SL 5.5 Integration

SL 5.8 Area under curve

Introduction

The purpose of this presentation is to review the chapters on integration.

The presentation is divided into three parts:

- Approximation of the area under the curve using trapezoidal rule.
- Calculating the anti-derivatives (indefinite integrals) of a given function.
- Using the definite integrals to calculate the exact area under the curve.

At the very end I've included a typical question you may expect to appear on the exam.

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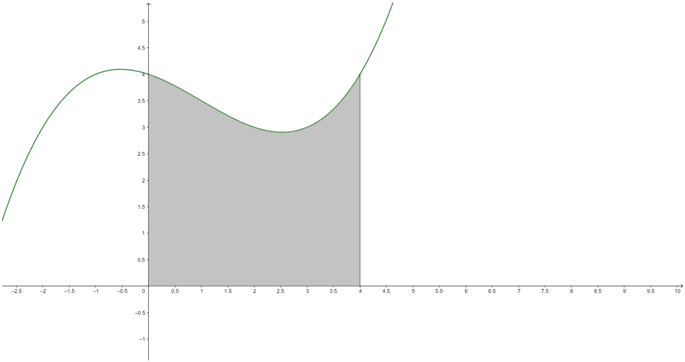
TRAPEZOIDAL RULE

Suppose we want to calculate the shaded area:



Which is the area under the graph of $y = \frac{1}{12}x^3 - \frac{1}{4}x^2 - \frac{1}{3}x + 4$ between $x = 0$ and $x = 4$.

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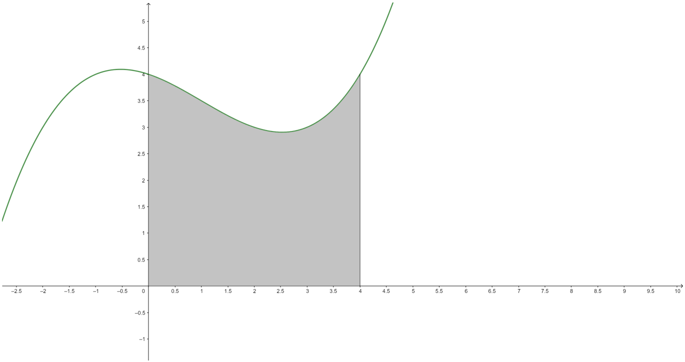

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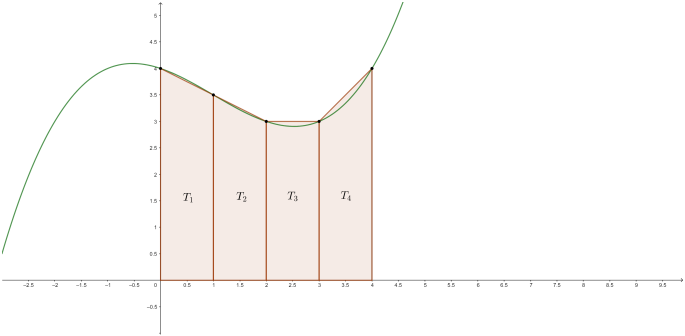
We will start by dividing the area into 4 trapeziums as follows



We can then say that:

$$Area \approx T_1 + T_2 + T_3 + T_4$$

Note that I've used $\approx$ because the area is not exactly equal to the sum of the areas of the trapeziums. You can see that on the diagram.

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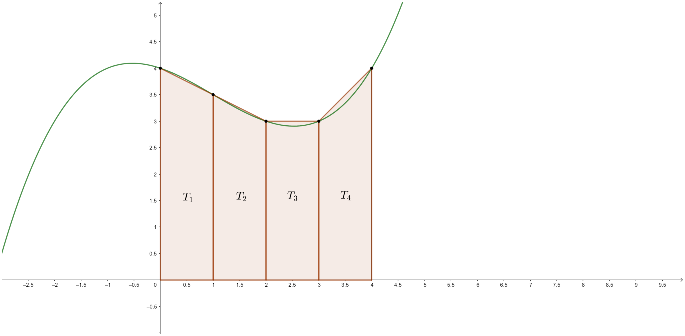

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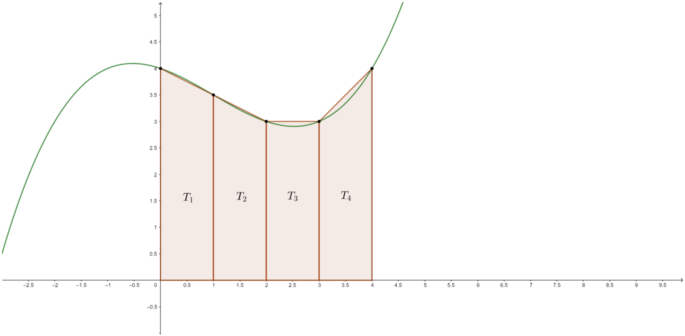

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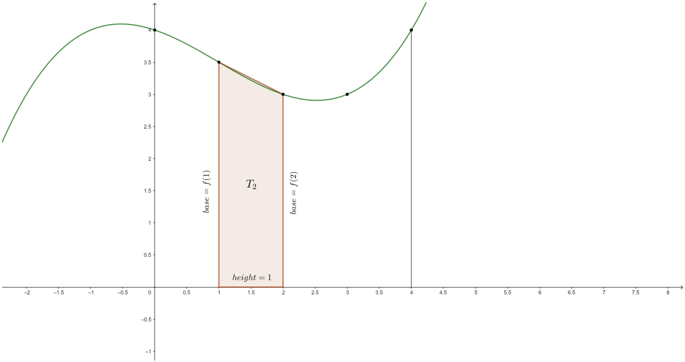
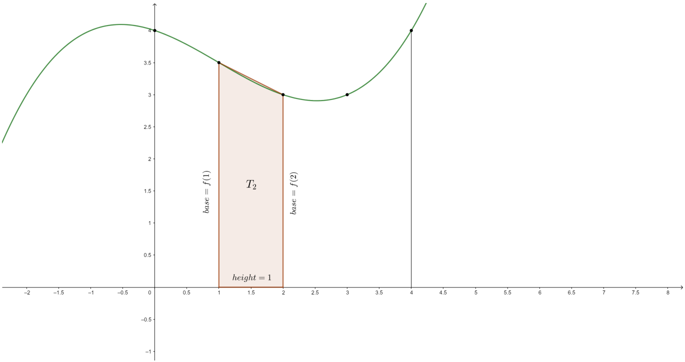
The area of each trapezium is easy to calculate. Let's take a look at T_2:



