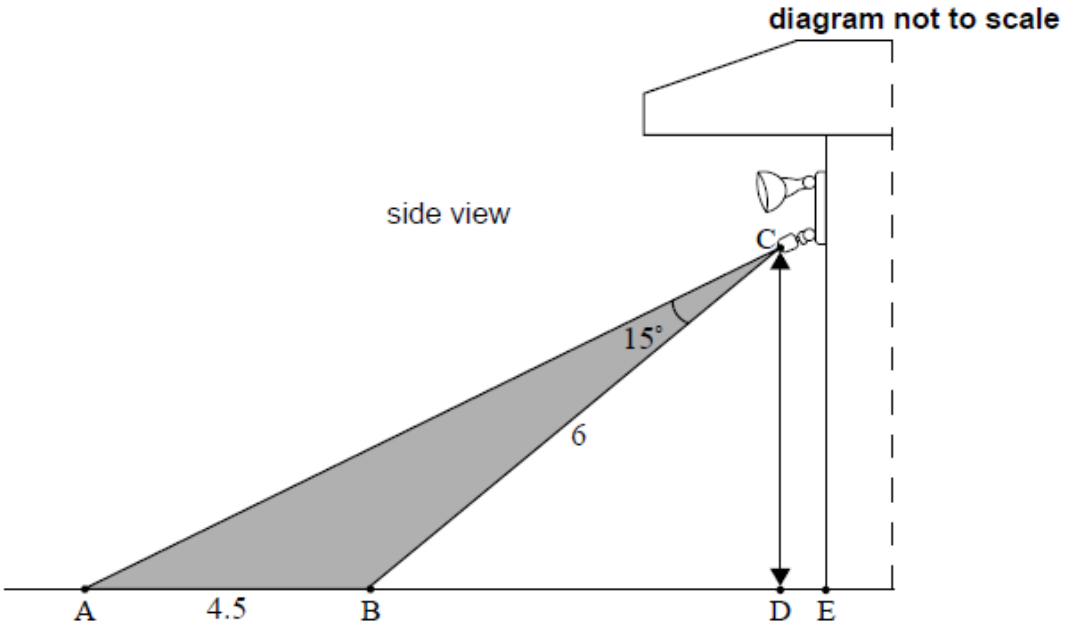
Geometry [152 marks]

1. [Maximum mark: 8] SPM.1.SL.TZ0.14

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle AĈB is 15°.



(a) Find CÂB. [3]

(b) Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

2. [Maximum mark: 9] EXN.1.SL.TZ0.11

A farmer owns a triangular field ABC. The length of side $[\mathrm{AB}]$ is $85\,\mathrm{m}$ and side $[\mathrm{AC}]$ is $110\,\mathrm{m}$. The angle between these two sides is 55°.

(a) Find the area of the field. [3]

(b) The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\mathrm{BC}]$.

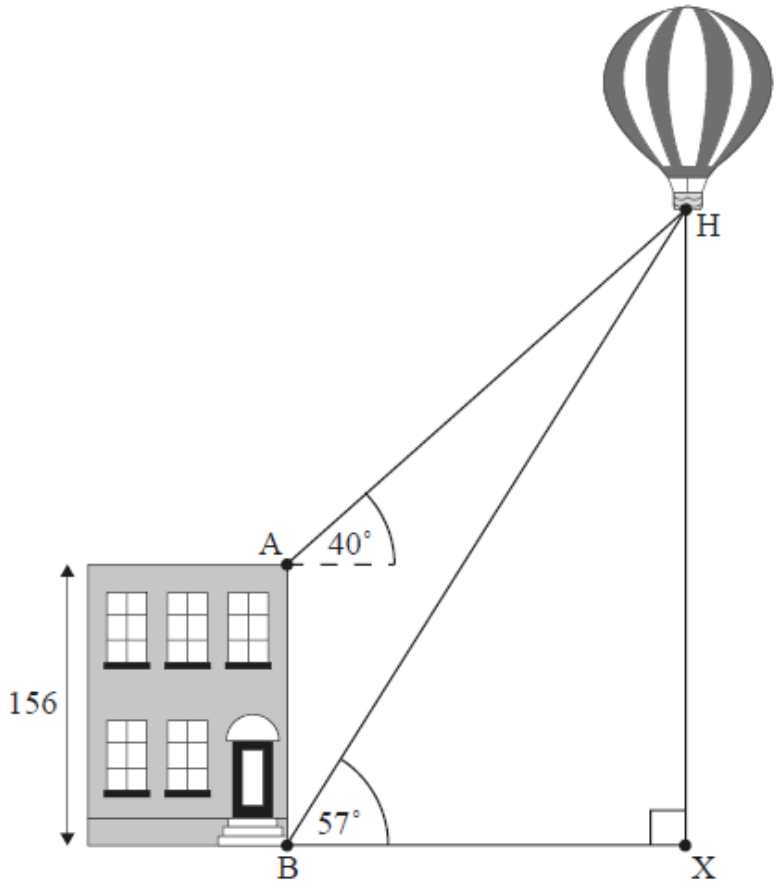
Find BD. Fully justify any assumptions you make. [6]

