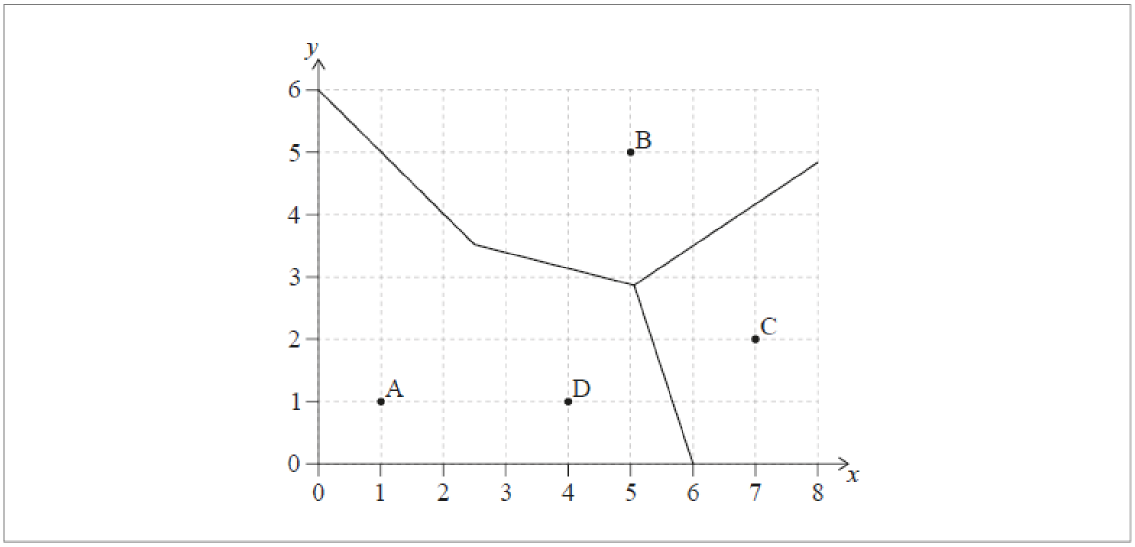
Review 3 [161 marks]

1. [Maximum mark: 6] 23M.1.SL.TZ2.7

Ani owns four cafes represented by points A, B, C and D. Ani wants to divide the area into delivery regions. This process has been started in the following incomplete Voronoi diagram, where 1 unit represents 1 kilometre.



The midpoint of CD is $(5.5,\ 1.5)$.

(a) Show that the equation of the perpendicular bisector of $[\mathrm{CD}]$ is $y = -3x + 18$. [3]

(b) Complete the Voronoi diagram shown above. [1]

Ani opens an office equidistant from three of the cafes, B, C and D. The equation of the perpendicular bisector of $[\mathrm{BC}]$ is $3y = 2x - 1.5$.

(c) Find the coordinates of the office. [2]

2. [Maximum mark: 5] 23M.1.SL.TZ2.12

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathrm{P}(X = x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in $\$$, is shown in the following table, where a negative gain means that the player loses money.

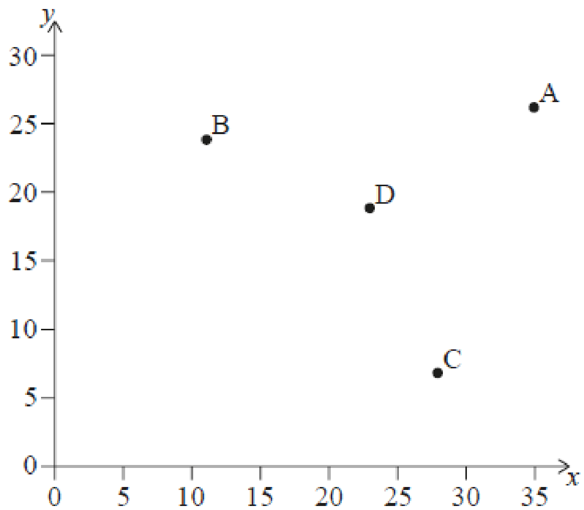
x	0	1	2	3	4	5
Player's gain ($\$$)	-4	-3	-1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

3. [Maximum mark: 5] 23M.1.AHL.TZ1.3

Three towns have positions $\mathrm{A}(35,\ 26)$, $\mathrm{B}(11,\ 24)$, and $\mathrm{C}(28,\ 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $\mathrm{D}(24,\ 19)$.