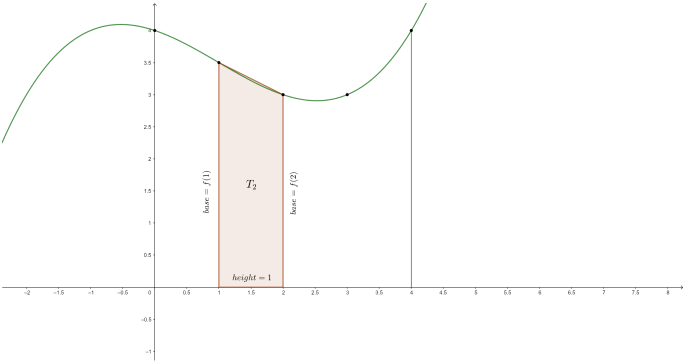
We should think of it as a trapezium rotated by 90°.
The formula for the area of the trapezium is:

$$A = \frac{(a+b)\cdot h}{2}$$



Note that in our case of T_2 we have $a = f(1)$, $b = f(2)$ and $h = 1$. We can calculate a and b be substituting 1 and 2 into the formula for our function. We have $a = 3.5$ and $b = 3$

$$T_2 = \frac{(3.5 + 3) \cdot 1}{2} = 3.25$$



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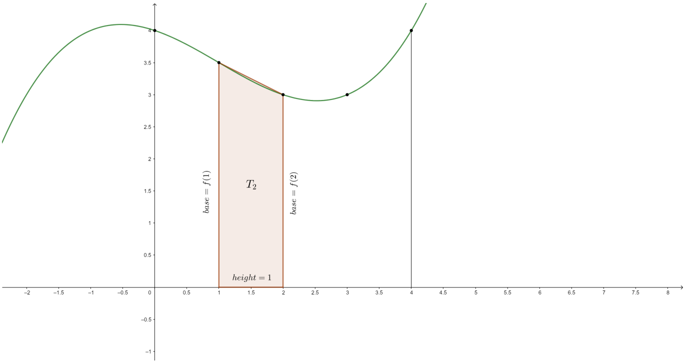

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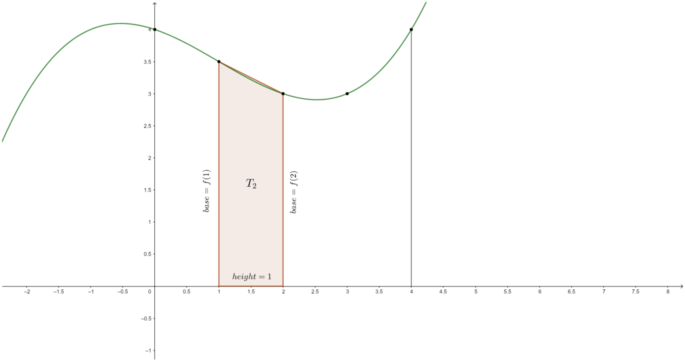

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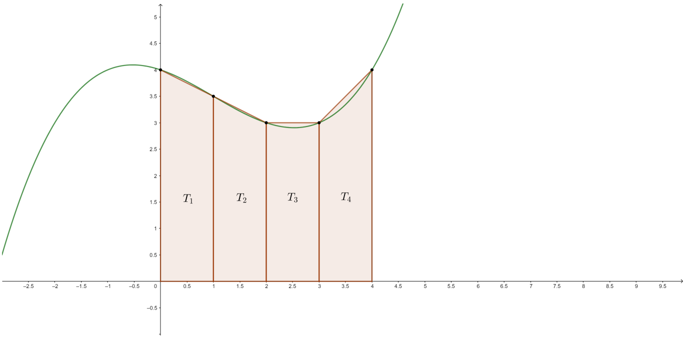
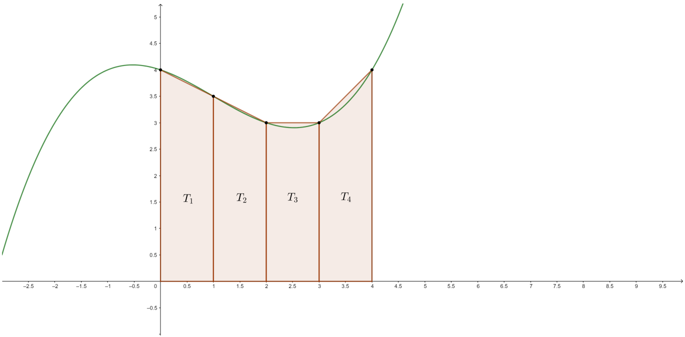


We can calculate the remaining trapeziums in an analogous way and we get $T_1 = 3.75$, $T_3 = 3$ and $T_4 = 3.5$.

So the area under the curve is approximately equal to:

$$Area \approx 3.75 + 3.25 + 3 + 3.5 = 13.5$$



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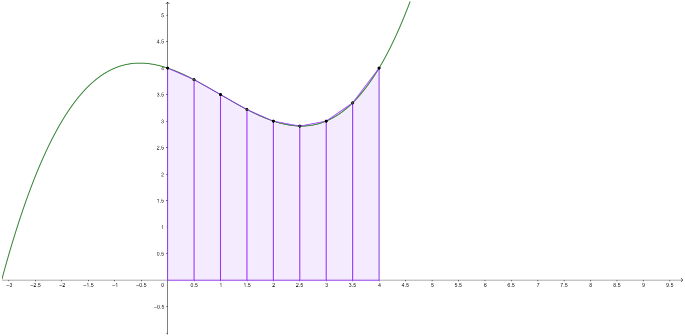
This is an approximation of the area and this approximation can be improved by considering more trapeziums. We chose 4, but we could have chosen 8 or any other number.

