Sequences and Financial Maths [92 marks]

1.   [Maximum mark: 5]

Give your answers to this question correct to two decimal places.

Gen invests $2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

(a)   Calculate the value of her savings after 10 years.   [2]

(b)   The rate of inflation during this 10 year period is 1.5% per year.

           Calculate the real value of her savings after 10 years.   [3]

2. [Maximum mark: 15]

Sophie is planning to buy a house. She needs to take out a mortgage for $120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay $1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

(a.i)   Calculate the monthly repayment using option 1. [2]

(a.ii) Calculate the total amount Sophie would pay, using option 1. [2]

(b.i)   Calculate the number of months it will take to repay the
        mortgage using option 2. [3]

(b.ii) Calculate the total amount Sophie would pay, using option 2. [2]

Give a reason why Sophie might choose

(c.i)   option 1. [1]

(c.ii) option 2. [1]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

(d.i)   Use your answer to part (a)(i) to calculate the amount
        remaining on her mortgage after the first 10 years. [2]

(d.ii) Hence calculate her monthly repayment for the final 10 years. [2]

3. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

4. [Maximum mark: 6]

Angel has $\$520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $\$520$. At the end of 5 years, the phone will have a value of $\$30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

5. [Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the $12\,\text{th}$ day. [3]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale

initial position

$u_1 = 17.1$

$u_2 = 16.245$

u_3
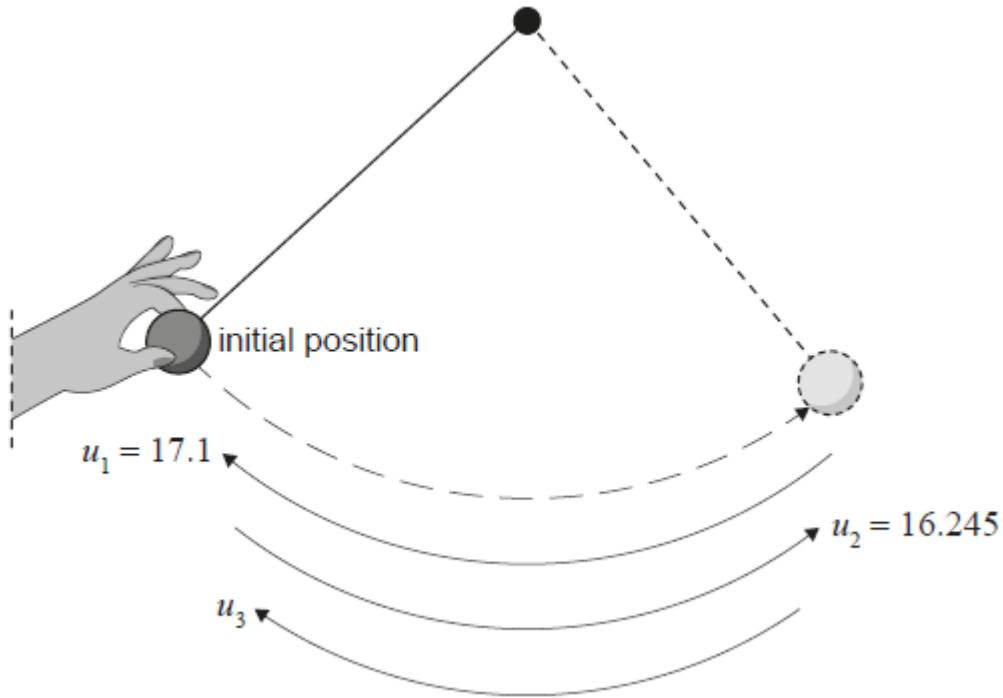
On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

(d)   Calculate the distance that the ball travelled during the $5\,th$ recorded swing.   [3]

(e)   Calculate the total distance that the ball travelled during the first 16 recorded swings.   [2]

(f)   Calculate the distance that the ball travelled before Daina started recording.   [2]

6. [Maximum mark: 7]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

(a) Find the number of trees to be planted in the 15th month. [3]

(b) Find the total number of trees to be planted in the first 15 months. [2]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

7.

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $A(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $C(4.5,\ 0)$.

This information is represented in the following diagram.
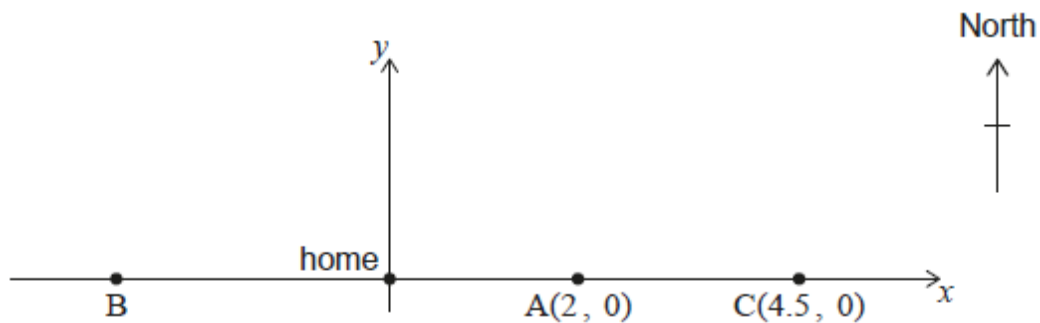
Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a)   Show that the common ratio, r, is -1.5.   [2]