3. [Maximum mark: 6] 23M.1.SL.TZ1.2

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is 40°, and the angle of elevation from point B (at the base of the building) to H is 57°. Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $\mathrm{A\hat{H}B}$. [2]

(b) Calculate the distance from point B to point H. [3]

The hot-air balloon remains at a constant height as it moves further away from the building.

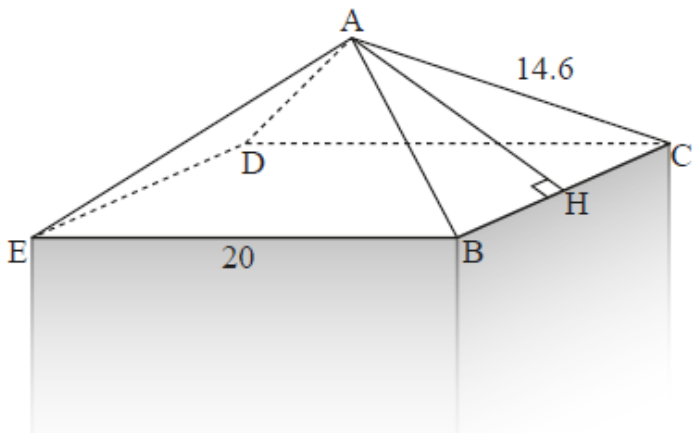
(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

4. [Maximum mark: 7] 23M.1.SL.TZ1.11

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly $20\ \text{cm}$. The length from the apex of the pyramid, A, to any corner of the base of the pyramid is $14.6\ \text{cm}$, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



(a) Write down the upper bound and lower bound for the possible lengths of edge AC. [2]

Point H is the midpoint of BC.

(b) Determine the upper bound and lower bound for AH, the slant height of the pyramid. [3]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than 22°.

(c) Show that this post is safe for children. Justify your answer. [2]

5. [Maximum mark: 5] 23M.1.SL.TZ1.13

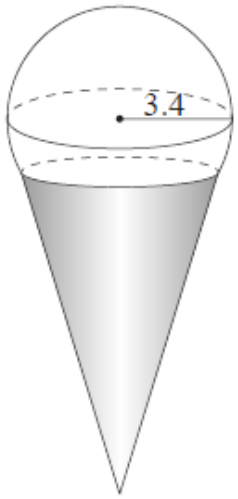
A boat travels $8\ \text{km}$ on a bearing of 315° and then a further $6\ \text{km}$ on a bearing of 045°.

Find the bearing on which the boat should travel to return directly to the starting point. [5]

6. [Maximum mark: 5] 23M.1.SL.TZ2.8

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\ \text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\ \text{cm}$ and a radius of $3\ \text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate.
Give your answer correct to the nearest cm^2. [2]

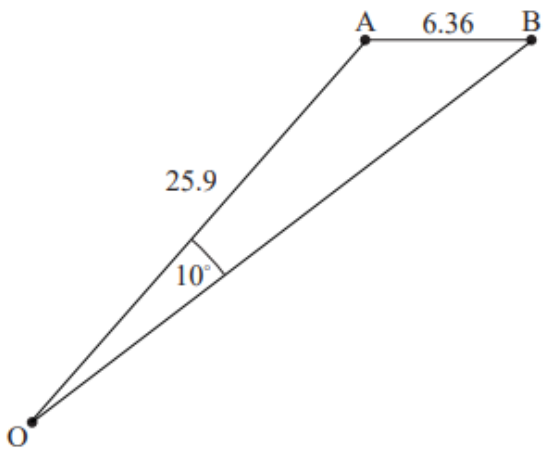
7. [Maximum mark: 17] 23M.2.SL.TZ2.1

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is $6.36\,\mathrm{m}$.

