1. In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

(a) Illustrate this information on a Venn diagram.

(2)

(b) Find the probability that a randomly selected student from this class is studying both Biology and History.

(1)

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History.

(1)
(Total 4 marks)

2. Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are defined as

$$\boldsymbol{A} = \begin{pmatrix} 1 & 5 & 1 \\ 3 & -1 & 3 \\ -9 & 3 & 7 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 0 \\ 0 & 3 & 1 \end{pmatrix}, \boldsymbol{C} = \begin{pmatrix} 8 \\ 0 \\ -4 \end{pmatrix}.$$

(a) Given that $\boldsymbol{AB} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$, find a.

(1)

(b) Hence, or otherwise, find $\boldsymbol{A}^{-1}$.

(2)

(c) Find the matrix $\boldsymbol{X}$, such that $\boldsymbol{AX} = \boldsymbol{C}$.

(2)
(Total 5 marks)

3. The company *Fresh Water* produces one-litre bottles of mineral water. The company wants to determine the amount of magnesium, in milligrams, in these bottles.

A random sample of ten bottles is analysed and the results are as follows:

6.7, 7.2, 6.7, 6.8, 6.9, 7.0, 6.8, 6.6, 7.1, 7.3.

Find unbiased estimates of the mean and variance of the amount of magnesium in the one-litre bottles.

(Total 4 marks)

4. The probability distribution of a discrete random variable X is defined by

$$P(X = x) = cx(5 - x),\ x = 1, 2, 3, 4.$$

(a) Find the value of c.

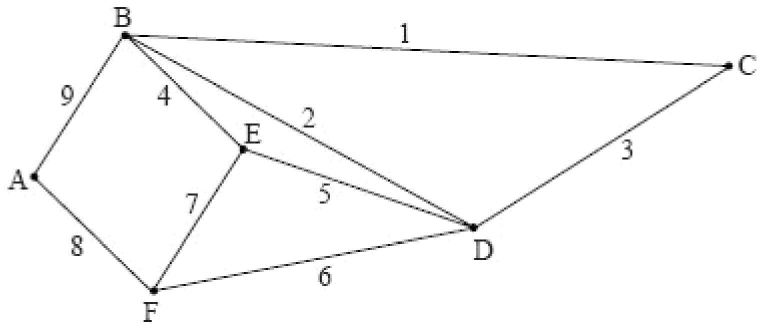
(3)

(b) Find $E(X)$.

(3)

(Total 6 marks)

5.



The above diagram shows the weighted graph *G*.

(a) (i) Write down the adjacency matrix for *G*.

(ii) Find the number of distinct walks of length 4 beginning and ending at A.

(4)

(b) Starting at A, use Prim's algorithm to find and draw the minimum spanning tree for *G*. Your solution should indicate clearly the way in which the tree is constructed.

(4)

(Total 8 marks)

6. Casualties arrive at an accident unit with a mean rate of one every 10 minutes.
Assume that the number of arrivals can be modelled by a Poisson distribution.

(a) Find the probability that there are no arrivals in a given half hour period.

(3)

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period.

(3)

(c) Six nurses work consecutive two hour periods between 8am and 8pm.
Find the probability that no more than three nurses have to attend to less than ten casualties during their working period.

(4)

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties.

(5)
(Total 15 marks)

7. The line L is given by the parametric equations $x = 1 - \lambda$, $y = 2 - 3\lambda$, $z = 2$.
Find the coordinates of the point on L that is nearest to the origin.

(Total 6 marks)