Binomial and normal distribution [77 marks]

1. [Maximum mark: 6] 19N.1.SL.TZ0.T_12

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

(a) Write down the probability that the runner completed the race in more than 28 minutes. [1]

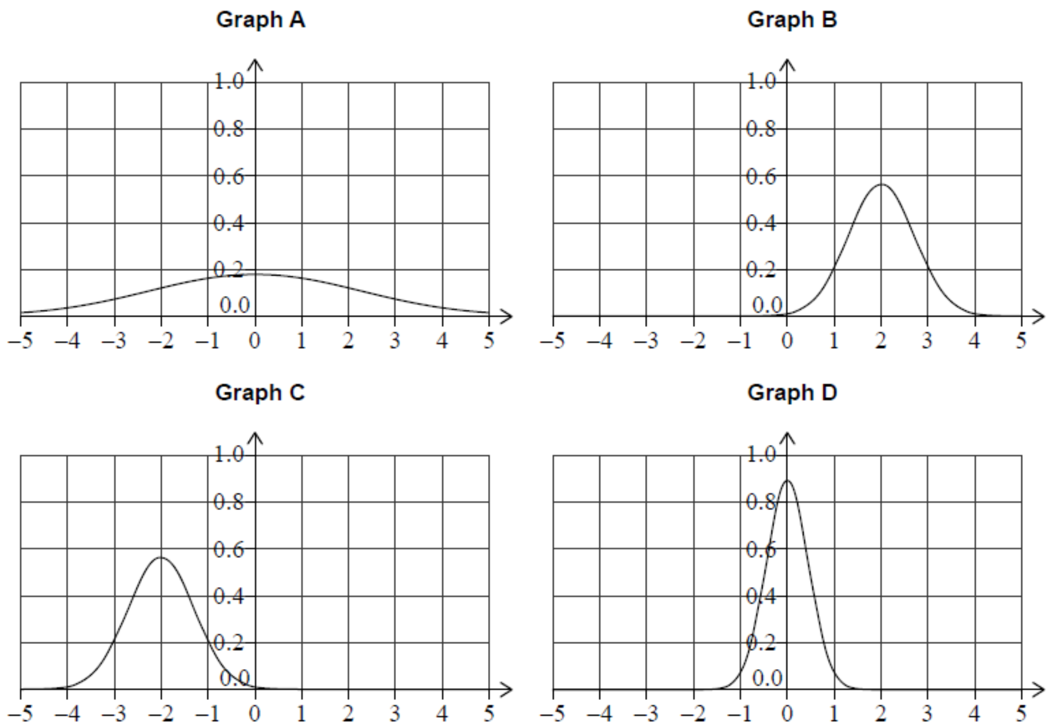
(b) Calculate the probability that the runner completed the race in less than 26 minutes. [2]

(c) It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race.

Find the value of k. [3]

2. [Maximum mark: 6] 19M.1.SL.TZ1.T_11

Consider the following graphs of normal distributions.



(a) In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation.

Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	
Mean $= 0$; standard deviation $= 0.447$	

[2]

At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

(b) Find the probability that a suitcase weighs less than 15 kg. [2]

(c) Any suitcase that weighs more than k kg is identified as excess baggage.
19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k. [2]

3. [Maximum mark: 7] 22M.1.SL.TZ2.5

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

4. [Maximum mark: 6] 23M.1.SL.TZ2.9

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of 4 cm and a standard deviation of 0.25 cm.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than 3.7 cm. [2]

It is known that 30% of the seeds have a length greater than $k\text{ cm}$.

(b) Find the value of k. [2]

For a seed of length $d\text{ cm}$, chosen at random, $\mathrm{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

5. [Maximum mark: 7] 23M.1.SL.TZ1.12

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\ \mathrm{km\ h^{-1}}$.

A speed of $75.7\ \mathrm{km\ h^{-1}}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

Speeding tickets are issued to all drivers travelling at a speed greater than $72\ \mathrm{km\ h^{-1}}$.