Odette is playing football in the park and is standing at point O, such that $\mathrm{A\hat{O}B} = 10^\circ$, $\mathrm{OA} = 25.9\,\mathrm{m}$ and $\mathrm{O\hat{A}B}$ is obtuse.

diagram not to scale

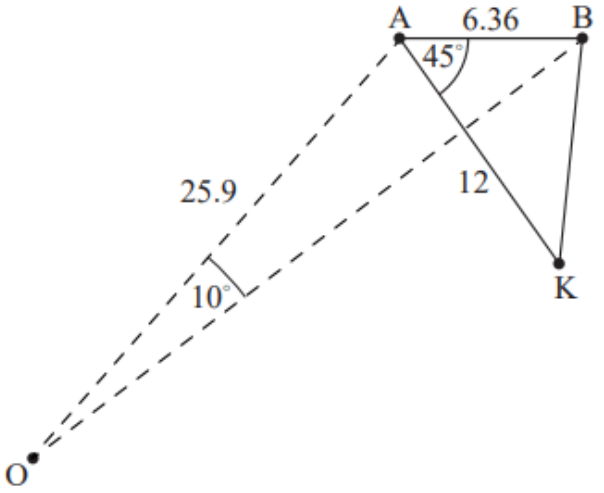


(a) Calculate the size of $\mathrm{A\hat{B}O}$. [3]

(b) Calculate the area of triangle AOB. [4]

Odette's friend, Khemil, is standing at point K such that he is $12\,\mathrm{m}$ from A and $\mathrm{K\hat{A}B} = 45^\circ$.

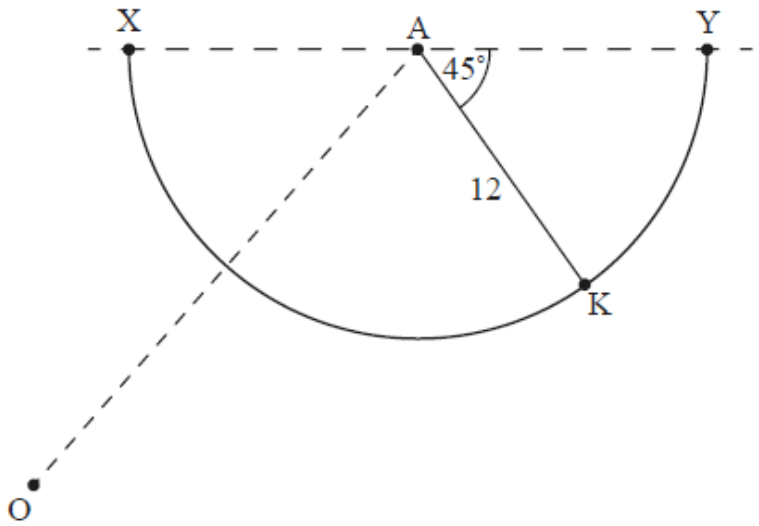
diagram not to scale



(c) Calculate Khemil's distance from B. [3]

XY is a semicircular path in the park with centre A, such that $\mathrm{K\widehat{A}Y} = 45^\circ$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $\mathrm{KX} = 22.2\,\mathrm{m}$, $\mathrm{K\widehat{O}X} = 53.8^\circ$ and $\mathrm{O\widehat{K}X} = 51.1^\circ$.

(d) Find whether Odette or Khemil is closer to the football. [4]

Khemil runs along the semicircular path to pick up the football.

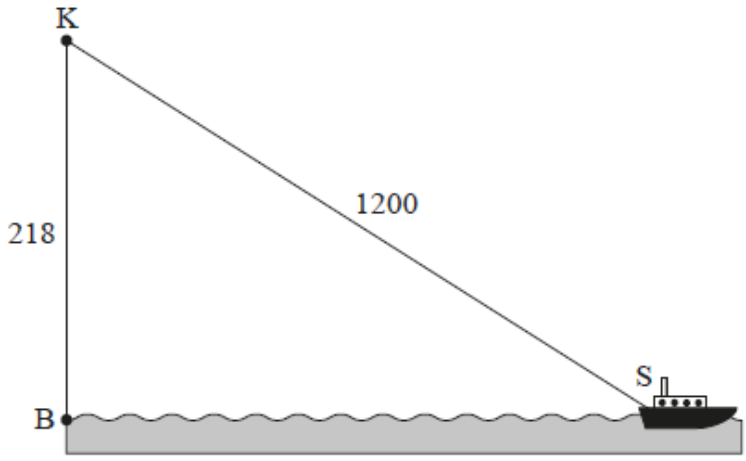
(e) Calculate the distance that Khemil runs. [3]

