Derivatives [191 marks]

1. [Maximum mark: 6] 19M.2.SL.TZ1.S_3

Consider the function $f\left(x\right)=x^{2}\mathrm{e}^{3x}$, $x\in\mathbb{R}$.

(a) Find $f'\left(x\right)$. [4]

(b) The graph of f has a horizontal tangent line at $x=0$ and at $x=a$. Find a. [2]

2. [Maximum mark: 7] 22M.1.SL.TZ1.9

The function f is defined by $f(x)=\frac{2}{x}+3x^{2}-3,\ x\neq 0$.

(a) Find $f\prime(x)$. [3]

(b) Find the equation of the normal to the curve $y=f(x)$ at $(1,\ 2)$ in the form $ax+by+d=0$, where $a,\ b,\ d\in\mathbb{Z}$. [4]

3. [Maximum mark: 7] EXN.1.SL.TZ0.7

Consider the curve $y=x^{2}-4x+2$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(b) Show that the normal to the curve at the point where $x=1$ is $2y-x+3=0$. [6]

4. [Maximum mark: 16] 18M.1.SL.TZ2.S_10

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

(a.i) Write down $f'(2)$. [2]

(a.ii) Find $f(2)$. [2]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

(b) Show that the graph of g has a gradient of 6 at P. [5]

(c) Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q.

Find the y-coordinate of Q. [7]

5. [Maximum mark: 8] EXN.1.AHL.TZ0.15

Consider the function $f(x) = \sqrt{-ax^2 + x + a},\ a \in \mathbb{R}^+$.

(a) Find $f\prime(x)$. [2]

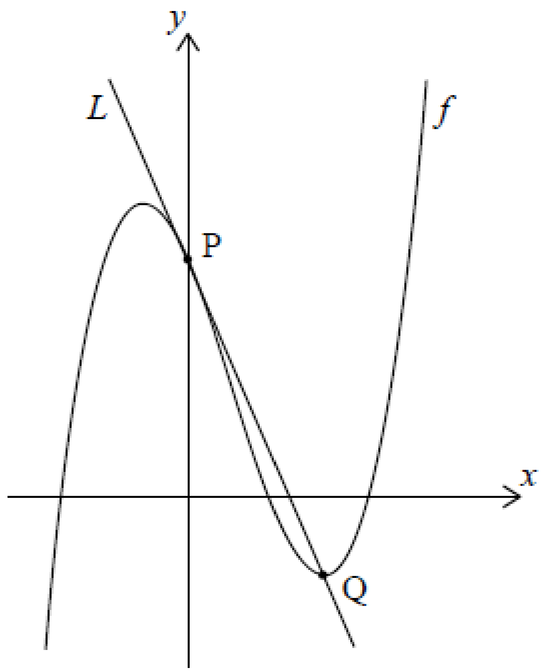
For $a > 0$ the curve $y = f(x)$ has a single local maximum.

(b) Find in terms of a the value of x at which the maximum occurs. [2]

(c) Hence find the value of a for which y has the smallest possible maximum value. [4]

6. [Maximum mark: 16] 18N.1.SL.TZ0.S_10

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y-axis at the point P. The line *L* is tangent to the graph of f at P.

(a) Find the coordinates of P. [2]

(b.i) Find $f'(x)$. [2]

(b.ii) Hence, find the equation of *L* in terms of a. [4]

(c) The graph of f has a local minimum at the point Q. The line *L* passes through Q.

Find the value of a. [8]

7. [Maximum mark: 7] 18M.1.SL.TZ1.S_7

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$. [7]

8. [Maximum mark: 6] 22M.1.AHL.TZ1.8

Consider the curve $y = 2x(4 - \mathrm{e}^{x})$.

(a.i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(a.ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

(b) The curve has a point of inflexion at $(a,\ b)$.

Find the value of a. [2]

9. [Maximum mark: 5] 23M.1.SL.TZ1.5

Line L_1 is tangent to the graph of a function $f(x)$ at the point $\mathrm{P}(3,\ -1)$.
Line L_2 is given by the equation $y = -\frac{1}{2}x - \frac{5}{2}$ and is perpendicular to L_1.

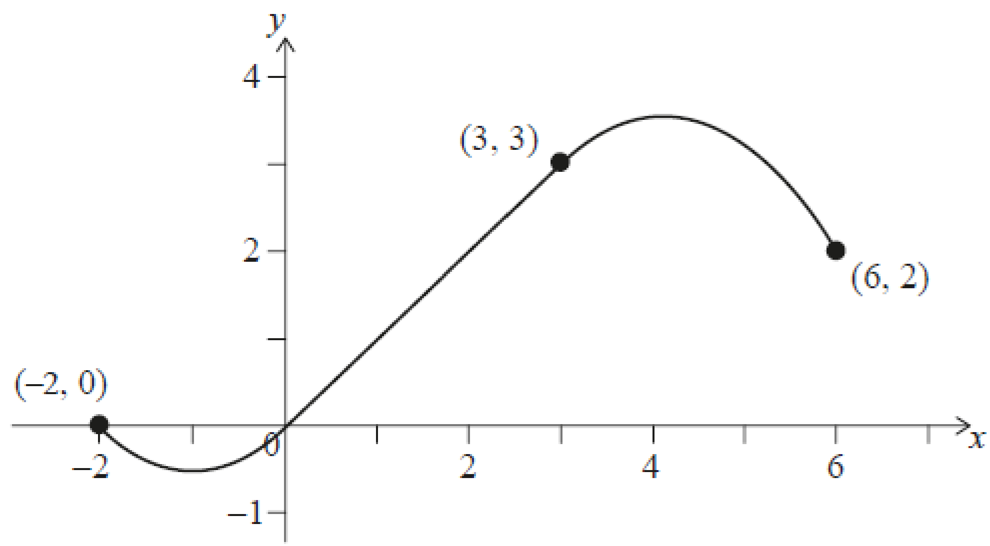
(a) Write down the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + c$. [2]