(a) Find AD. [2]

On a particular day, the mean temperatures recorded in each of towns A, B and C are $34\ ^{\circ}\mathrm{C}$, $29\ ^{\circ}\mathrm{C}$ and $30\ ^{\circ}\mathrm{C}$ respectively.

(b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

4. [Maximum mark: 5] 23M.1.AHL.TZ1.12

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6\ \sin\ (at + 30^{\circ})$ and $V_2 = 6\ \sin\ (at + 90^{\circ})$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V\ \sin\ (at + \theta^{\circ})$.

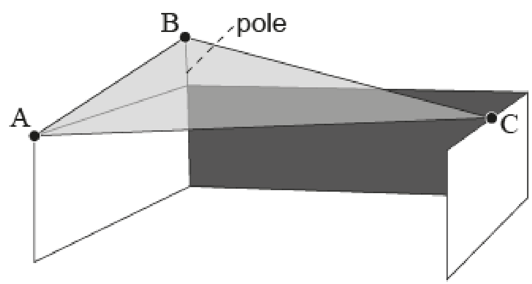
Determine the value of V and the value of θ. [5]

5. [Maximum mark: 9] 23M.1.AHL.TZ1.7

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\mathrm{m}$ wall, and at a point B, located at the top of a $1\,\mathrm{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. [2]

(b) Hence find the area of the triangular cover. [2]

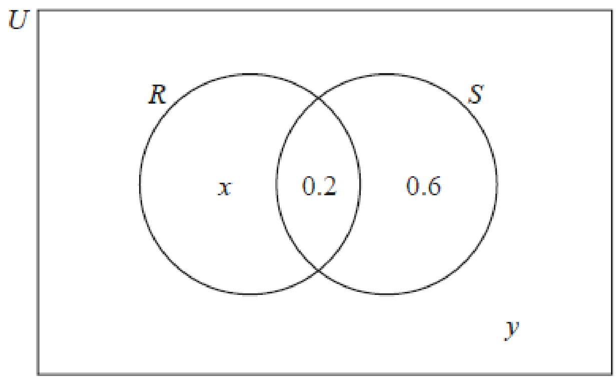
The point X on $[\mathrm{AC}]$ is such that $[\mathrm{BX}]$ is perpendicular to $[\mathrm{AC}]$.

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

6. [Maximum mark: 7] 23M.1.AHL.TZ2.3

The following Venn diagram shows two independent events, R and S. The values in the diagram represent probabilities.



(a) Find the value of x. [3]

(b) Find the value of y. [2]

(c) Find $\mathrm{P}(R\prime|S\prime)$. [2]

7. [Maximum mark: 6] 23M.1.AHL.TZ2.7

Akar starts a new job in Australia and needs to travel daily from Wollongong to Sydney and back. He travels to work for 28 consecutive days and therefore makes 56 single journeys. Akar makes all journeys by bus.

The probability that he is successful in getting a seat on the bus for any single journey is 0.86.