Note that with 8 trapeziums the area fits much better:



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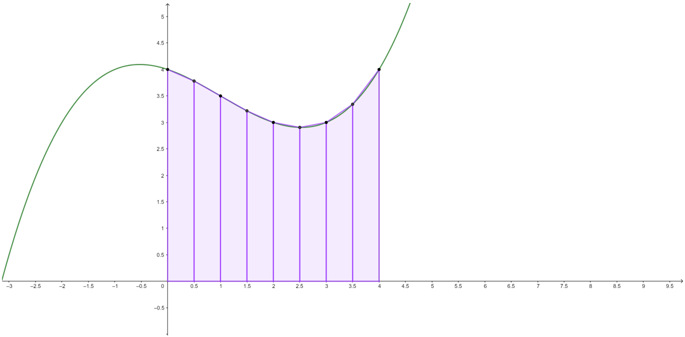

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Formula

Instead of calculating the area of each trapezium separately we can apply the formula:

$$Area \approx \frac{h}{2}\Big(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)\Big)$$

Here h is the height of each trapezium and x_is are the x-coordinates of the vertices of the trapezium on the horizontal axis.

Note that this formula is what we would get if we added the formulas for the areas of each trapezium, so there is no need to calculate the areas separately.

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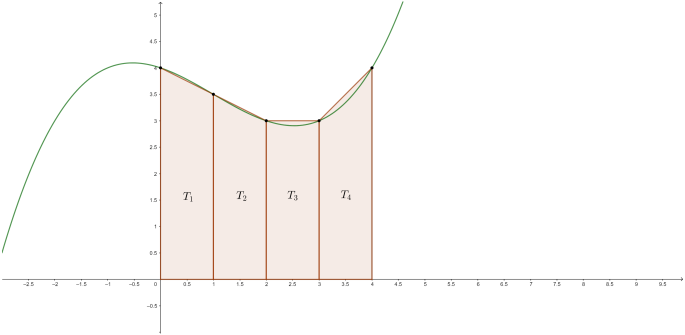

We have $h = 1$ and the x_is are $0, 1, 2, 3$ and 4, so the area is:

$$Area \approx \frac{1}{2}\left(f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right) = \frac{1}{2}\left(4 + 7 + 6 + 6 + 4\right) = 13.5$$

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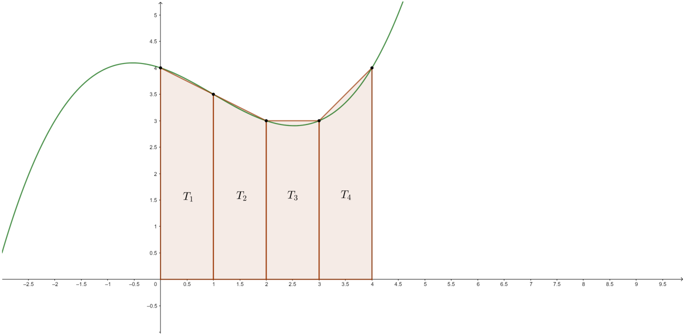

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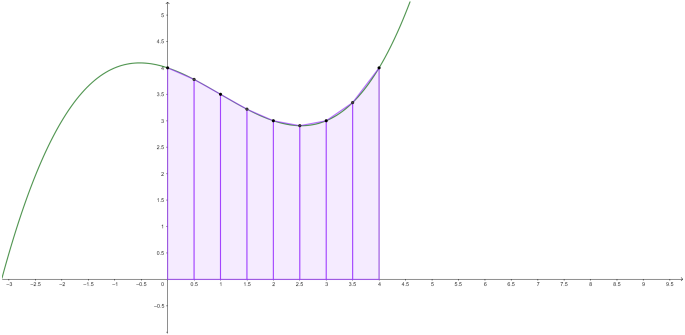

We would have $h = 0.5$ and the x_is are $0, 0.5, 1, 1.5, \ldots, 4$ and the area would be:

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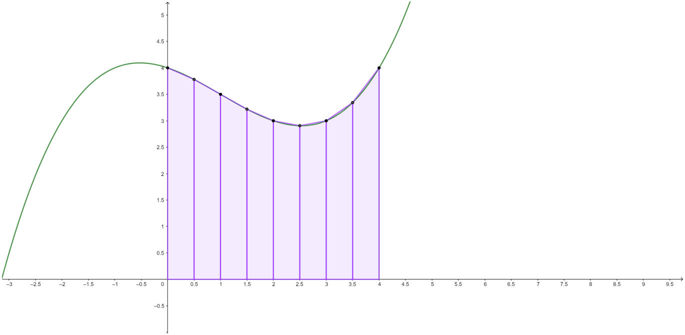

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GDC

Quick tip on the use of GDC.

If you need to evaluate function several times (i.e. calculate $f(1)$, then $f(2)$, then $f(3)$ etc.), it makes sense to store the function under $Y1$ (in the GRAPH option) and then use VARS $\to$ GRPH $\to$ Y1. Then you can do $Y1(1)$ to evaluate the function at $x = 1$, $Y1(2)$ to evaluate at $x = 2$ etc.

