(a) (i) the median height is 1.18 A1

(ii) the interquartile range is UQ – LQ
= 1.22 – 1.13 = 0.09 (accept answers that round to 0.09) A1A1

Note: Award A1 for the quartiles, A1 for final answer.

(b) (i)

$1.00 < h \le 1.05$	$1.05 < h \le 1.10$	$1.10 < h \le 1.15$	$1.15 < h \le 1.20$	$1.20 < h \le 1.25$	$1.25 < h \le 1.30$
5	9	13	24	19	10

A1A1

Note: Award A1 for entries within ±1 of the above values and A1 for a total of 80.

(ii) unbiased estimate of the population mean

$$\left(\frac{5\times1.025+9\times1.075+13\times1.125+24\times1.175+19\times1.225+10\times1.275}{80}\right) = 1.17$$

A1

unbiased estimate of the population variance

use of $s_{n-1}^2 = \left(\frac{n}{n-1}\right)s_n^2$ or GDC (M1)

obtain 0.00470 A1

(c) (i) $\mathrm{P}(h \le 1.15\text{ m}) = \frac{27}{80}$ (0.3375 or 0.338) (allow $\frac{26}{80}$ (0.325)) A1

(ii) use of the conditional probability formula $\mathrm{P}(A \mid B) = \mathrm{P}(A \cap B)/\mathrm{P}(B)$ (M1)

obtain $\frac{18}{80} \div \frac{27}{80}$ (A1)(A1)

$= \frac{2}{3}$ (0.667) $\left(\text{allow } \frac{18}{26} \quad (0.692)\right)$ A1

[13]

(a) mean for 30 days: $30 \times 0.2 = 6$. (A1)

$\text{P}(X=4)=\frac{6^4}{4!}\text{e}^{-6}=0.134$ (M1)A1 N3

(b) $\text{P}(X > 3) = 1 - \text{P}(X \le 3) = 1 - \text{e}^{-6}(1 + 6 + 18 + 36) = 0.849$ (M1)A1 N2

(c) **EITHER**

mean for five days: $5 \times 0.2 = 1$ (A1)

$\text{P}(X = 0) = \text{e}^{-1}$ $(= 0.368)$ A1 N2

OR

mean for one day: 0.2 (A1)

$\text{P}(X = 0) = (\text{e}^{-0.2})^5 = \text{e}^{-1}$ $(= 0.368)$ A1 N2

(d) Required probability $= \text{e}^{-0.2} \times \text{e}^{-0.2} \times (1 - \text{e}^{-0.2})$ M1A1

$= 0.122$ A1 N3

(e) Expected cost is $1850 \times 6 = 11100$ euros A1

(f) On any one day $\text{P}(X = 0) = \text{e}^{-0.2}$

Therefore, $\binom{5}{1}(\text{e}^{-0.2})^4(1-\text{e}^{-0.2})=0.407$ M1A1 N2

[13]

(a) $X \sim \text{Po}(0.6)$
$\text{P}(X \ge 1) = 1 - \text{P}(X = 0)$ M1
$= 0.451$ A1 N1

(b) $Y \sim \text{Po}(2.4)$ (M1)
$\text{P}(Y = 3) = 0.209$ A1

(c) $Z \sim \text{Po}(0.6n)$ (M1)
$\text{P}(Z \ge 3) = 1 - \text{P}(Z \le 2) > 0.8$ (M1)

Note: Only one of these M1 marks may be implied.

$n \ge 7.132...$ (hours)
so, Mr Lee needs to fish for at least 8 complete hours A1 N2

Note: Accept a shown trial and error method that leads to a correct solution.