8. [Maximum mark: 6] 22N.1.SL.TZ0.1

Kacheena stands at point K, the top of a $218\,\mathrm{m}$ vertical cliff. The base of the cliff is located at point B. A ship is located at point S, $1200\,\mathrm{m}$ from Kacheena.

This information is shown in the following diagram.

diagram not to scale



(a) Find the angle of elevation from the ship to Kacheena. [2]

(b) Find the horizontal distance from the base of the cliff to the ship. [2]

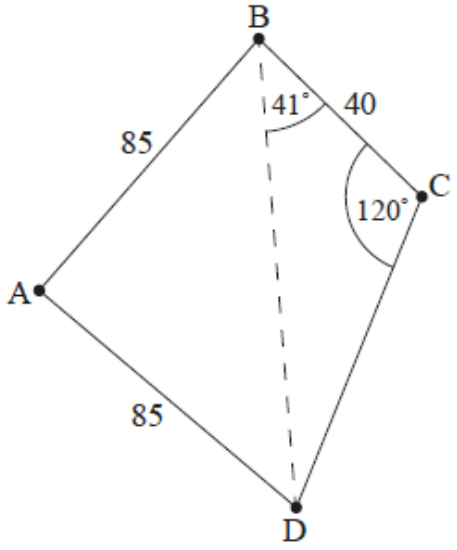
(c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

9. [Maximum mark: 17] 22N.2.SL.TZ0.2

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB} = 85\,\mathrm{m},\ \mathrm{AD} = 85\,\mathrm{m},\ \mathrm{BC} = 40\,\mathrm{m},\ \mathrm{C\widehat{B}D} = 41^\circ,\ \mathrm{B\widehat{C}D} = 120^\circ$$

diagram not to scale



(a.i) Write down the value of angle BDC. [1]

(a.ii) Hence use triangle BDC to find the length of path BD. [3]

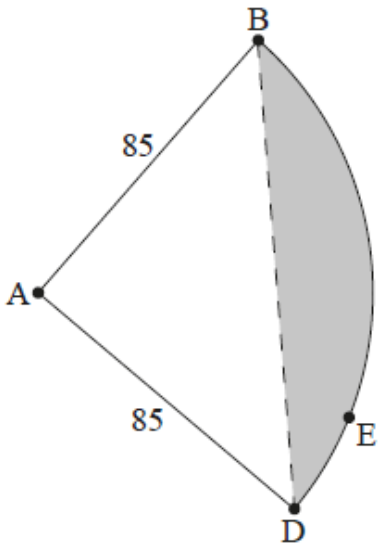
(b) Calculate the size of angle $\mathrm{B\widehat{A}D}$, correct to five significant figures. [3]

The size of angle $\mathrm{B\widehat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\widehat{A}D} = 77^\circ$ for the rest of this question.

(c) Find the area bounded by the path BD, and fences AB and AD. [3]

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

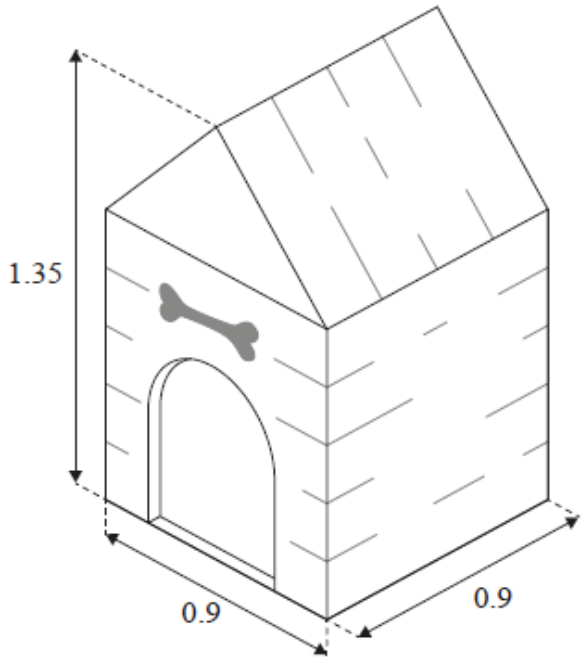
(f) Find the area of the shaded region in the proposed park. [3]

10. [Maximum mark: 5] 22M.1.SL.TZ1.1

The front view of a doghouse is made up of a square with an isosceles triangle on top.

