Sequences and Financial Maths [92 marks]

1. [Maximum mark: 5] EXM.1.SL.TZ0.1

Give your answers to this question correct to two decimal places.

Gen invests $2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

(a) Calculate the value of her savings after 10 years. [2]

Markscheme

$2400(1.04)^{10} = \$3552.59$ ***M1A1***

[2 marks]

(b) The rate of inflation during this 10 year period is 1.5% per year.

Calculate the real value of her savings after 10 years. [3]

Markscheme

real interest rate = $4 - 1.5 = 2.5\%$ ***A1***

$2400(1.025)^{10} = \$3072.20$ ***M1A1***

[3 marks]

2. [Maximum mark: 15] EXM.2.SL.TZ0.2

Sophie is planning to buy a house. She needs to take out a mortgage for \$120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay \$1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

(a.i) Calculate the monthly repayment using option 1. [2]

Markscheme

evidence of using Finance solver on GDC ***M1***

Monthly payment = \$785 (\$784.60) ***A1***

[2 marks]

(a.ii) Calculate the total amount Sophie would pay, using option 1. [2]

Markscheme

$240 \times 785 = \$188000$ ***M1A1***

[2 marks]

(b.i) Calculate the number of months it will take to repay the mortgage using option 2. [3]

Markscheme

$N = 180.7$ ***M1A1***

It will take 181 months ***A1***

[3 marks]

(b.ii) Calculate the total amount Sophie would pay, using option 2. [2]

Markscheme

$181 \times 1000 = \$\ 181000$ ***M1A1***

[2 marks]

Give a reason why Sophie might choose

(c.i) option 1. [1]

Markscheme

The monthly repayment is lower, she might not be able to afford $1000 per month. ***R1***

[1 mark]

(c.ii) option 2. [1]

Markscheme

the total amount to repay is lower. ***R1***

[1 mark]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

(d.i) Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years.

[2]

Markscheme

\$74400 (accept \$74300) ***M1A1***

[2 marks]

(d.ii) Hence calculate her monthly repayment for the final 10 years. [2]

Markscheme

Use of finance solver with $N = 120$, $PV = \$74400$, $I = 7\%$ ***A1***

\$855 (accept \$854 – \$856) ***A1***

[2 marks]

3. [Maximum mark: 5] 23M.1.SL.TZ1.3

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

Markscheme

$N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).

Approaches that use the compound interest formula receive no marks.

Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

Markscheme

$N = 36.5 \ \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

Note: Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

4. [Maximum mark: 6] 23M.1.SL.TZ2.2

Angel has $\$520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Markscheme

METHOD 1 (use of financial app in GDC)

$$N = 5 \quad I\% = 1.2 \quad PV = \pm 520 \quad P/Y = 1 \quad C/Y = 4$$

OR

$$N = 20 \quad I\% = 1.2 \quad PV = \pm 520 \quad P/Y = 4 \quad C/Y = 4$$

(M1)(A1)

Note: Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

$(\$)\ 552.11$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula ***(M1)***

$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4}$ ***(A1)***

$(\$)\ 552.11$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

[3 marks]

Instead of investing the money, Angel decides to buy a phone that costs $\$520$. At the end of 5 years, the phone will have a value of $\$30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

Markscheme

EITHER

$N = 5$
$I\% = 43.5\ (43.4772\ldots(\%))$
$PV = \pm 520$
$FV = \mp 30$ ***(M1)(A1)A1***

Note: Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$3 = 520\left(1 - \frac{r}{100}\right)^5$ (or equivalent) ***(M1)(A1)***

$(r =)\ 43.5\%\ (43.477\ldots\%)$ ***A1***

Note: Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =)\ -43.5\%$.

Award ***M1A1A0*** for a final answer of 56.5%.