(b) Find the probability that a randomly selected driver who passes the speed camera receives a speeding ticket. [2]

It is found that 82% of cars on this road travel at speeds between $p\ \mathrm{km\ h^{-1}}$ and $q\ \mathrm{km\ h^{-1}}$, where $p < q$. This interval includes cars travelling at a speed of $74\ \mathrm{km\ h^{-1}}$.

(c) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

6. [Maximum mark: 5] 22N.1.SL.TZ0.8

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

(a) Find the probability that Roy completes a lap in less than 55 seconds. [2]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

(b) Find the value of t. [3]

7. [Maximum mark: 6] 22M.1.SL.TZ1.8

A factory produces bags of sugar with a labelled weight of $500\,\mathrm{g}$. The weights of the bags are normally distributed with a mean of $500\,\mathrm{g}$ and a standard deviation of $3\,\mathrm{g}$.

(a) Write down the percentage of bags that weigh more than $500\,\mathrm{g}$. [1]

A bag that weighs less than $495\,\mathrm{g}$ is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight. [2]

(c) A bag that weighs more than k grams is rejected by the factory for being overweight. The factory rejects 2% of bags for being overweight.

Find the value of k. [3]

8. [Maximum mark: 5] 22M.1.SL.TZ2.10

The masses of Fuji apples are normally distributed with a mean of $163\,\mathrm{g}$ and a standard deviation of $6.83\,\mathrm{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\,\mathrm{g}$ and $183\,\mathrm{g}$.

(a) Determine the probability that a Fuji apple selected at random will be a large apple. [2]

Approximately 68% of Fuji apples have a mass within the medium-sized category, which is between k and $172\,\mathrm{g}$.

(b) Find the value of k. [3]

9. [Maximum mark: 15] 22M.2.SL.TZ1.5

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

(a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]

(b.i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]

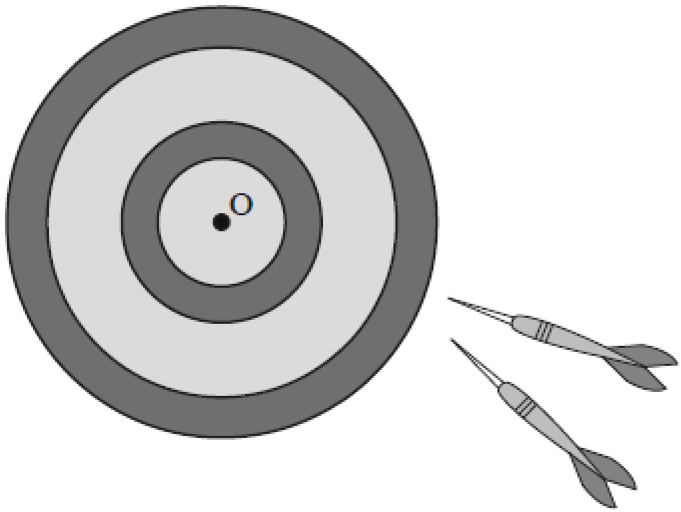
(b.ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays $\$150$ for a ticket. If too many passengers arrive, then the airline will give $\$300$ in compensation to each passenger that cannot board.

(c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

10. [Maximum mark: 14] 21N.2.SL.TZ0.5

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean $10\,\mathrm{cm}$ and standard deviation $3\,\mathrm{cm}$.

Find the probability that

(a.i) a dart lands less than $13\,\mathrm{cm}$ from O. [2]

(a.ii) a dart lands more than $15\,\mathrm{cm}$ from O. [1]

Each of Arianne's throws is independent of her previous throws.

(b) Find the probability that Arianne throws two consecutive darts that land more than $15\,\mathrm{cm}$ from O. [2]

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

(c) Find the probability that Arianne does **not** score a point on a turn of three darts.

[2]

In the competition Arianne has ten turns, each with three darts.

(d.i) Find the probability that Arianne scores at least 5 points in the competition. [3]

(d.ii) Find the probability that Arianne scores at least 5 points and less than 8 points. [2]

(d.iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points. [2]