The doghouse is $1.35\,\text{m}$ high and $0.9\,\text{m}$ wide, and sits on a square base.

diagram not to scale

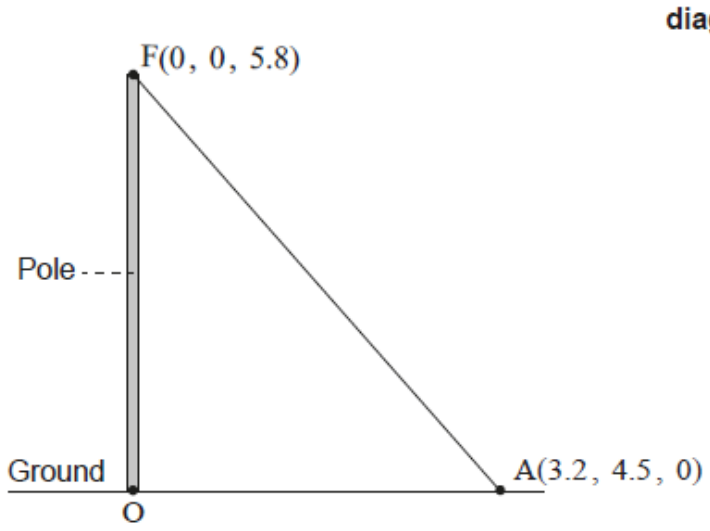


The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted. [5]

11. [Maximum mark: 4] 22M.1.SL.TZ1.2

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0,\ 0,\ 5.8)$. All units are in metres.



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2,\ 4.5,\ 0)$. The rope forms a straight line from A to F.

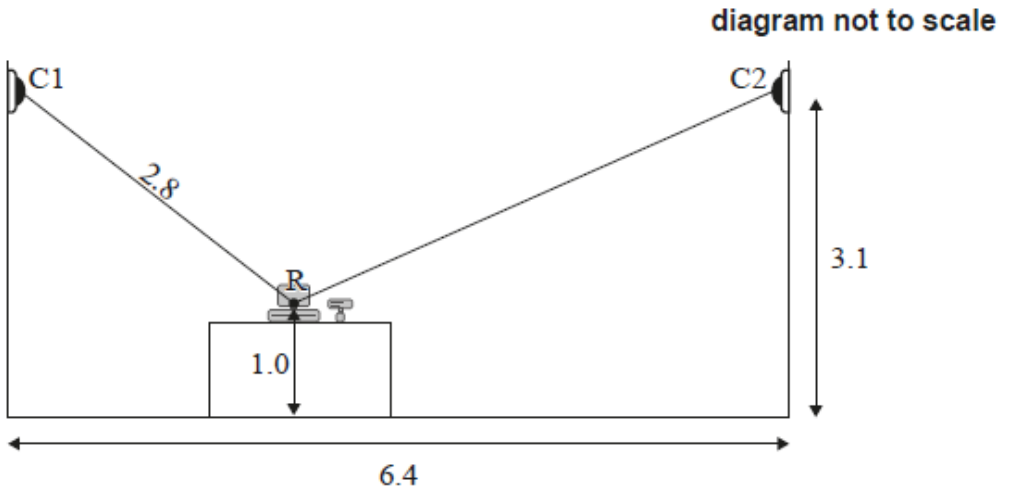
(a) Find the length of the rope connecting A to F. [2]

(b) Find $\mathrm{F\hat{A}O}$, the angle the rope makes with the ground. [2]

12. [Maximum mark: 8] 22M.1.SL.TZ2.3

The owner of a convenience store installs two security cameras, represented by points $C1$ and $C2$. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

