

Paper 3 (21.02) [27 marks]

1. [Maximum mark: 27]

22M.3.AHL.TZ1.2

This question is about a metropolitan area council planning a new town and the location of a new toxic waste dump.

A metropolitan area in a country is modelled as a square. The area has four towns, located at the corners of the square. All units are in kilometres with the x -coordinate representing the distance east and the y -coordinate representing the distance north from the origin at $(0, 0)$.

- Edison is modelled as being positioned at $E(0, 40)$.
- Fermitown is modelled as being positioned at $F(40, 40)$.
- Gaussville is modelled as being positioned at $G(40, 0)$.
- Hamilton is modelled as being positioned at $H(0, 0)$.

- (a) The model assumes that each town is positioned at a single point. Describe possible circumstances in which this modelling assumption is reasonable.

[1]

Markscheme

the size of each town is small (in comparison with the distance between the towns)

OR

if towns have an identifiable centre

OR

the centre of the town is at that point ***R1***

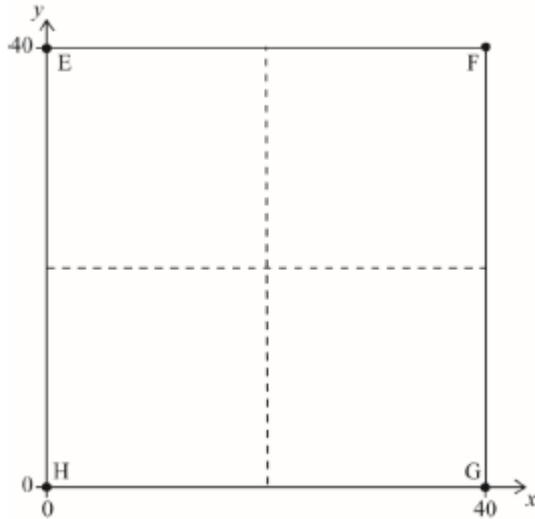
Note: Accept a geographical landmark in place of "centre", e.g. "town hall" or "capitol".

[1 mark]

- (b) Sketch a Voronoi diagram showing the regions within the metropolitan area that are closest to each town.

[1]

Markscheme



A1

Note: There is no need for a scale / coordinates here. Condone boundaries extending beyond the metropolitan area.

[1 mark]

The metropolitan area council decides to build a new town called Isaacopolis located at $I(30, 20)$.

A new Voronoi diagram is to be created to include Isaacopolis. The equation of the perpendicular bisector of $[IE]$ is $y = \frac{3}{2}x + \frac{15}{2}$.

- (c.i) Find the equation of the perpendicular bisector of $[IF]$.

[4]

Markscheme

the gradient of IF is $\frac{40-20}{40-30} = 2$ *(A1)*

negative reciprocal of any gradient (M1)

gradient of perpendicular bisector = $\frac{1}{2}$

Note: Seeing $-\frac{2}{3}$ (for example) used clearly as a gradient anywhere is evidence of the “negative reciprocal” method despite being applied to an inappropriate gradient.

midpoint is $(\frac{40+30}{2}, \frac{40+20}{2}) = (35, 30)$ (A1)

equation of perpendicular bisector is $y - 30 = -\frac{1}{2}(x - 35)$ A1

Note: Accept equivalent forms e.g. $y = -\frac{1}{2}x + \frac{95}{2}$ or $2y + x - 95 = 0$.

Allow **FT** for the final **A1** from their midpoint and gradient of perpendicular bisector, as long as the **M1** has been awarded

[4 marks]

- (c.ii) Given that the coordinates of one vertex of the new Voronoi diagram are $(20, 37.5)$, find the coordinates of the other two vertices within the metropolitan area.