[7]

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

(a) let T be Tim's score

(i) $\mathrm{P}(T = 6) = \frac{1}{9}$ (= 0.111 to 3 s.f.) A1

(ii) $\mathrm{P}(T \geq 3) = 1 - \mathrm{P}(T \leq 2) = 1 - \frac{1}{9} = \frac{8}{9}$ (= 0.889 to 3 s.f.) (M1)A1

(b) let B be Bill's score

(i) $\mathrm{P}(T = 6 \text{ and } B = 6) = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$ (= 0.012 to 3 s.f.) (M1)A1

(ii) $\mathrm{P}(B = T) = \mathrm{P}(2)\mathrm{P}(2) + \mathrm{P}(3)\mathrm{P}(3) + \ldots + \mathrm{P}(6)\mathrm{P}(6)$

$= \frac{1}{9} \times \frac{1}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{3}{9} \times \frac{3}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{1}{9} \times \frac{1}{9}$ M1

$= \frac{19}{81}$ (= 0.235 to 3 s.f.) A1

(c) (i) **EITHER**

$\mathrm{P}(X \leq 2) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ M1A1

because $\mathrm{P}(X \leq 2) = \mathrm{P}((a, b, c, d) \mid a, b, c, d = 1, 2)$ R1
or equivalent

$\mathrm{P}(X \leq 2) = \frac{16}{81}$ AG

OR

there are sixteen possible permutations, which are

Combinations	Number
1111	1
1112	4
1122	6
1222	4
2222	1

M1A1

Note: This information may be presented in a variety of forms.

$\mathrm{P}(X \leq 2) = \frac{1+4+6+4+1}{81}$ A1

$= \frac{16}{81}$ AG

(ii)

x	1	2	3
$\mathrm{P}(X=x)$	$\frac{1}{81}$	$\frac{15}{81}$	$\frac{65}{81}$

A1A1

(iii) $\mathrm{E}(X)=\sum_{x=1}^{3} x\mathrm{P}(X=x)$ (M1)

$=\frac{1}{81}+\frac{30}{81}+\frac{195}{81}$

$=\frac{226}{81}$ (= 2.79 to 3 sf) A1

$\mathrm{E}(X^2)=\sum_{x=1}^{3} x^2\mathrm{P}(X=x)$

$=\frac{1}{81}+\frac{60}{81}+\frac{585}{81}$

$=\frac{646}{81}$ (= 7.98 to 3 sf) A1

$\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ (M1)

= 0.191 (to 3 s.f.) A1

Note: Award M1A0 for answers obtained using rounded values (e.g. Var(X) = 0.196).

d)

Combinations	Number
3311	6
3221	12

$\mathrm{P}(\text{total is } 8 \cap (X=3))=\frac{18}{81}$ M1A1

since $\mathrm{P}(X=3)=\frac{65}{81}$

$\mathrm{P}(\text{total is } 8 \mid (X=3))=\frac{\mathrm{P}((\text{total is } 8)\cap(X=3))}{\mathrm{P}(X=3)}$ M1

$=\frac{18}{65}$ (= 0.277) A1

[21]

(a) $p + q = 0.44$ A1

$2.5p + 3.5q = 1.25$ (M1)A1

$p = 0.29$, $q = 0.15$ A1

(b) use of $\text{Var}(X) = \text{E}(X^2) - \text{E}(X)^2$ (M1)

$\text{Var}(X) = 2.10$ A1

[6]

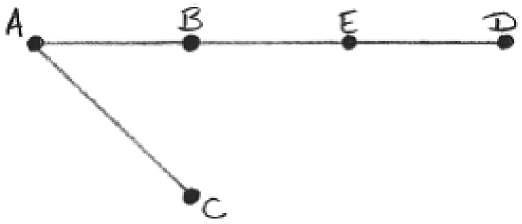
(a) (i) the edges are joined in the order

AC

BE

AB

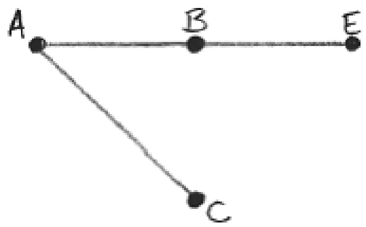
ED A2



A1

Note: Final A1 independent of the previous A2.