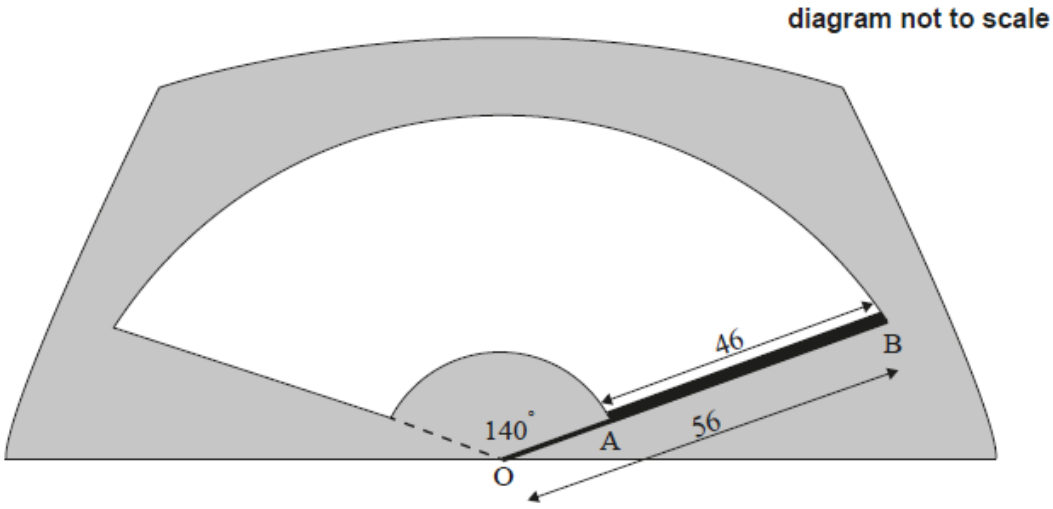
(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

13. [Maximum mark: 5] 22M.1.SL.TZ2.1

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of 140°. The windscreen is cleared by a rubber blade of length $46\,\mathrm{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\,\mathrm{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.



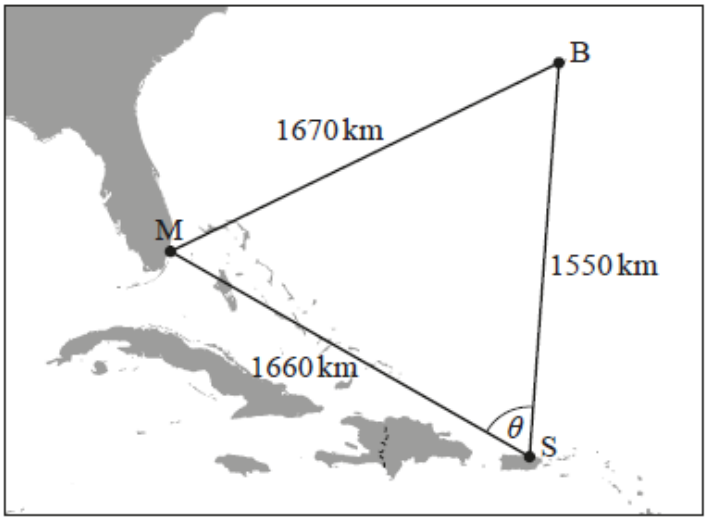
(a) Calculate the length of the arc made by B, the end of the rubber blade. [2]

(b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

14. [Maximum mark: 5] 21N.1.SL.TZ0.2

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

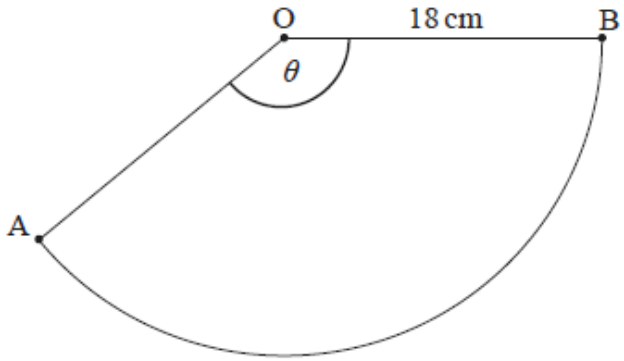
(a) Calculate the value of θ, the measure of angle MSB. [3]

(b) Find the area of the Bermuda Triangle. [2]

