

Paper 3 (21.02) [27 marks]

1. [Maximum mark: 27]

22M.3.AHL.TZ1.2

This question is about a metropolitan area council planning a new town and the location of a new toxic waste dump.

A metropolitan area in a country is modelled as a square. The area has four towns, located at the corners of the square. All units are in kilometres with the x -coordinate representing the distance east and the y -coordinate representing the distance north from the origin at $(0, 0)$.

- Edison is modelled as being positioned at $E(0, 40)$.
- Fermitown is modelled as being positioned at $F(40, 40)$.
- Gaussville is modelled as being positioned at $G(40, 0)$.
- Hamilton is modelled as being positioned at $H(0, 0)$.

(a) The model assumes that each town is positioned at a single point. Describe possible circumstances in which this modelling assumption is reasonable. [1]

(b) Sketch a Voronoi diagram showing the regions within the metropolitan area that are closest to each town. [1]

The metropolitan area council decides to build a new town called Isaacopolis located at $I(30, 20)$.

A new Voronoi diagram is to be created to include Isaacopolis. The equation of the perpendicular bisector of $[IE]$ is $y = \frac{3}{2}x + \frac{15}{2}$.

(c.i) Find the equation of the perpendicular bisector of $[IF]$. [4]

(c.ii) Given that the coordinates of one vertex of the new Voronoi diagram are $(20, 37.5)$, find the coordinates of the other two vertices within the metropolitan area. [4]

(c.iii) Sketch this new Voronoi diagram showing the regions within the metropolitan area which are closest to each town. [2]

The metropolitan area is divided into districts based on the Voronoi regions found in part (c).

- (d) A car departs from a point due north of Hamilton. It travels due east at constant speed to a destination point due North of Gaussville. It passes through the Edison, Isaacopolis and Fermitown districts. The car spends 30% of the travel time in the Isaacopolis district.

Find the distance between Gaussville and the car's destination point.

[4]

A toxic waste dump needs to be located within the metropolitan area. The council wants to locate it as far as possible from the nearest town.

- (e.i) Find the location of the toxic waste dump, given that this location is not on the edge of the metropolitan area.

[4]

- (e.ii) Make one possible criticism of the council's choice of location.

[1]

The toxic waste dump, T , is connected to the towns via a system of sewers.

The connections are represented in the following matrix, M , where the order of rows and columns is (E, F, G, H, I, T).

$$M = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

A leak occurs from the toxic waste dump and travels through the sewers. The pollution takes one day to travel between locations that are directly connected.

The digit 1 in M represents a direct connection. The values of 1 in the leading diagonal of M mean that once a location is polluted it will stay polluted.

(f.i) Find which town is last to be polluted. Justify your answer. [3]

(f.ii) Write down the number of days it takes for the pollution to reach the last town. [1]

(f.iii) A sewer inspector needs to plan the shortest possible route through each of the connections between different locations. Determine an appropriate start point and an appropriate end point of the inspection route.

Note that the fact that each location is connected to itself does not correspond to a sewer that needs to be inspected. [2]