(a) Determine the expected number of these 56 journeys for which Akar gets a seat on the bus. [1]

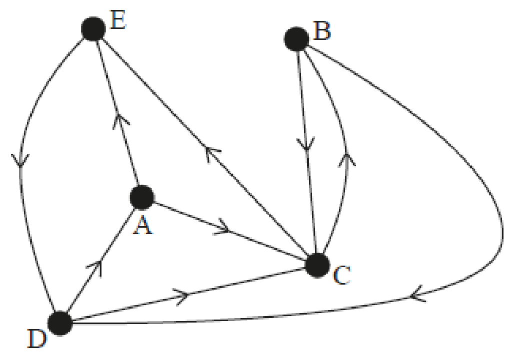
(b) Find the probability that Akar gets a seat on at least 50 journeys during these 28 days. [3]

The probability that Akar gets a seat on at most n journeys is at least 0.25.

(c) Find the smallest possible value of n. [2]

8. [Maximum mark: 7] 23M.1.AHL.TZ2.8

The following directed, unweighted, graph shows a simplified road network on an island, connecting five small villages marked A to E.



(a) Construct the adjacency matrix $\boldsymbol{M}$ for this network. [3]

Beatriz the bus driver starts at village E and drives to seven villages, such that the seventh village is A.

(b.i) Determine how many possible routes Beatriz could have taken, to travel from E to A. [2]

(b.ii) Describe one possible route taken by Beatriz, by listing the villages visited in order. [2]

9. [Maximum mark: 9] 23M.1.AHL.TZ2.9

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run 100 m. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks.

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

[2]

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

(e.i) Find the coefficient of determination for the data from the original table. [2]

(e.ii) Interpret this value in the context of the question. [1]

10. [Maximum mark: 6] 23M.1.AHL.TZ2.11

Two AC (alternating current) electrical sources of equal frequencies are combined.

The voltage of the first source is modelled by the equation $V = 30\ \sin\ (t + 60^\circ)$.

The voltage of the second source is modelled by the equation $V = 60\ \sin\ (t + 10^\circ)$.

(a) Determine the maximum voltage of the combined sources. [2]

(b) Using your graphic display calculator, find a suitable equation for the combined voltages, giving your answer in the form $V = V_0\ \sin\ (at + b)$, where a, b and V_0 are constants, $a > 0$ and $0^\circ \leq b \leq 180^\circ$. [4]

11. [Maximum mark: 7] 23M.1.AHL.TZ2.13

The matrices $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{Q} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}$ represent two transformations.

A triangle T is transformed by $\boldsymbol{P}$, and this image is then transformed by $\boldsymbol{Q}$ to form a new triangle, $T\prime$.

(a) Find the single matrix that represents the transformation $T\prime \rightarrow T$, which will undo the transformation described above. [4]

The area of $T\prime$ is $273\ \text{cm}^2$.

(b) Using your answer to part (a), or otherwise, determine the area of T. [3]

12. [Maximum mark: 8] 23M.1.AHL.TZ2.14

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

13. [Maximum mark: 15] 23M.2.SL.TZ1.1

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Year $^\circ \text{C}$ (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates $(1708,\ 8.73)$ and $(1958,\ 9.45)$.

(a) Calculate the gradient of the straight line that passes through these two points. [2]

(b.i) Interpret the meaning of the gradient in the context of the question. [1]

(b.ii) State appropriate units for the gradient. [1]

(c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]

(d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

(e.i) Find the equation of the regression line y on x. [2]

(e.ii) Find the value of r, the Pearson's product-moment correlation coefficient. [1]

(f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses his regression line to predict the year when the mean annual temperature will first exceed $15\,^\circ\text{C}$.

(g) State two reasons why Thandizo's prediction may not be valid. [2]

14. [Maximum mark: 13] 23M.2.AHL.TZ1.1

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Year $^\circ\mathrm{C}$ (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates $(1708,\ 8.73)$ and $(1958,\ 9.45)$.