[4]

Markscheme

the perpendicular bisector of **EH** is $y = 20$ (A1)

Note: Award this **A1** if seen in the y -coordinate of any final answer or if **20** is used as the y -value in the equation of any other perpendicular bisector.

attempt to use symmetry **OR** intersecting two perpendicular bisectors
(M1)

$$\left(\frac{25}{3}, 20\right) \quad A1$$

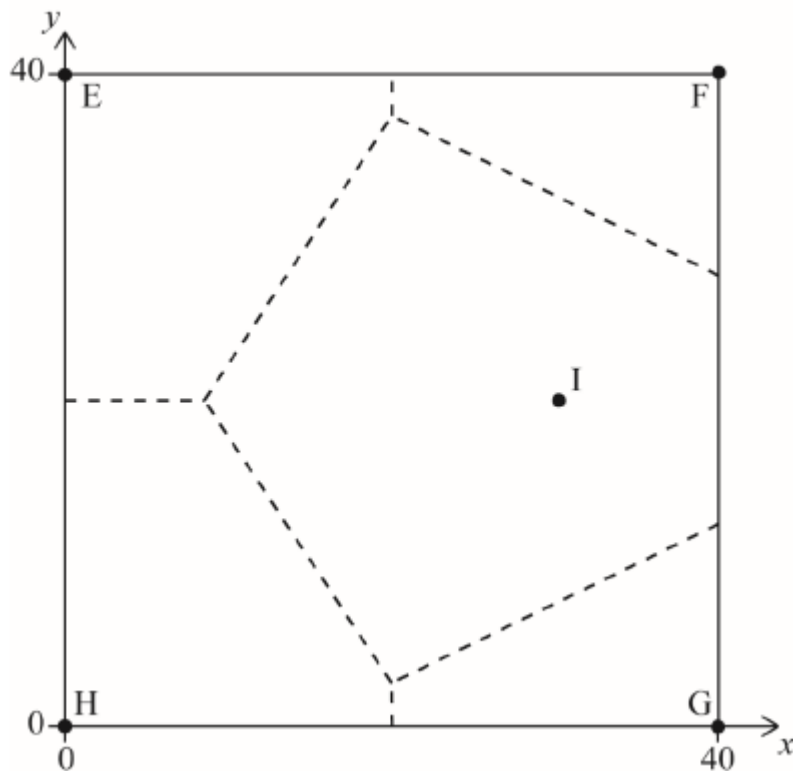
$$(20, 2.5) \quad A1$$

[4 marks]

(c.iii) Sketch this new Voronoi diagram showing the regions within the metropolitan area which are closest to each town.

[2]

Markscheme



M1A1

Note: Award *M1* for exactly four perpendicular bisectors around I (IE, IF, IG and IH) seen, even if not in exactly the right place.

Award *A1* for a completely correct diagram. Scale / coordinates are NOT necessary. Vertices should be in approximately the correct positions but only penalized if clearly wrong (condone northern and southern vertices appearing to be very close to the boundary).

Condone the Voronoi diagram extending outside of the square.

Do not award follow-through marks in this part.

[2 marks]

The metropolitan area is divided into districts based on the Voronoi regions found in part (c).

- (d) A car departs from a point due north of Hamilton. It travels due east at constant speed to a destination point due North of Gaussville. It passes through the Edison, Isaacopolis and Fermitown districts. The car spends 30% of the travel time in the Isaacopolis district.

Find the distance between Gaussville and the car's destination point.

[4]

Markscheme

30% of 40 is 12 (A1)

recognizing line intersects bisectors at $y = c$ (or equivalent) but different x -values (M1)

$$c = \frac{3}{2}x_1 + \frac{15}{2} \text{ and } c = -\frac{1}{2}x_2 + \frac{95}{2}$$

finding an expression for the distance in Isaacopolis in terms of one variable (M1)

$$x_2 - x_1 = (95 - 2c) - \frac{2c-15}{3} = 100 - \frac{8c}{3}$$

equating their expression to 12

$$100 - \frac{8c}{3} = 0.3 \times 40 = 12$$

$$c = 33$$

$$\text{distance} = 33 \text{ (km)} \quad A1$$

[4 marks]

A toxic waste dump needs to be located within the metropolitan area. The council wants to locate it as far as possible from the nearest town.

