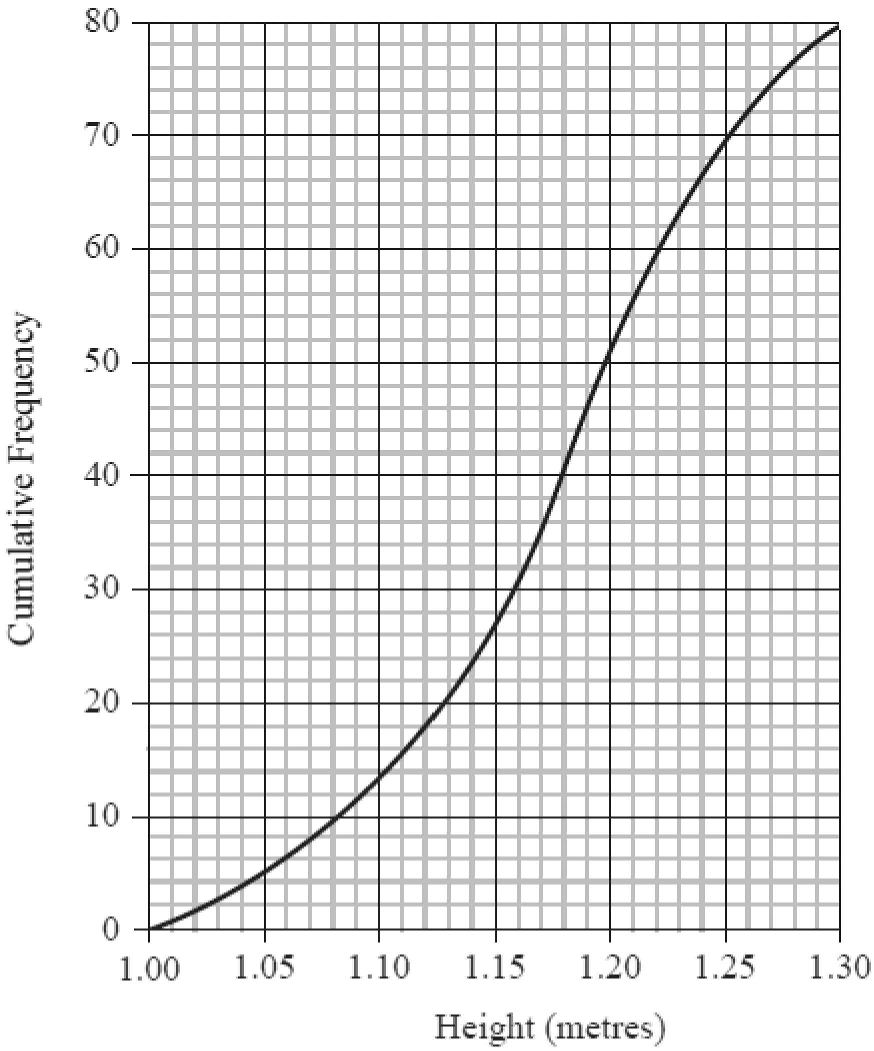
1. The heights in metres of a random sample of 80 boys in a certain age group were measured and the following cumulative frequency graph obtained.



(a) (i) Estimate the median of these data.

(ii) Estimate the interquartile range for these data.

(3)

(b) (i) Produce a frequency table for these data, using a class width of 0.05 metres.

(ii) Calculate unbiased estimates of the mean and variance of the heights of the population of boys in this age group.

(5)

(c) A boy is selected at random from these 80 boys.

(i) Find the probability that his height is less than or equal to 1.15 metres.

(ii) Given that his height is less than or equal to 1.15 metres, find the probability that his height is less than or equal to 1.12 metres.

(5)
(Total 13 marks)

2. The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days).

(3)

(b) Determine the probability that there are more than three breakdowns during the month of June.

(2)

(c) Determine the probability that there are no breakdowns during the first five days of June.

(2)

(d) Find the probability that the first breakdown in June occurs on June 3^{rd}.

(3)

(e) It costs 1850 euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June.

(1)

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June.

(2)
(Total 13 marks)

3. Mr Lee is planning to go fishing this weekend. Assuming that the number of fish caught per hour follows a Poisson distribution with mean 0.6, find

(a) the probability that he catches at least one fish in the first hour;

(2)

(b) the probability that he catches exactly three fish if he fishes for four hours;

(2)

(c) the number of **complete** hours that Mr Lee needs to fish so that the probability of catching more than two fish exceeds 80 %.

(3)
(Total 7 marks)

4. Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice.

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3.

(3)

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score.

(4)

(c) Let X denote the largest number shown on the four dice.