(ii)



A1

the weight of this spanning tree is 33 A1

to find a lower bound for the travelling salesman problem,
we add to that the two smallest weights of edges to D,
i.e. 15 + 16 , giving 64 M1A1

(b) an upper bound is the weight of any Hamiltonian cycle,
e.g. ABCDEA has weight 75, so 80 is certainly an upper bound M1A1

[9]

(a)	$AB = \sqrt{1^2 + (2-\sqrt{3})^2}$	M1
	$= \sqrt{88 - 4\sqrt{3}}$	A1
	$= 2\sqrt{2-\sqrt{3}}$	A1
(b)	**METHOD 1**	
	$\arg z_1 = -\frac{\pi}{4}$, $\arg z_2 = -\frac{\pi}{3}$	A1A1

Note: Allow $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

Note: Allow degrees at this stage.

$A\hat{O}B = \frac{\pi}{3} - \frac{\pi}{4}$	
$= \frac{\pi}{12}$ (accept $-\frac{\pi}{12}$)	A1

Note: Allow FT for final A1.

METHOD 2	
attempt to use scalar product or cosine rule	M1
$\cos A\hat{O}B = \frac{1+\sqrt{3}}{2\sqrt{2}}$	A1
$A\hat{O}B = \frac{\pi}{12}$	A1

[6]

(a)	det $\boldsymbol{M} = a^2 + b^2$	A1
	$a^2 + b^2 > 0$, therefore $\boldsymbol{M}$ is non-singular or equivalent statement	R1
(b)	$\boldsymbol{M}^2 = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = \begin{pmatrix} a^2-b^2 & 2ab \\ -2ab & a^2-b^2 \end{pmatrix}$	M1A1
(c)	**EITHER**	
	$\det(\boldsymbol{M}^2) = (a^2-b^2)(a^2-b^2) + (2ab)(2ab)$	A1

$\det(\boldsymbol{M}^2) = (a^2-b^2)^2 + (2ab)^2 \quad (= (a^2+b^2)^2)$

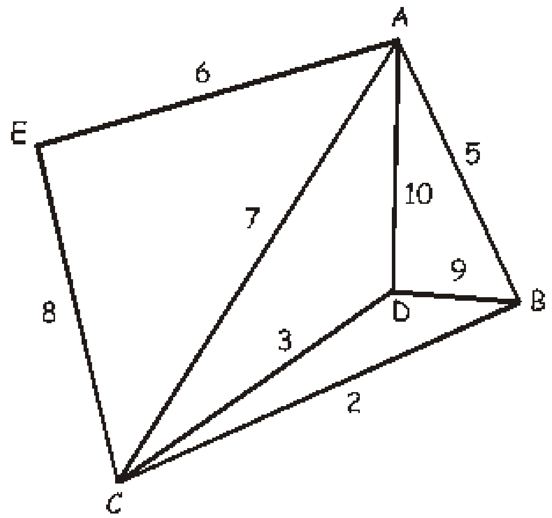
since the first term is non-negative and the second is positive	R1
therefore $\det(\boldsymbol{M}^2) > 0$	

Note: Do not penalise first term stated as positive.

OR	
$\det(\boldsymbol{M}^2) = (\det \boldsymbol{M})^2$	A1
since det $\boldsymbol{M}$ is positive so too is det $(\boldsymbol{M}^2)$	R1

[6]

(a) (i)



A1A1A1

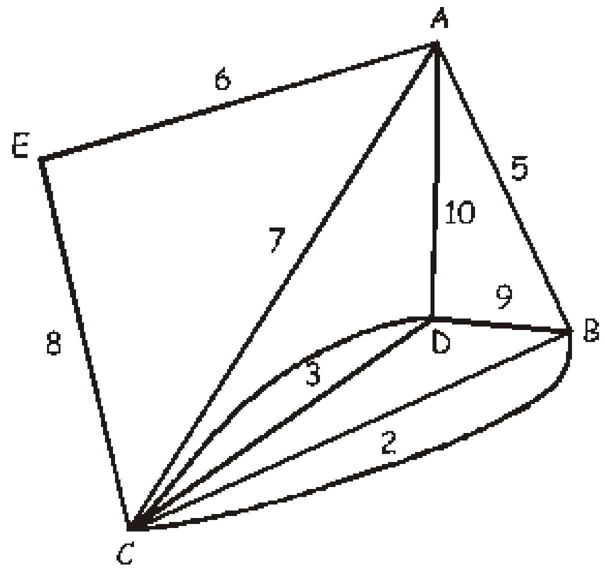
Note: Award A1 for the vertices, A1 for edges and A1 for planar form.