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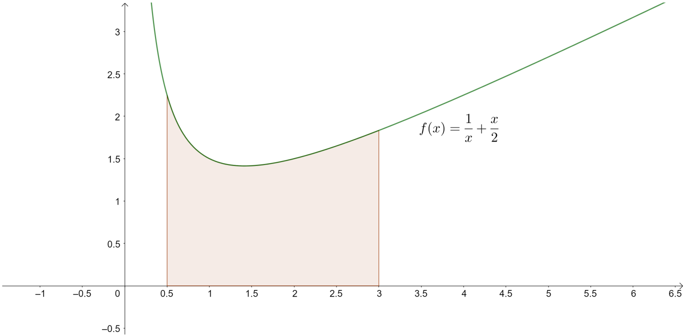
Exercise

Approximate the given area using 5 trapeziums.



The x is between 0.5 and 3 and we want 5 trapeziums so each trapezium will have height of 0.5 and the xs that we will use will be 0.5, 1, 1.5, 2, 2.5 and 3.

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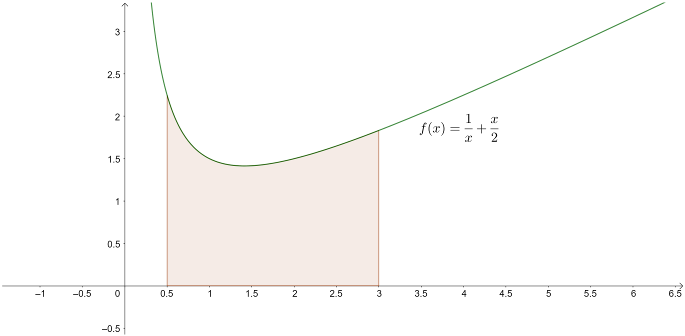

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INDEFINITE INTEGRALS

Anti-differentiation

Given a function $f(x)$ its anti-derivative is a function $F(x)$ such that $F'(x) = f(x)$.

Note that the anti-derivative is not unique.

For example:

$(2x^3)' = 6x^2$ but also $(2x^3 + 17)' = 6x^2$ and $(2x^3 - 100)' = 6x^2$.

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Terminology

Another name for anti-derivative is **integral** and the process of finding the anti-derivative is called anti-differentiation or more commonly **integration**. We use $\int$ to denote the process of integration.

Important formula

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$$

Examples:

- $\int x^2 \, dx = \frac{1}{3} x^3 + c$,
- $\int x^5 \, dx = \frac{1}{6} x^6 + c$,
- $\int x \, dx = \int x^1 \, dx = \frac{1}{2} x^2 + c$,
- $\int 1 \, dx = \int x^0 \, dx = x + c$,
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Integration rules

Recall that we have the following rules for differentiation:

$$(cf(x))' = cf'(x) \qquad \text{and} \qquad (f(x) \pm g(x))' = f'(x) \pm g'(x)$$

So for example: $(7x^3)' = 21x^2$ and $(5x^2 - x^4)' = 10x - 4x^3$.

It follows from these two that we also must have:

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So if we want to integrate $f(x) = x^3 + 4x - 5x^2$, we can do this term by term:

$$\begin{aligned}\int x^3 + 4x - 5x^2\ dx &= \int x^3\ dx + 4\int x\ dx - 5\int x^2\ dx = \\ &= \frac{1}{4}x^4 + 4\cdot\frac{1}{2}x^2 - 5\cdot\frac{1}{3}x^3 + c = \\ &= \frac{1}{4}x^4 + 2x^2 - \frac{5}{3}x^3 + c\end{aligned}$$

Of course when you calculate the integral you do not need to write down all the intermediate step, you can go directly to the last line.

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Calculate the following integrals:

$$\int 3x^2 - 2x \; dx = x^3 - x^2 + c$$

$$\int 2x^4 + x^{-2} \; dx = \frac{2}{5}x^5 - x^{-1} + c$$

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Note that in all our examples we had to include c.

This is because the derivative of a constant is 0, so the derivative of $x^2 + 7$ is the same as that of $x^2 - 5$ or simply x^2. In all these cases the derivative is $2x$, so the anti-derivative (integral) of $2x$ can be any of these.

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Integrals with intitial conditions

Consider the following example.

Find $f(x)$ if $f'(x) = x^3 - x$ and $f(2) = 5$.

We are given that $f'(x) = x^3 - x$, so we're given the derivative, we need to find the anti-derivative, so we integrate:

$$f(x) = \int x^3 - x \, dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$$

So our function is $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$, but we also know that $f(2) = 5$. So we substitute this into our formula and get:

$$5 = \frac{1}{4} \cdot 2^4 - \frac{1}{2} \cdot 2^2 + c$$

This allows us to calculate that $c = 3$.

Integrals with intitial conditions

Consider the following example.

Find $f(x)$ if $f'(x) = x^3 - x$ and $f(2) = 5$.

We are given that $f'(x) = x^3 - x$, so we're given the derivative, we need to find the anti-derivative, so we integrate:

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So our function is $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$, but we also know that $f(2) = 5$. So we substitute this into our formula and get:

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So the answer to the problem:

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Integrals with intitial conditions - practice

Find $f(x)$ if $f'(x) = 4x^3 - 5$ and $f(1) = 2$.

$$f(x) = \int 4x^3 - 5 \, dx = x^4 - 5x + c$$

Using $f(1) = 2$:

$$2 = 1^4 - 5 \cdot 1 + c$$

So $c = 6$, which means that $f(x) = x^4 - 5x + 6$.

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Find y as a function of x given that $\frac{dy}{dx} = x^2 - 4$ and $y = 2$, when $x = 3$.

Note that this is exactly the same problem, only different notation is used.

$$y = \int x^2 - 4 \, dx = \frac{1}{3}x^3 - 4x + c$$

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Find y as a function of x given that $\frac{dy}{dx} = \frac{2}{x^3} + x$ and the point $(1, 5)$ lies on the graph of this function.