(i) Show that $\mathrm{P}(X \le 2) = \frac{16}{81}$.

(ii) Copy and complete the following probability distribution table.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$		

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ and hence find $\mathrm{Var}(X)$.

(10)

(d) Given that $X = 3$, find the probability that the sum of the numbers shown on the four dice is 8.

(4)
(Total 21 marks)

5. A discrete random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$\mathrm{P}(X = x)$	0.15	0.21	p	q	0.13	0.07

(a) If $\mathrm{E}(X) = 2.61$, determine the value of p and of q.

(4)

(b) Calculate Var (X) to three significant figures.

(2)
(Total 6 marks)

6. A graph G with vertices A, B, C, D, E has the following cost adjacency matrix.

	A	B	C	D	E
A	–	12	10	17	19
B	12	–	13	20	11
C	10	13	–	16	14
D	17	20	16	–	15
E	19	11	14	15	–

(a) (i) Use Kruskal's algorithm to find and draw the minimum spanning tree for G.

(ii) The graph H is formed from G by removing the vertex D and all the edges connected to D. Draw the minimum spanning tree for H and use it to find a lower bound for the travelling salesman problem for G.

(7)

(b) Show that 80 is an upper bound for this travelling salesman problem.

(2)
(Total 9 marks)

7. The complex numbers $z_1 = 2 - 2\text{i}$ and $z_2 = 1 - \text{i}\sqrt{3}$ are represented by the points A and B respectively on an Argand diagram. Given that O is the origin,

(a) find AB, giving your answer in the form $a\sqrt{b-\sqrt{3}}$, where $a, b \in \mathbb{Z}^+$;

(3)

(b) calculate $\text{A}\hat{\text{O}}\text{B}$ in terms of π.

(3)
(Total 6 marks)

8. Let $\boldsymbol{M} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ where a and b are non-zero real numbers.

(a) Show that $\boldsymbol{M}$ is non-singular.

(2)

(b) Calculate $\boldsymbol{M}^2$.

(2)

(c) Show that $\det(\boldsymbol{M}^2)$ is positive.

(2)
(Total 6 marks)

9. (a) The matrix below shows the distances between towns A, B, C, D and E.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} - & 5 & 7 & 10 & 6 \\ 5 & - & 2 & 9 & - \\ 7 & 2 & - & 3 & 8 \\ 10 & 9 & 3 & - & - \\ 6 & - & 8 & - & - \end{pmatrix} \end{array}$$

(i) Draw the graph, in its planar form, that is represented by the matrix.

(ii) Write down with reasons whether or not it is possible to find an Eulerian trail in this graph.

(iii) Solve the Chinese postman problem with reference to this graph if A is to be the starting and finishing point. Write down the walk and determine the length of the walk.

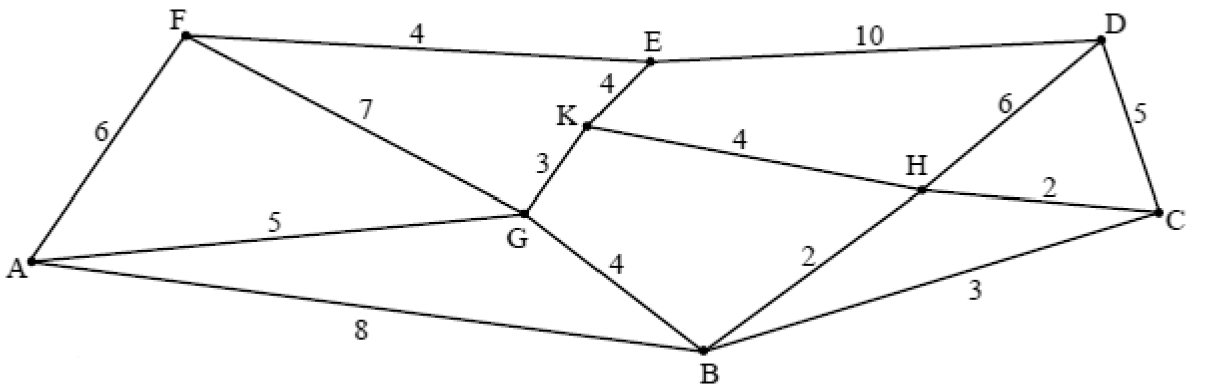
(9)

(b) Show that a graph cannot have exactly one vertex of odd degree.

(2)
(Total 11 marks)

10. (a) Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph shown below. State the weight of the tree.

(5)



(b) For the travelling salesman problem defined by this graph, find

(i) an upper bound;

(ii) a lower bound.

(8)
(Total 13 marks)