(a) Calculate the gradient of the straight line that passes through these two points. [2]

(b.i) Interpret the meaning of the gradient in the context of the question. [1]

(b.ii) State appropriate units for the gradient. [1]

(c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]

(d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

(e.i) Find the equation of the regression line y on x. [2]

(e.ii) Find the value of r, the Pearson's product-moment correlation coefficient. [1]

(f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

15. [Maximum mark: 17] 23M.2.AHL.TZ1.3

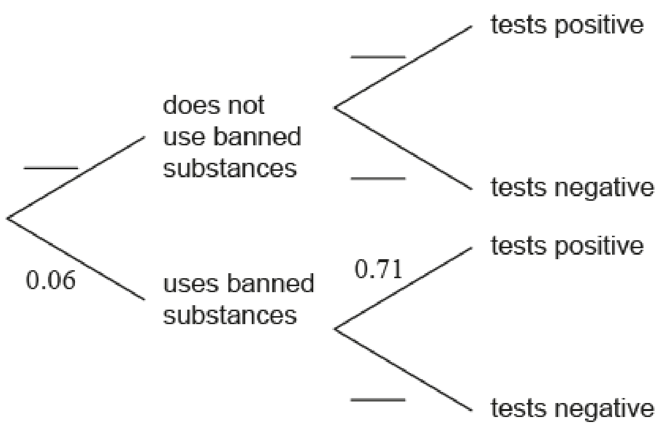
A large international sports tournament tests their athletes for banned substances.
They interpret a positive test result as meaning that the athlete uses banned substances.
A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

(a) Using the information given, complete the following tree diagram.



[2]

(b.i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative. [2]

(b.ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [2]

(c.i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result. [3]

(c.ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [2]

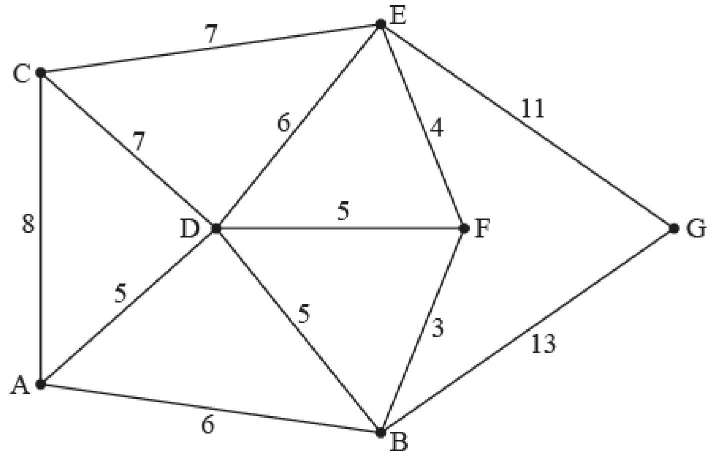
Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

(d) Calculate the probability that none of the athletes in Team X will test positive. [4]

(e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

16. [Maximum mark: 17] 23M.2.AHL.TZ1.4

The vertices in the following graph represent seven towns. The edges represent their connecting roads. The weight on each edge represents the distance, in kilometres, between the two connected towns.



(a) Determine whether it is possible to complete a journey that starts and finishes at different towns that also uses each of the roads exactly once. Give a reason for your answer. [2]

The shortest distance, in kilometres, between any two towns is given in the table.

	A	B	C	D	E	F	G
A		6	8	5	11	9	19
B	6		12	5	7	3	13
C	8	12		7	7	a	b
D	5	5	7		6	5	c
E	11	7	7	6		4	11
F	9	3	a	5	4		d
G	19	13	b	c	11	d	

(b) Find the value of

(i) a;

(ii) b;

(iii) c;

(iv) d. [2]

(c) Use the nearest neighbour algorithm, starting at vertex G to find an upper bound for the travelling salesman problem. [3]

(d.i) Sketch a minimum spanning tree for the subgraph with vertices $\mathrm{A},\ \mathrm{B},\ \mathrm{C},\ \mathrm{D},\ \mathrm{E},\ \mathrm{F}$. [2]

(d.ii) Write down the total weight of the minimum spanning tree. [2]

(e) Hence find a lower bound for the travelling salesman problem. [2]