[3 marks]

5. [Maximum mark: 15] 23M.2.SL.TZ2.2

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12th day. [3]

Markscheme

recognizing arithmetic sequence (may be seen in part (b)) ***(M1)***

$(u_{12} =)\ 10 + (12 - 1) \times 6$ ***(A1)***

76 ***A1***

[3 marks]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

(b) Calculate the total number of pendulums she would have available at the market. [2]

Markscheme

correct substitution into either arithmetic series formula ***(A1)***

$\left(S_{15} =\right)\ \frac{15}{2}\left(2 \times 10 + \left(15 - 1\right) \times 6\right)$ **OR**
$\left(S_{15} =\right)\ \frac{15}{2}\left(10 + 94\right)$

780 ***A1***

[2 marks]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Markscheme

attempt to use either arithmetic series formula equated to 1000 ***(M1)***

$\frac{15}{2}\left(2\times 10+\left(15-1\right)\times x\right)=1000$ **OR** $\frac{15}{2}\left(10+u_{15}\right)=1000$

$x=8.09523\ldots$ ***(A1)***

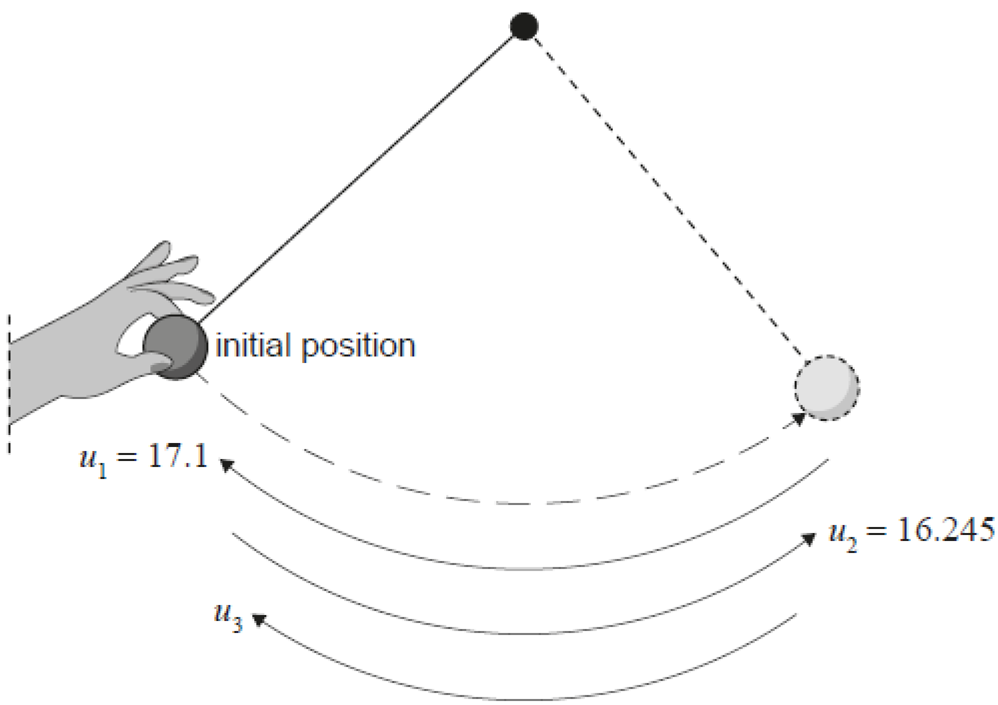
$x=9$ ***A1***

Note: Follow through within question part for final ***A1*** for candidates correctly rounding their value of x up to the nearest integer. Award ***(M0)(A0)A0*** for a response of $x=8$ with no working shown.

[3 marks]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

(d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]

Markscheme

recognizing geometric sequence (may be seen in part (e)) ***(M1)***

$17.1 \times 0.95^{5-1}$ ***(A1)***

$13.9\ (\text{cm})\ (13.9280\ldots)$ ***A1***

[3 marks]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

Markscheme

correct substitution into geometric series formula ***(A1)***

$\frac{17.1\left(1-0.95^{16}\right)}{1-0.95}$

$191\ (\mathrm{cm})\ (191.476\ldots\ (\mathrm{cm}))$ ***A1***

[2 marks]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

Markscheme

correct method to find u_0 ***(M1)***

$u_0 = 17.1 \times (0.95)^{0-1}$ **OR** $17.1 = 0.95x$ **OR** $\frac{17.1}{0.95}$ (seen)

Note: Award ***(M0)A0*** for any attempt to find answer using 0.05 or 1.05.