15. [Maximum mark: 5] 21N.1.SL.TZ0.8

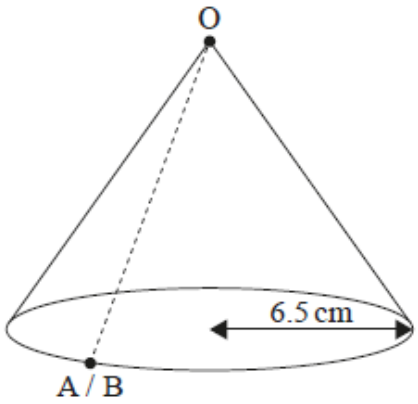
Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of $18\ \mathrm{cm}$ and $\mathrm{AOB} = \theta$ as shown in the diagram.

diagram not to scale



To make the hat, sides $[\mathrm{OA}]$ and $[\mathrm{OB}]$ are joined together. The hat has a base radius of $6.5\ \mathrm{cm}$.

diagram not to scale



(a.i) Write down the perimeter of the base of the hat in terms of π. [1]

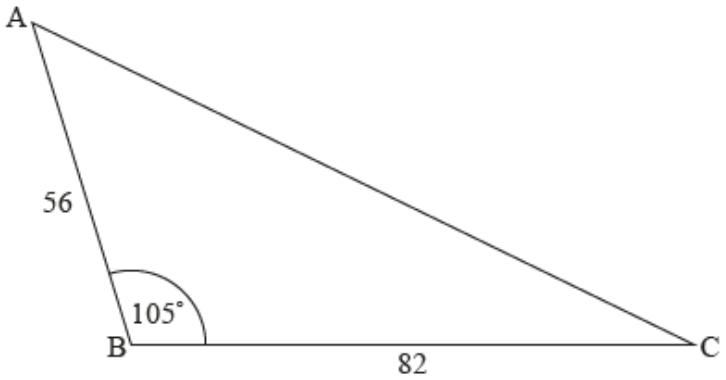
(a.ii) Find the value of θ. [2]

(b) Find the surface area of the outside of the hat. [2]

16. [Maximum mark: 5] 21M.1.SL.TZ1.9

A triangular field ABC is such that $\mathrm{AB} = 56\,\mathrm{m}$ and $\mathrm{BC} = 82\,\mathrm{m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

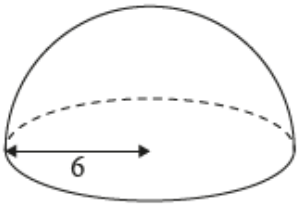
diagram not to scale



Calculate the maximum possible area of the field. [5]

17. [Maximum mark: 6] 21M.1.SL.TZ1.3

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is $6\,\mathrm{mm}$.



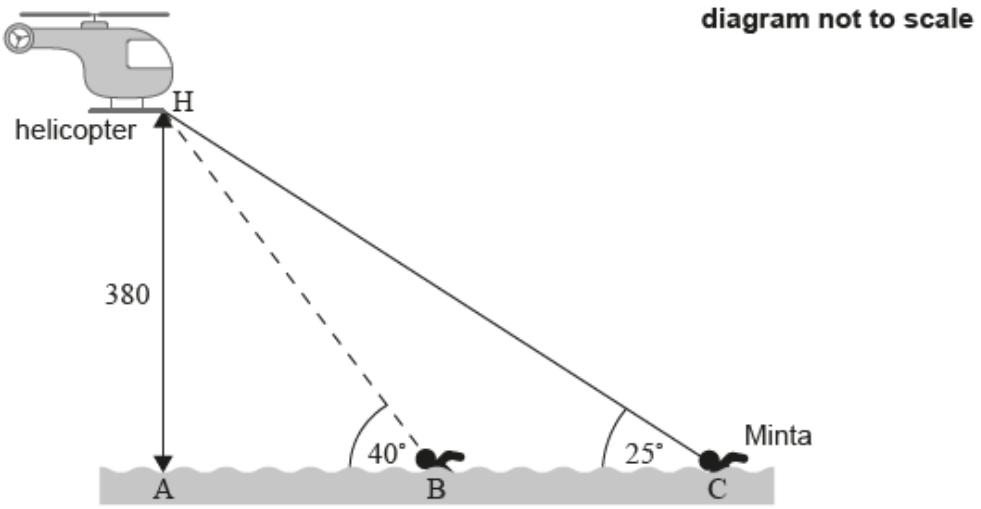
(a) Calculate the **total** surface area of one piece of candy. [4]

(b) The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\,\mathrm{mm}^2$.