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DEFINITE INTEGRALS

Definite integrals

A definite integral is an integral over a specific interval. It is denoted by including the boundaries of the interval above and below the integral sign. The definite integral of x^2 between 1 and 3 (so over the interval $[1, 3]$) is denoted:

$$\int_1^3 x^2 \, dx$$

A definite integral is calculated as follows:

$$\int_1^3 x^2 \, dx = \left[\frac{1}{3}x^3\right]_1^3 = \left(\frac{1}{3} \cdot 3^3\right) - \left(\frac{1}{3} \cdot 1^3\right) = 26\frac{2}{3}$$

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Definite integrals

Let's go through this step by step:

$$\int_1^3 x^2\,dx = \left[\frac{1}{3}x^3\right]_1^3$$

First we calculate the indefinite integral. Note that c is omitted, we will come back to this later.

$$\left(\frac{1}{3}\cdot 3^3\right) \qquad \left(\frac{1}{3}\cdot 1^3\right)$$

Then we substitute the boundary values into the indefinite integrals. And finally we subtract these values:

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The definite integrals can be calculated on the GDC:

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Ti - 84

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Then you need to enter the function you want to integrate and the boundaries. Check our answer to $\int_1^3 x^2\, dx$.

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Fundamental Theorem of Calculus

Now we come to the part that will link all these pieces together.

Recall that what we want to be able to do is to calculate the area under the graph of some function $f(x)$ (here we will assume that the graph of the function lies above the x-axis).

The Fundamental Theorem of Calculus states that the area under the graph of $f(x)$ between $x = a$ and $x = b$ is equal to the definite integral

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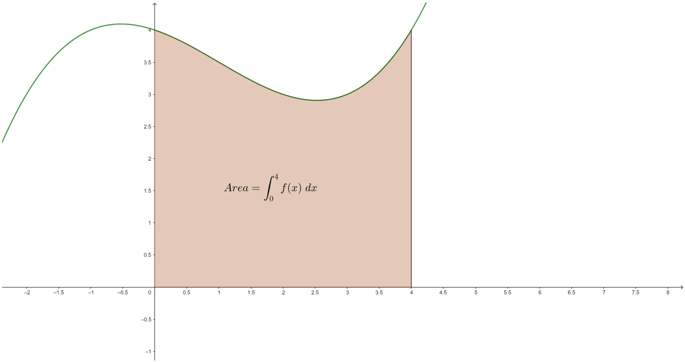
Fundamental Theorem of Calculus

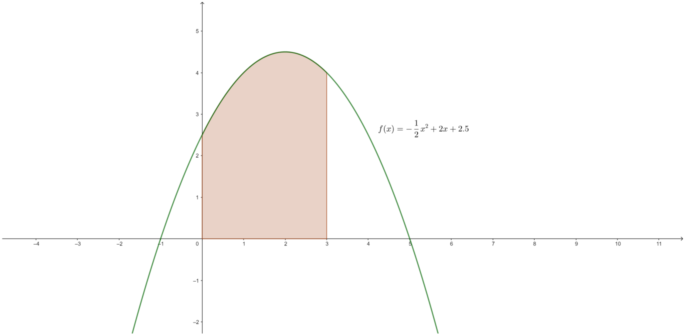
Coming back to our initial example:



The area can be calculated **exactly** using integral. We can do this by hand (or better using GDC) and we get that the area is equal to $13\frac{1}{3}$ units^2.

Practice problems

Calculate the shaded area:



The area is given by $Area = \int_0^3 -\frac{1}{2}x^2 + 2x + 2.5\, dx = 12.$

Practice problems

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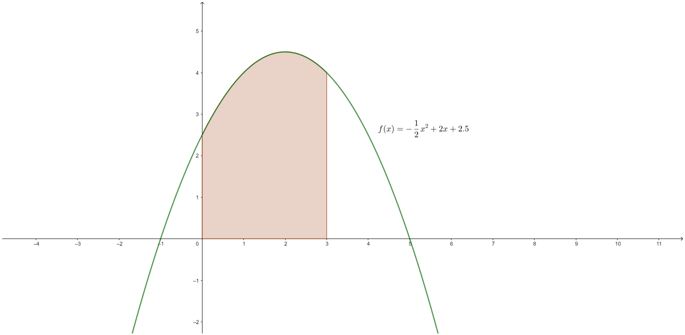

The area is given by $Area = \int_0^3 -\frac{1}{2}x^2 + 2x + 2.5 \; dx =$

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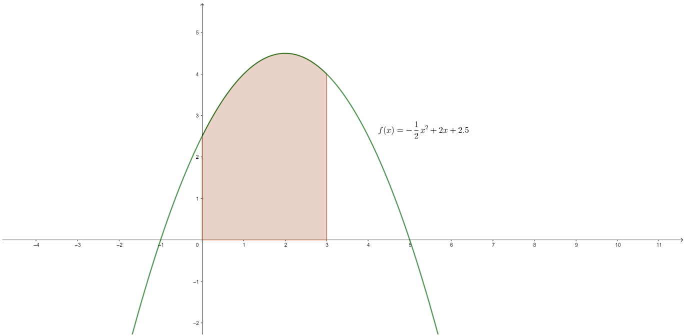

The area is given by $Area = \int_0^3 -\frac{1}{2}x^2 + 2x + 2.5\ dx = 12.$

GDC

We can calculate the areas on the GDC directly, but we can also use graphs to do this.

CASIO MENU → GRAPH → y1 = ... → DRAW (F6) → G-SOLVE (F5) → F6 → $\int$ (F3)

Then you need to input the boundaries for example: 0 EXE, 3 EXE.

Ti - 84
Y= → y1=... → GRAPH → CALC → 7:$\int f(x)dx$

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On the next slides you will be asked to calculate the shaded areas.

Make sure to write down the integral you are trying to calculate. Then calculate it on GDC directly and by sketching the graph.