$18\ (\mathrm{cm})$ ***A1***

[2 marks]

6. [Maximum mark: 7] 22N.1.SL.TZ0.2

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

(a) Find the number of trees to be planted in the 15th month. [3]

Markscheme

use of the n^{th} term of an arithmetic sequence formula ***(M1)***

$u_{15} = 85 + (15 - 1) \times 30$ ***(A1)***

505 ***A1***

[3 marks]

(b) Find the total number of trees to be planted in the first 15 months. [2]

Markscheme

use of the sum of n terms of an arithmetic sequence formula ***(M1)***

$S_{15} = \frac{15}{2}(85 + 505)$ **OR** $\frac{15}{2}(2 \times 85 + (15 - 1) \times 30)$

$4430 \ (4425)$ ***A1***

[2 marks]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

Markscheme

$\frac{4425}{15}$ **OR** $85 + (8 - 1) \times 30$ ***(M1)***

295 ***A1***

Note: Accept $295.333\ldots$ from use of 3sf value from part (b).

[2 marks]

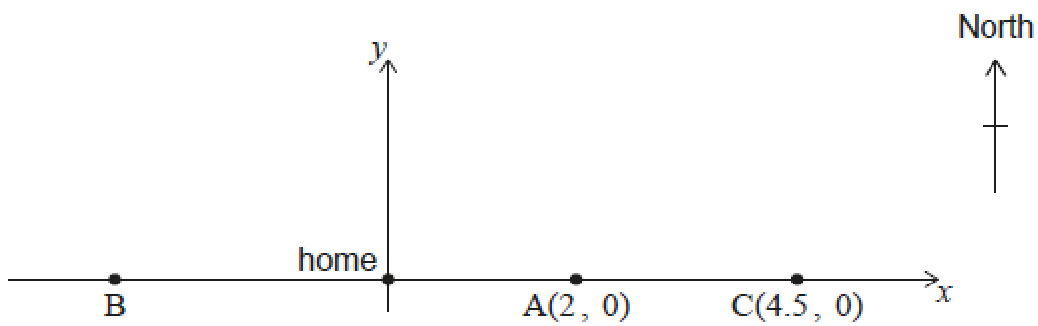
7. [Maximum mark: 7] 22N.1.SL.TZ0.11

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

Markscheme

$4.5 = 2(r)^{3-1}$ ***(M1)***

$r = \pm 1.5,$ ***R1***

(Some x-values are negative or direction from house changes each day)

$r = -1.5$ ***AG***

Note: Award ***M0R0AG*** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

[2 marks]

On the 6th day, Kristi runs to point F.

(b) Find the location of point F. [2]

Markscheme

$2(-1.5)^{6-1}$ ***(M1)***

EITHER

$(-15.2,\ 0)\ \ (-15.1875\ldots,\ 0)$ ***A1***

OR

$x = -15.2\,\text{km}$ ***A1***

OR

15.2 km west (of the origin) ***A1***

Note: Award ***(M1)A0*** for an answer of "$-15.2\,(\text{km})$" without indicating that it is the x-value.

[2 marks]

(c) Find the total distance Kristi runs during the first 7 days of training. [3]

Markscheme

choosing, $r = 1.5$ ***(A1)***

$\frac{2\left((1.5)^7 - 1\right)}{1.5 - 1}$ ***(M1)***

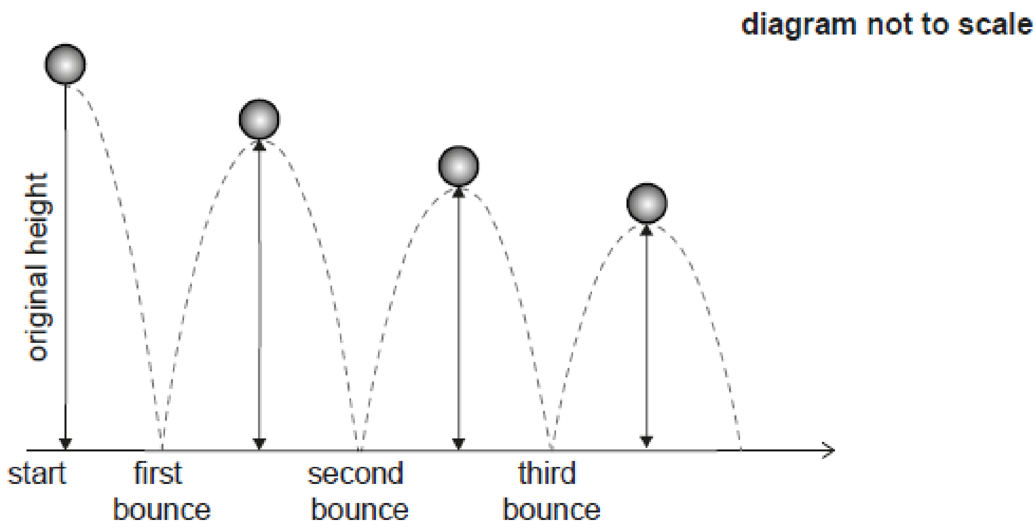
Note: Award ***M1*** for an attempt at a substituted GP formula with $n = 7$. Award ***A0M1A0*** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of $14.4687\ldots$).