Calculate the weight of chocolate required to coat one piece of candy. [2]

18. [Maximum mark: 7] 21M.1.SL.TZ2.4

The diagram below shows a helicopter hovering at point H, $380\,\mathrm{m}$ vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Write down the size of the angle of depression from H to C. [1]

(b) Find the distance from A to C. [2]

(c) Find the distance from B to C. [3]

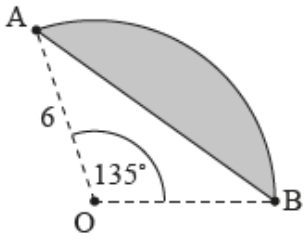
(d) Find Minta's speed, in metres per hour. [1]

19. [Maximum mark: 7] 21M.1.SL.TZ2.9

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius $6\,\mathrm{m}$, such that $\mathrm{AOB} = 135^\circ$. The straight border of the lawn is defined by chord $[\mathrm{AB}]$.

The lawn is shown as the shaded region in the following diagram.

diagram not to scale

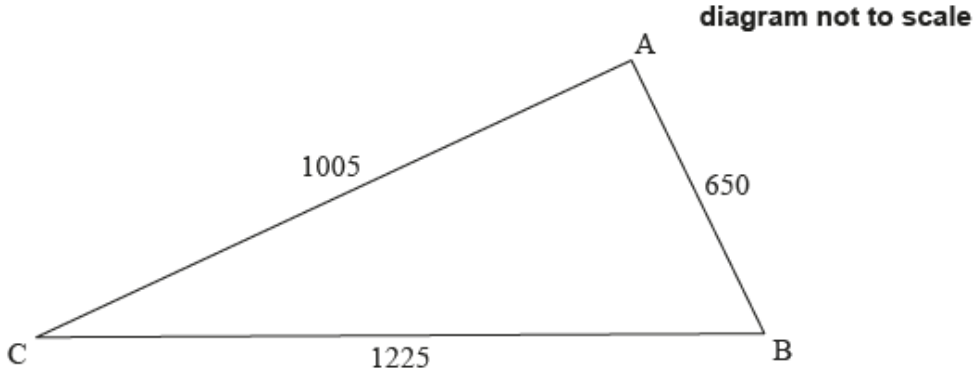


(a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]

(b) Find the area of the lawn. [4]

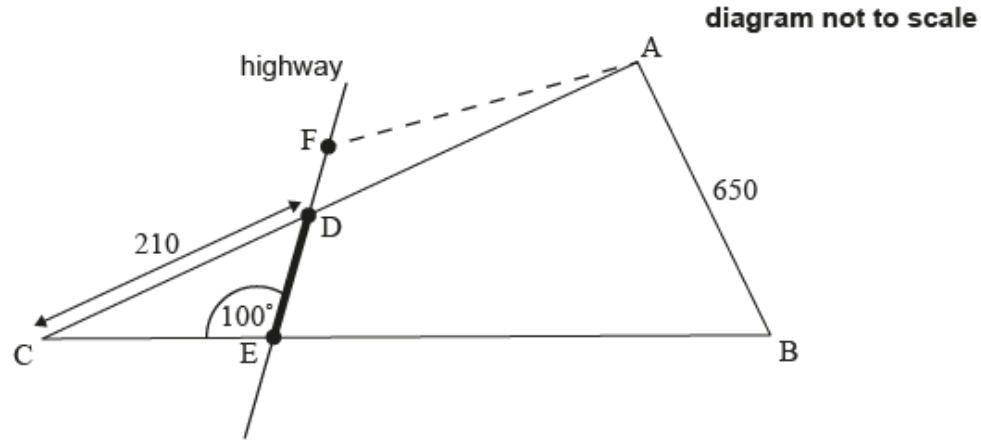
20. [Maximum mark: 15] 21M.2.SL.TZ2.2

A farmer owns a field in the shape of a triangle ABC such that $\mathrm{AB} = 650\,\mathrm{m},\ \mathrm{AC} = 1005\,\mathrm{m}$ and $\mathrm{BC} = 1225\,\mathrm{m}$.



(a) Find the size of ACB. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $\mathrm{DC} = 210\,\mathrm{m}$ and $\mathrm{CED} = 100^\circ$, as shown in the diagram below.



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

(d) Estimate DF. You may assume the highway has a width of zero. [4]