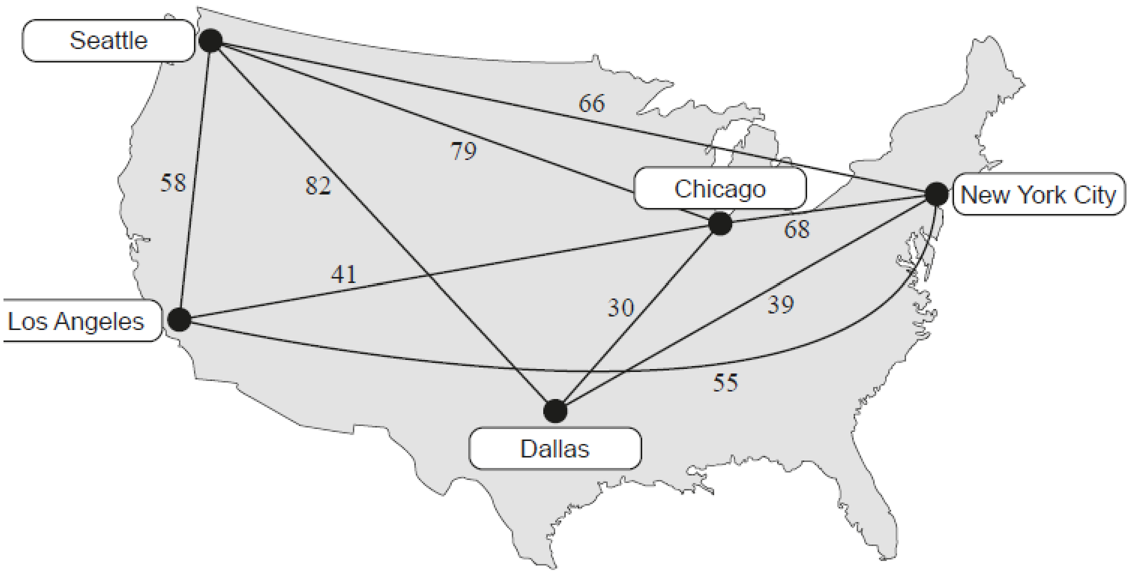
(f) Explain one way in which an improved lower bound could be found. [1]

It is found that the optimum solution starting at A is actually $\mathrm{A–C–E–G–B–F–D–A}$.

(g) Given that the length of each road shown on the graph is given to the nearest kilometre, find the lower bound for the total distance in the optimal solution. [3]

17. [Maximum mark: 19] 23M.2.AHL.TZ2.4

The following graph shows five cities of the USA connected by weighted edges representing the cheapest direct flights in dollars ($\$$) between cities.



(a) Explain why the graph can be described as "connected", but not "complete". [2]

(b) Find a minimum spanning tree for the graph using Kruskal's algorithm.

State clearly the order in which your edges are added, and draw the tree obtained. [3]

(c) Using only the edges obtained in your answer to part (b), find an upper bound for the travelling salesman problem. [2]

Ronald lives in New York City and wishes to fly to each of the other cities, before finally returning to New York City. After some research, he finds that there exists a direct flight between Los Angeles and Dallas costing $\$26$. He updates the graph to show this.

(d) By using the nearest neighbour algorithm and starting at Los Angeles, determine a better upper bound than that found in part (c).

State clearly the order in which you are adding the vertices. [3]

(e.i) By deleting the vertex which represents Chicago, use the deleted vertex algorithm to determine a lower bound for the travelling salesman problem. [3]

(e.ii) Similarly, by instead deleting the vertex which represents Seattle, determine another lower bound. [2]

(f) Hence, using your previous answers, write down your best inequality for the **least** expensive tour Ronald could take. Let the variable C represent the total cost, in dollars, for the tour. [2]

(g) Write down a tour that is strictly greater than your lower bound and strictly less than your upper bound. [2]