$64.3\ldots\ \text{km}\ \ (64.3437\ldots)$ ***A1***

[3 marks]

8. [Maximum mark: 7] 22M.1.SL.TZ1.13

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

Markscheme

use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68\,\text{m}$

$= 68\,\text{cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68\,\text{cm}$ ***AG***

[2 marks]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

Markscheme

EITHER

$(0.85)^{n}(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If $1.8\,\text{m}$ (or $180\,\text{cm}$) is used then ***(M1)*** only awarded for use of n in $(0.85)^{n}(1.8) > 0.1$.
If $1.53\,\text{m}$ (or $153\,\text{cm}$) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114\,\text{m}$ and $(0.85)^{18}(1.8) = 0.0966\,\text{m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^{n}(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

Markscheme

EITHER

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8 \times 0.85)(1-0.85^3)}{1-0.85}$ $(= 3.935925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$18 + 2(3.935925)$

$= 9.67\,\text{m}\ \ (9.67185\ldots\ \text{m})$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1-0.85^4)}{1-0.85}$ $(= 5.735925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67\,\text{m}\ \ (9.67185\ldots\ \text{m})$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8) \quad (= 3.935925\ldots)$$

(A1)

recognizing distances are travelled twice except first distance ***(M1)***

$$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$$

$$= 9.67\,\text{m} \quad (9.67185\ldots\,\text{m})$$ ***A1***

Note: Answers may be given in cm.

[3 marks]

9. [Maximum mark: 6] 22M.1.SL.TZ2.13

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account

at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

Markscheme

METHOD 1 – *(with* $FV = 4000$*)*

EITHER

$\mathrm{N} = 10$
$\mathrm{I} = 1.5$
$\mathrm{FV} = 4000$
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ ***(A1)(M1)***

Note: Award ***A1*** for $(3.5 - 2 =)\ 1.5$ seen and ***M1*** for all other entries correct.

OR

$4000 = A(1 + 0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

$(\text{PV} =)\ \$3447$ ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*

calculate FV with inflation

4000×1.02^{10} ***(A1)***

$(= 4875.977\ldots)$

EITHER

$4000 \times 1.02^{10} = \text{PV} \times 1.035^{10}$ ***(M1)***

OR

$\text{N} = 10$
$\text{I} = 3.5$
$\text{FV} = 4875.977\ldots$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN

$(\text{PV} =)\ \$3457$ ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 3 – *(Using formula to calculate real rate of return)*

(real rate of return =) $1.47058\ldots(\%)$ ***(A1)***

EITHER

$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***

OR

$\text{N} = 10$
$\text{I} = 1.47058\ldots$
$\text{FV} = 4000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN

$(\text{PV} =)\ \$3457$ ***A1***

[3 marks]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

Markscheme

METHOD 1 – *(Finding the future value of the investment using PV from part (a))*

$\mathrm{N} = 10$
$\mathrm{I} = 3.5$
$\mathrm{PV} = 3446.66\ldots$ (from Method 1) OR $3456.67\ldots$ (from Methods 2, 3)
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

$(\mathrm{FV} =)\ \$4861.87$ **OR** $\$4875.97$ ***(A1)***

so payment required (from TVM) will be $\$294$ **OR** $\$295$ ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*

$\mathrm{N} = 10$
$\mathrm{I} = 3.5$
$\mathrm{PV} = -1000$
$\mathrm{FV} = 4875.977\ldots$
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ ***(A1)(M1)***

Note: Award ***A1*** for $\mathrm{I} = 3.5$ **and** $\mathrm{FV} = \pm 4875.977\ldots$, ***M1*** for all other entries correct and opposite PV and FV signs.