On the 6th day, Kristi runs to point F.
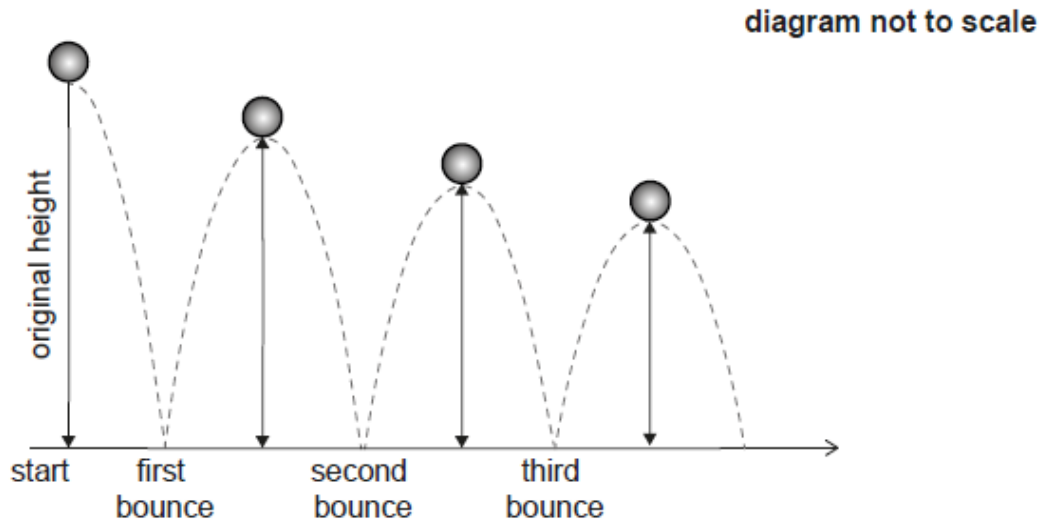
(b)   Find the location of point F.   [2]

(c)   Find the total distance Kristi runs during the first 7 days of training.   [3]

8. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale

(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

9. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account
at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would
     need to invest. Give your answer to the nearest dollar.   [3]

(b) For option 2, find the minimum value of x that Juliana would
   need to invest each year. Give your answer to the nearest dollar.   [3]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\,\text{cm}$ wide and $20\,\text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.

diagram not to scale

$n = 1$

tile

$l_1 = 30$

$w_1 = 20$

$n = 3$

tile

$l_3 = 50$

$w_3 = 40$
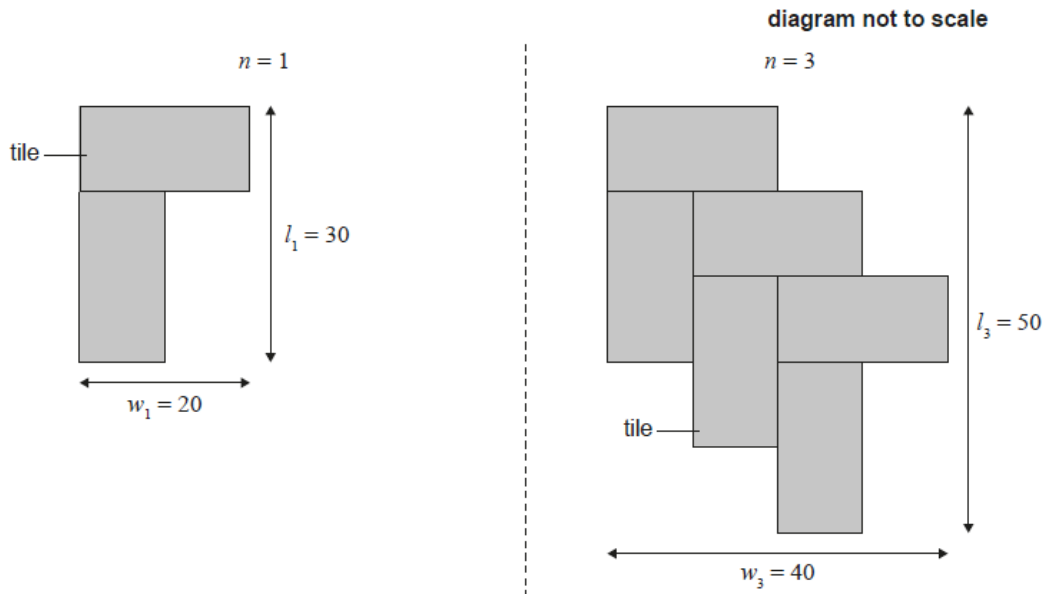
The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

(a.i) a.   [1]

(a.ii) b.   [1]

Write down an expression in terms of n for

(b.i) w_n.   [2]

(b.ii) l_n.   [1]

Eddie's lawn has a length $740\,\text{cm}$.

(c.i)   Show that Eddie needs 144 tiles.   [2]

(c.ii)   Find the value of w_n for this path.   [1]

(d)   Find the total area of the tiles in Eddie's path. Give your answer
      in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer.   [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e)   Find the cost of a single pack of five tiles.   [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he
needs.

(f)   Find the minimum number of packs of tiles Eddie will need to
      order.   [3]

There is a fixed delivery cost of $\$35$.

(g)   Find the total cost for Eddie's order.   [2]