- (e.i) Find the location of the toxic waste dump, given that this location is not on the edge of the metropolitan area.

[4]

Markscheme

must be a vertex (award if vertex given as a final answer) (R1)

attempt to calculate the distance of at least one town from a vertex (M1)

Note: This must be seen as a calculation or a value.

correct calculation of distances A1

$$\frac{65}{3} \text{ OR } 21.7 \text{ AND } \sqrt{406.25} \text{ OR } 20.2$$

$$\left(\frac{25}{3}, 20\right) \quad A1$$

Note: Award *R1M0A0A0* for a vertex written with no other supporting calculations.

Award *R1M0A0A1* for correct vertex with no other supporting calculations. The final *A1* is not dependent on the previous *A1*. There is no follow-through for the final *A1*.

Do not accept an answer based on “uniqueness” in the question.

[4 marks]

(e.ii) Make one possible criticism of the council’s choice of location.

[1]

Markscheme

For example, any one of the following:

decision does not take into account the different population densities

closer to a city will reduce travel time/help employees

it is closer to some cities than others *R1*

Note: Accept any correct reason that engages with the scenario.

Do not accept any answer to do with ethical issues about whether toxic waste should ever be dumped, or dumped in a metropolitan area.

[1 mark]

The toxic waste dump, **T**, is connected to the towns via a system of sewers.

The connections are represented in the following matrix, M , where the order of rows and columns is (E, F, G, H, I, T).

$$M = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

A leak occurs from the toxic waste dump and travels through the sewers. The pollution takes one day to travel between locations that are directly connected.

The digit 1 in M represents a direct connection. The values of 1 in the leading diagonal of M mean that once a location is polluted it will stay polluted.

(f.i) Find which town is last to be polluted. Justify your answer.

[3]

Markscheme

METHOD 1

attempting M^3 **M1**

attempting M^4 **M1**

e.g.

last row/column of $M^3 = (3 \ 5 \ 1 \ 6 \ 0 \ 7)$

last row/column of $M^4 = (10 \ 12 \ 4 \ 16 \ 1 \ 18)$

hence Isaacopolis is the last city to be polluted **A1**

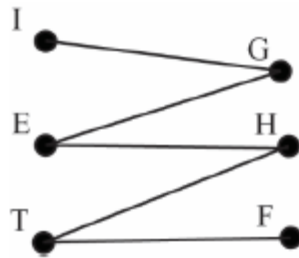
Note: Do not award the **A1** unless both M^3 and M^4 are considered.

Award **M1M0A0** for a claim that the shortest distance is from T to I and that it is 4, without any support.

METHOD 2

attempting to translate M to a graph or a list of cities polluted on each day (M1)

correct graph or list A1



hence Isaacopolis is the last city to be polluted A1

Note: Award *M1A1A1* for a clear description of the graph in words leading to the correct answer.

[3 marks]

(f.ii) Write down the number of days it takes for the pollution to reach the last town.

[1]

Markscheme

it takes 4 days A1

[1 mark]

- (f.iii) A sewer inspector needs to plan the shortest possible route through each of the connections between different locations. Determine an appropriate start point and an appropriate end point of the inspection route.

Note that the fact that each location is connected to itself does not correspond to a sewer that needs to be inspected.

[2]

Markscheme

EITHER

the orders of the different vertices are:

E	2	
F	1	
G	2	
H	2	
I	1	
T	2	(A1)

Note: Accept a list where each order is 2 greater than listed above.

OR

a correct diagram/graph showing the connections between the locations

(A1)

Note: Accept a diagram with loops at each vertex.

This mark should be awarded if candidate is clearly using their correct diagram from the previous part.

THEN

*"Start at F and end at I" OR "Start at I and end at F" **A1***

Note: Award **A1A0** for *"it could start at either F or I"*.

Award **A1A1** for *"IGEHTF" OR "FTHEGI"*.

Award **A1A1** for *"F and I" OR "I and F"*.

[2 marks]