(ii) It is possible to find an Eulerian trail in this graph since exactly two of the vertices have odd degree — R1

(iii) B and D are the odd vertices — M1

$BC + CD = 3 + 2 = 5$ and $BD = 9$, — A1

since $5 < 9$, BC and CD must be traversed twice — R1



A possible walk by inspection is ACBDABCDCEA — A1

This gives a total length of

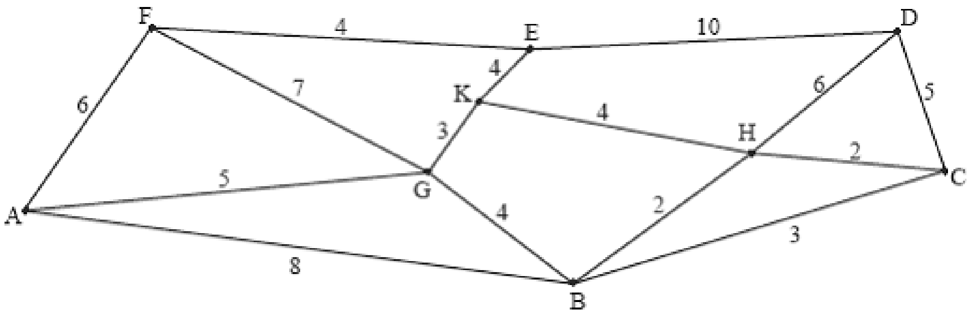
$2(2 + 3) + 8 + 9 + 5 + 7 + 10 + 6 = 55$ for the walk — A1

(b) The sum of all the vertex degrees is twice the number of edges, i.e. an even number.

Hence a graph cannot have exactly one vertex of odd degree. — M1R1

[11]

(a)



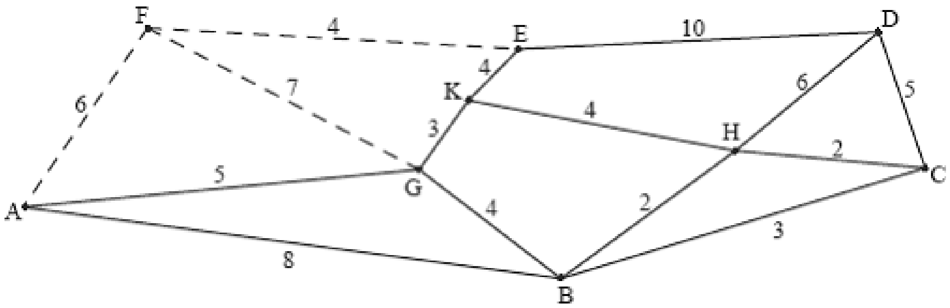
Start at an edge with weight 2, say BH, add other edges of weight 2 such that a cycle is not formed. Continue to add edges of increasing weight until all vertices have been included. M1

thus the minimum spanning tree is
BH + HC + GK + KH + KE + EF + GA + CD A3
total weight = 2 + 2 + 3 + 4 + 4 + 4 + 5 + 5 = 29 A1

Note: GB may replace KH and other orders are possible.

(b) (i) upper bound = 2 × weight of minimum spanning tree M1
= 58 A1

(ii) deleting vertex F M1



the minimum spanning tree is
BH + HC + GK + KE + KH + GA + CD A2
total weight = 2 + 2 + 3 + 4 + 4 + 5 + 5 = 25 A1

adding the two edges of least weight from F M1
lower bound = 25 + 4 + 6 = 35 A1

Note: Alternative solutions may be given by deleting a different vertex.

[13]