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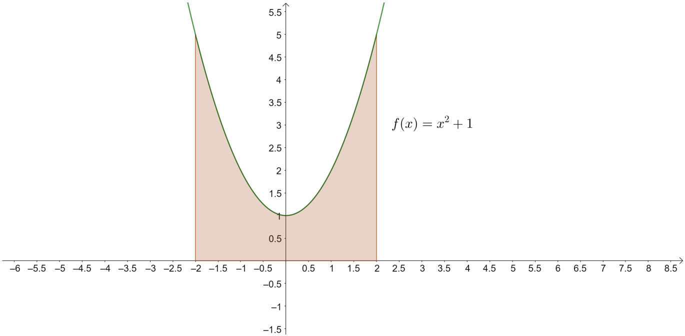
Area under graph - practice

Calculate shaded area:



$$Area = \int_{-2}^{2} x^2 + 1 \, dx = \frac{28}{3}$$

Area under graph - practice

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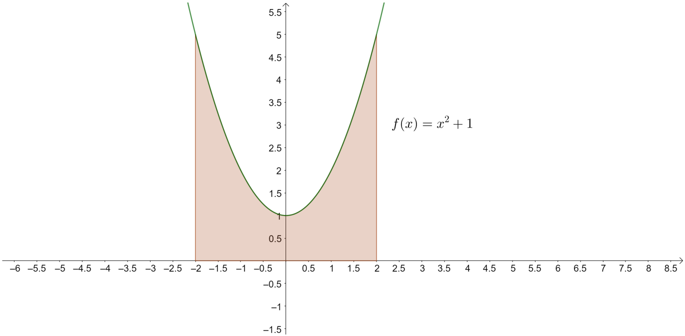

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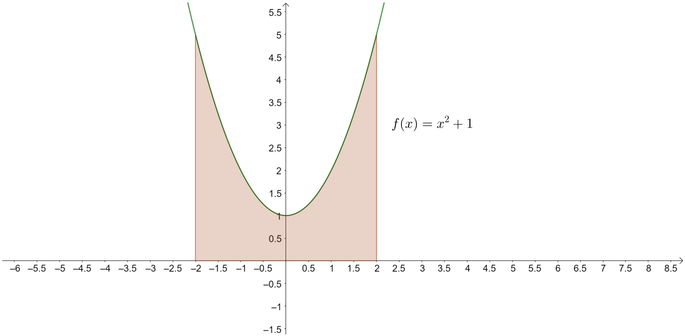

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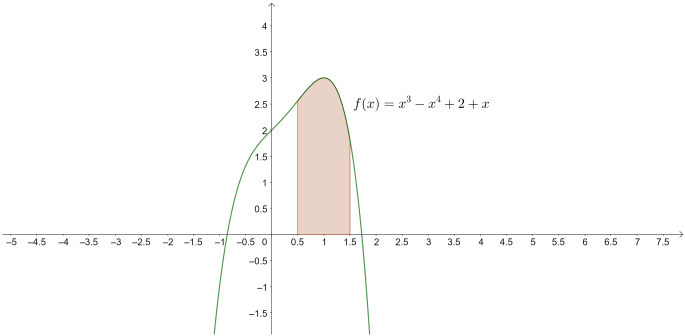

$$Area = \int_{0.5}^{1.5} x^3 - x^4 + 2 + x \, dx = 2.7375$$

Area under graph - practice

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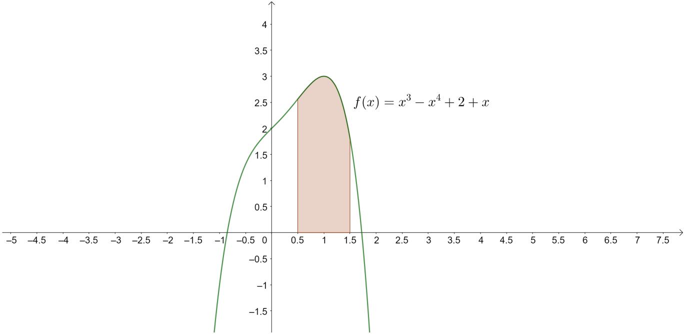

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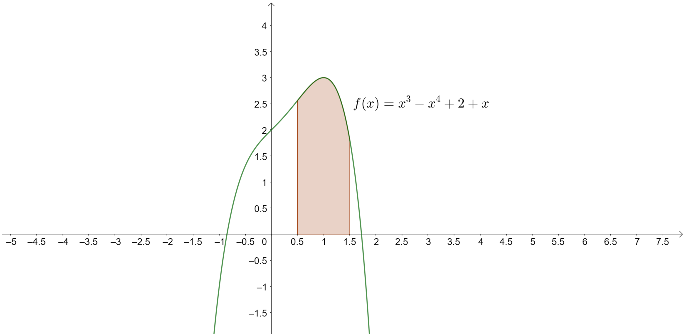

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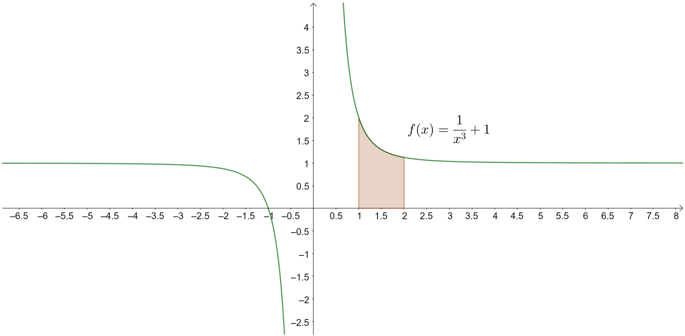
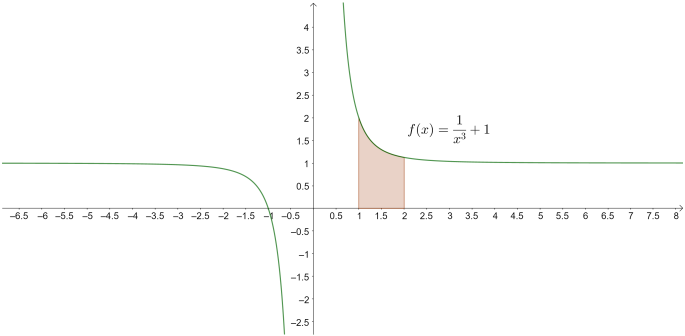


$$Area = \int_1^2 \frac{1}{x^3} + 1 \, dx = \frac{11}{8}$$

Area under graph - practice

Calculate shaded area:



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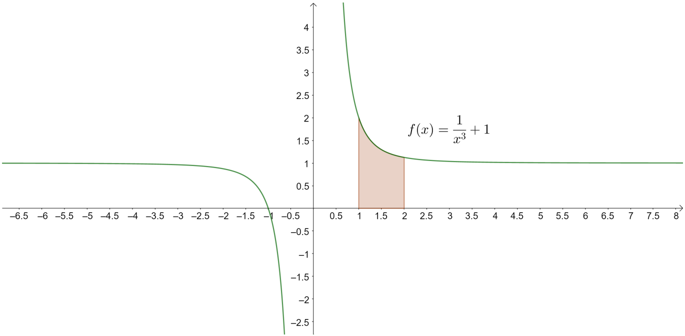

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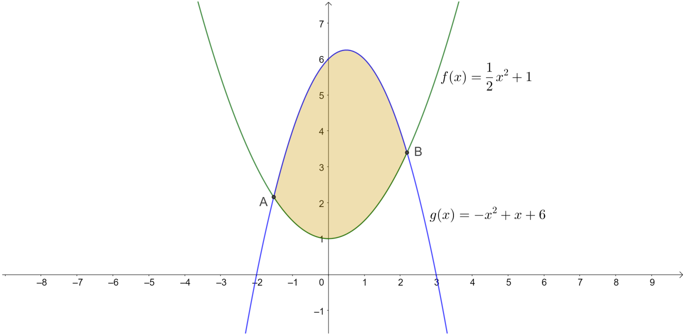
Area between graphs

You may also be asked to calculate the area between graphs:



Here we need to do 4 steps.

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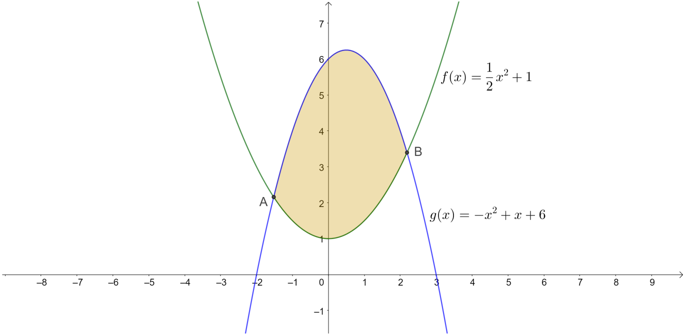

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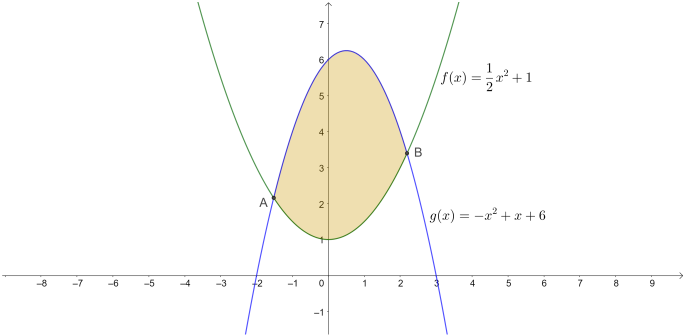
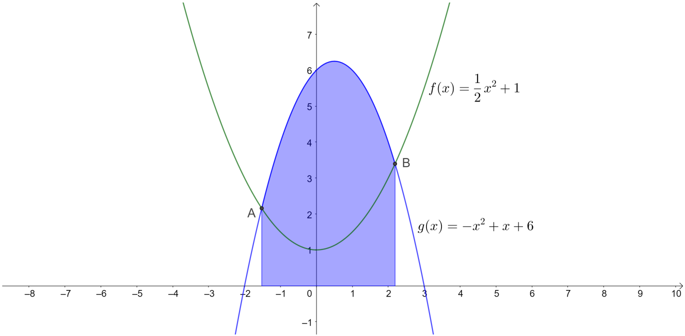
Area between graphs - step 1



Step 1: Calculate where the graphs intersect. This can be done using graphs on GDC. In this example the graphs intersect at $x \approx -1.5226$ and at $x \approx 2.1893$.

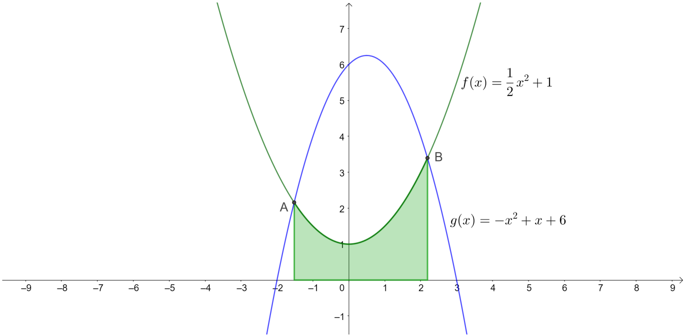
Area between graphs - step 2



Step 2: Calculate the area under the top function between the values you found in step 1.:

$$Area_g = \int_{-1.5226}^{2.1893} -x^2 + x + 6 \, dx \approx 18.83435$$

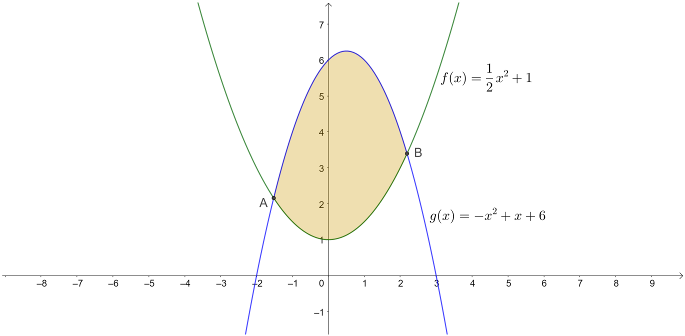
Area between graphs - step 3



Step 3: Calculate the area under the bottom function between the values you found in step 1.:

$$Area_f = \int_{-1.5226}^{2.1893} \frac{1}{2}x^2 + 1\, dx \approx 6.049108$$

Area between graphs - step 4

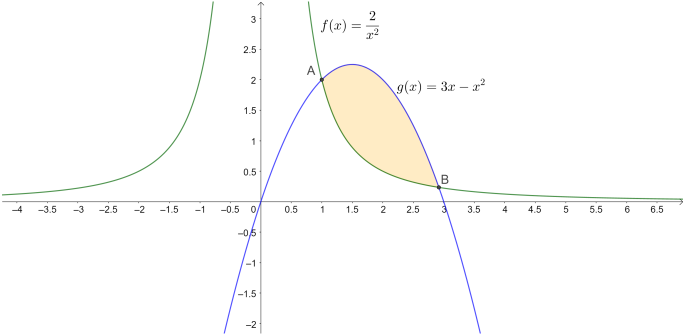


Step 4: The required area is the difference between the areas found in steps 2 and 3:

$$Area \approx 18.83435 - 6.049108 \approx 12.8$$

Area between graphs - practice

Find the shaded area:



$$\text{Area} = \int_1^{2.9196} 3x - x^2 \, dx - \int_1^{2.9196} \frac{2}{x^2} \, dx \approx 2.01$$

Area between graphs - practice

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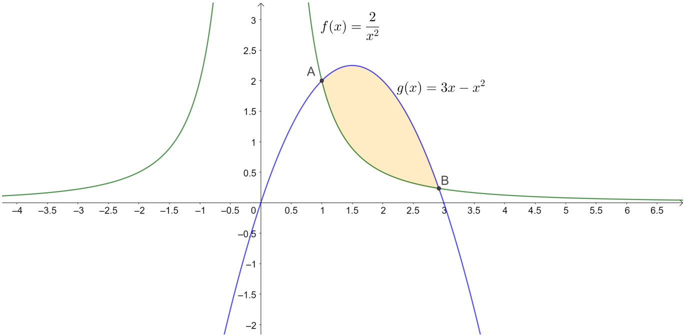

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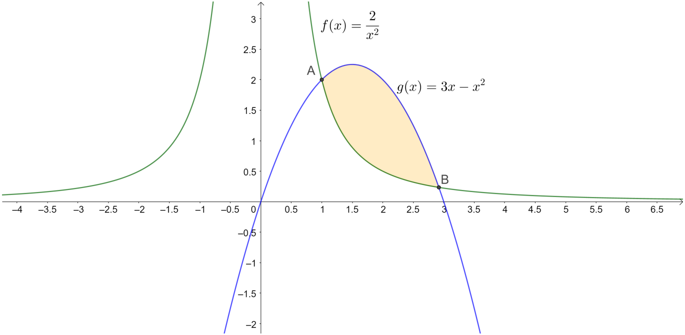

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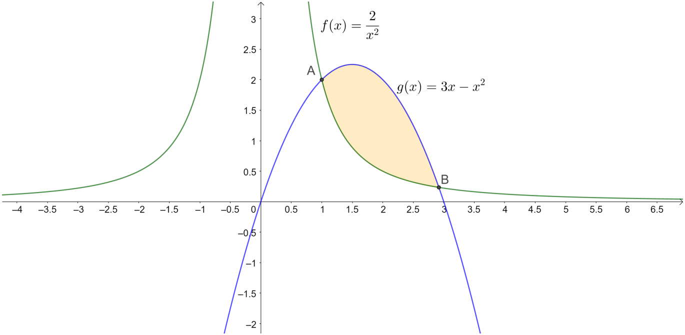

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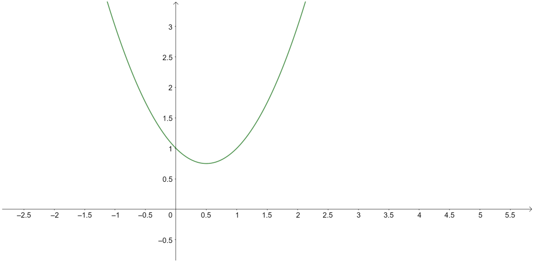
EXAM-STYLE QUESTION

Exam-style question

Let $f(x) = x^2 - x + 1$. The graph of $y = f(x)$ is shown below.



Let A be the area under the graph between $x = 0$ and $x = 2$.

a) Approximate A using trapezoidal rule using 4 trapeziums.

b) Write down the integral that represents the exact value of A and calculate it.

c) Calculate the percentage error of your approximation in part (a).

Exam-style question (a)

Because we have to use 4 trapeziums and the area is between $x = 0$ and $x = 2$, this means that the height of each trapezium is 0.5.

We can use the formula:

$$A \approx \frac{h}{2}(f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + f(2))$$
$$= \frac{1}{4}(1 + 1.5 + 2 + 3.5 + 3) = 2.75$$

Exam-style question (b)

We want the definite integral of $f(x)$ between $x = 0$ and $x = 2$:

$$A = \int_0^2 x^2 - x + 1 \, dx = \frac{8}{3}$$

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Exam-style question (c)

The exact area was calculated in part (b), the approximated area (using trapezoidal rule) was calculated in part (a).

The percentage error of the approximation is equal to:

$$\epsilon_{\%} = \left| \frac{2.75 - \frac{8}{3}}{\frac{8}{3}} \right| \cdot 100\% = 3.125\%$$

Make sure you go through all the examples in this presentation.

If there are any questions, ask me in person or on Teams chat.

We will do some more exam-style questions in class.