$(\mathrm{PMT} =)\ \$295 \quad (295.393)$ ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

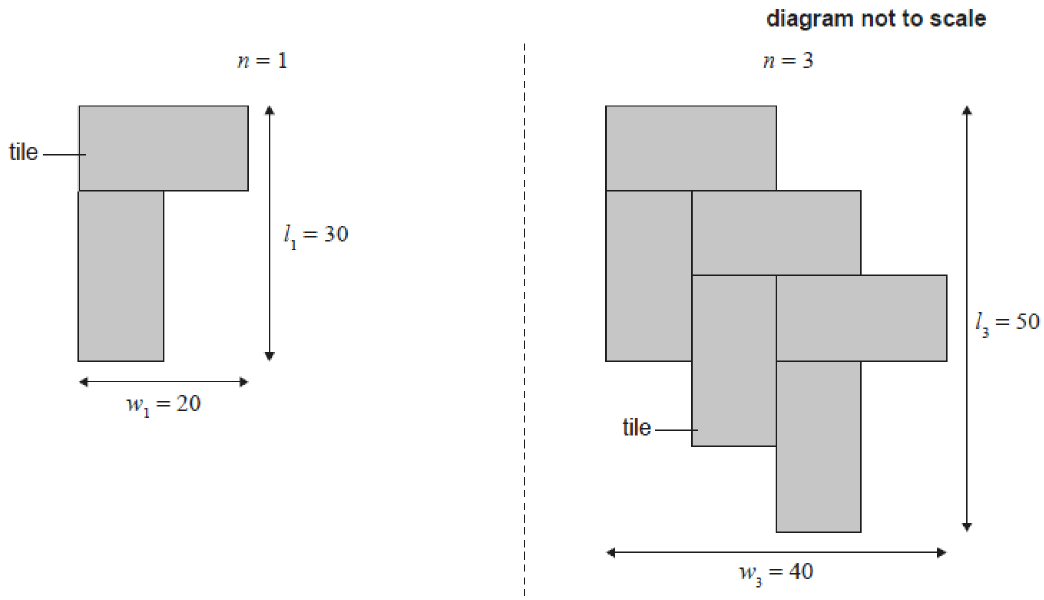
[3 marks]

10. [Maximum mark: 19] 22M.2.SL.TZ1.2

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

(a.i) a. [1]

Markscheme

30 ***A1***

[1 mark]

(a.ii) b. [1]

Markscheme

40 ***A1***

[1 mark]

Write down an expression in terms of n for

(b.i) w_n. [2]

Markscheme

arithmetic formula chosen ***(M1)***

$w_n = 20 + (n - 1)10 \quad (= 10 + 10n)$ ***A1***

[2 marks]

(b.ii) l_n. [1]

Markscheme

arithmetic formula chosen

$l_n = 30 + (n - 1)10 \quad (= 20 + 10n)$ ***A1***

[1 mark]

Eddie's lawn has a length $740\,\mathrm{cm}$.

(c.i) Show that Eddie needs 144 tiles. [2]

Markscheme

$740 = 30 + (n - 1)10$ **OR** $740 = 20 - 10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

[2 marks]

(c.ii) Find the value of w_n for this path. [1]

Markscheme

$w_{72} = 730$ ***A1***

[1 mark]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

Markscheme

$(10 \times 20) \times 144$ ***(M1)***

$= 28800$ ***(A1)***

$2.88 \times 10^4 \text{ cm}^2$ ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

Markscheme

EITHER

1 square metre $= 100\,\text{cm} \times 100\,\text{cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10 \text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2\,\mathrm{m} \times 0.1\,\mathrm{m} \times 5 =)\ 0.1\,\mathrm{m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$\$2.45$ per pack (of 5 tiles) ***A1***

[3 marks]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

Markscheme

$\frac{1.08 \times 144}{5}\ (= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

Markscheme

$35 + (32 \times 2.45)$ ***(M1)***

$\$113 \ \ (113.4)$ ***A1***

[2 marks]