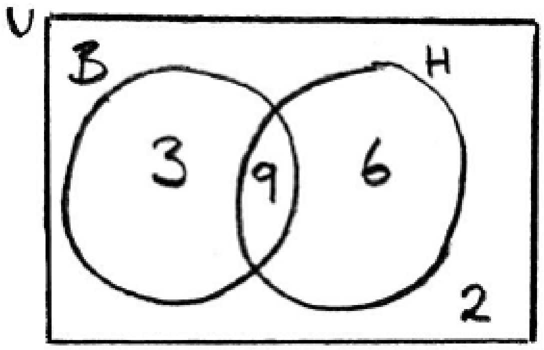
(a)



A1A1

Note: Award A1 for a diagram with two intersecting regions and at least the value of the intersection.

(b) $\frac{9}{20}$ A1

(c) $\frac{9}{12}\left(=\frac{3}{4}\right)$ A1

[4]

(a) $a = 16$ A1

(b) $\boldsymbol{A}^{-1} = \frac{1}{16}\begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$ (M1)A1

(c) $\boldsymbol{AX} = \boldsymbol{C} \Rightarrow \boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{C}$ (M1)

$= \frac{1}{16}\begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 0 \\ 0 & 3 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 0 \\ -4 \end{pmatrix}$

$= \frac{1}{16}\begin{pmatrix} 12 \\ 24 \\ -4 \end{pmatrix} \left(= \begin{pmatrix} 0.75 \\ 1.5 \\ -0.25 \end{pmatrix}\right)$ A1

[5]

$\overline{m} = \frac{6.7 + 7.2 + \ldots + 7.3}{10} = 6.91$ (M1)A1

$s_{n-1}^2 = \frac{1}{9}((6.7 - 6.91)^2 + \ldots + (7.3 - 6.91)^2)$ (M1)

$= \frac{0.489}{9} = 0.0543$ (3 sf) A1

Note: Award M1A0 for 0.233.

[4]

(a) Using $\sum P(X = x) = 1$ (M1)

$4c + 6c + 6c + 4c = 1 \quad (20c = 1)$ A1

$c = \frac{1}{20} \quad (= 0.05)$ A1 N1

(b) Using $E(X) = \sum xP(X = x)$ (M1)

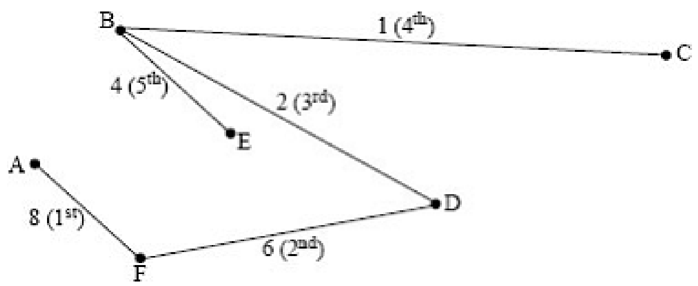
$= (1 \times 0.2) + (2 \times 0.3) + (3 \times 0.3) + (4 \times 0.2)$ (A1)

$= 2.5$ A1 N1

(b) (i) $\boldsymbol{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$ A1

(ii) We require the (A, A) element of $\boldsymbol{M}^4$ which is 13. M1A2

(c)



A1A1A1A1A1

(a) number of patients in 30 minute period = X
$X \sim \text{Po}(3)$ (A1)
$P(X = 0) = 0.0498$ (M1)A1

(b) number of patients in working period = Y
$Y \sim \text{Po}(12)$ (A1)
$P(X < 10) = P(X \le 9) = 0.242$ (M1)A1

(c) number of working period with less than 10 patients = W
$W \sim B(6, 0.2424\ldots)$ (M1)(A1)
$P(W \le 3) = 0.966$ (M1)A1

Note: Accept exact answers in parts (a) to (c).

(d) number of patients in t minute interval = X
$X \sim \text{Po}(T)$
$P(X \ge 2) = 0.95$
$P(X = 0) + P(X = 1) = 0.05$ (M1)(A1)
$e^{-T}(1 + T) = 0.05$ (M1)
$T = 4.74$ (A1)
$t = 47.4$ minutes A1

[15]

EITHER

Let s be the distance from the origin to a point on the line, then

$s^2 = (1-\lambda)^2 + (2-3\lambda)^2 + 4$ (M1)

$= 10\lambda^2 - 14\lambda + 9$ A1

$\frac{d(s^2)}{d\lambda} = 20\lambda - 14$ A1

For minimum $\frac{d(s^2)}{d\lambda} = 0, \Rightarrow \lambda = \frac{7}{10}$ A1

OR

The position vector for the point nearest to the origin is perpendicular to the direction of the line. At that point:

$\begin{pmatrix} 1-\lambda \\ 2-3\lambda \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} = 0$ (M1)A1

Therefore, $10\lambda - 7 = 0$ A1

Therefore, $\lambda = \frac{7}{10}$ A1

THEN

$x = \frac{3}{10}, y = -\frac{1}{10}$ (A1)(A1)

The point is $\left(\frac{3}{10}, \frac{-1}{10}, 2\right)$. N3

[6]