(c) Show that L_2 is not the line that is normal to $f(x)$ at point P. [2]

10. [Maximum mark: 9] 23M.1.AHL.TZ1.10

A decorative hook can be modelled by the curve with equation $y = f(x)$. The graph of $y = f(x)$ is shown and consists of a line segment from $(0,\ 0)$ to $(3,\ 3)$ and two sections formed by quadratic curves.



(a) Write down the equation of the line segment for $0 \leq x \leq 3$. [1]

The quadratic curve, with endpoints $(-2,\ 0)$ and $(0,\ 0)$, has the same gradient at $(0,\ 0)$ as the line segment.

(b) Find the equation of the curve between $(-2,\ 0)$ and $(0,\ 0)$. [3]

The second quadratic curve, with endpoints $(3,\ 3)$ and $(6,\ 2)$, has the same gradient at $(3,\ 3)$ as the line segment.

(c) Find the equation of this curve. [4]

(d) Write down f as a piecewise function. [1]

11. [Maximum mark: 7] 22M.1.SL.TZ2.11

Consider the function $f(x) = x^2 - \frac{3}{x},\ x \neq 0.$

(a) Find $f\prime(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1,\ \ -2)$.

(b) Use your answer to part (a) to find the gradient of L. [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]

12. [Maximum mark: 14] 19M.2.SL.TZ1.T_6

The function $f\left(x\right) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + kx + 5$ has a local maximum and a local minimum. The local maximum is at $x = -3$.

(a) Show that $k = -6$. [5]

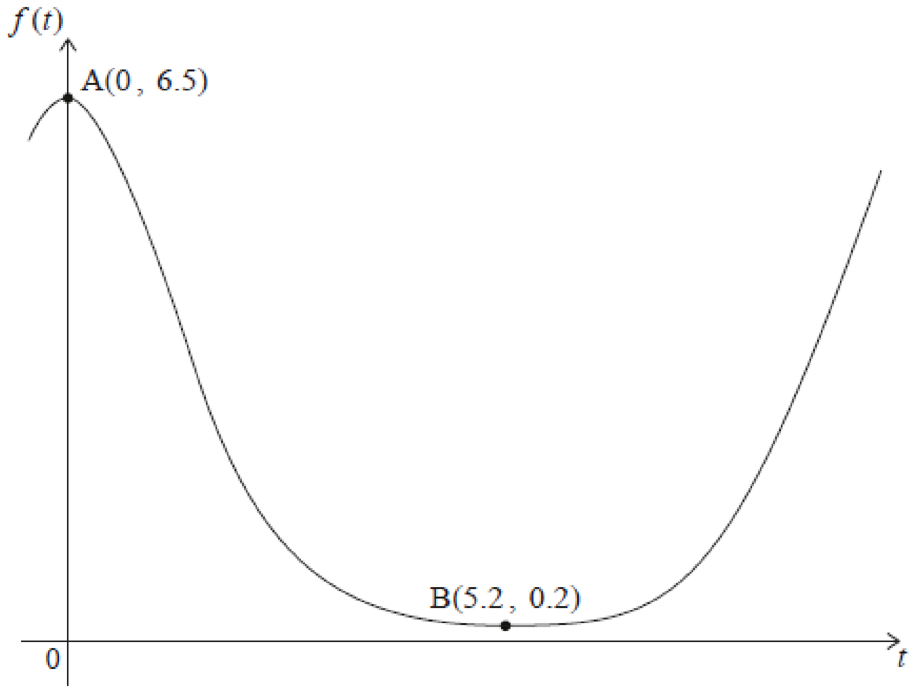
(b) Find the coordinates of the local **minimum**. [2]

(c) Write down the interval where the gradient of the graph of $f\left(x\right)$ is negative. [2]

(d) Determine the equation of the normal at $x = -2$ in the form $y = mx + c$. [5]

13. [Maximum mark: 8] 22M.1.AHL.TZ1.17

A function f is of the form $f(t) = p\mathrm{e}^{q\cos(rt)},\ p,\ q,\ r \in \mathbb{R}^+$. Part of the graph of f is shown.



The points A and B have coordinates $\mathrm{A}(0,\ 6.5)$ and $\mathrm{B}(5.2,\ 0.2)$, and lie on f.

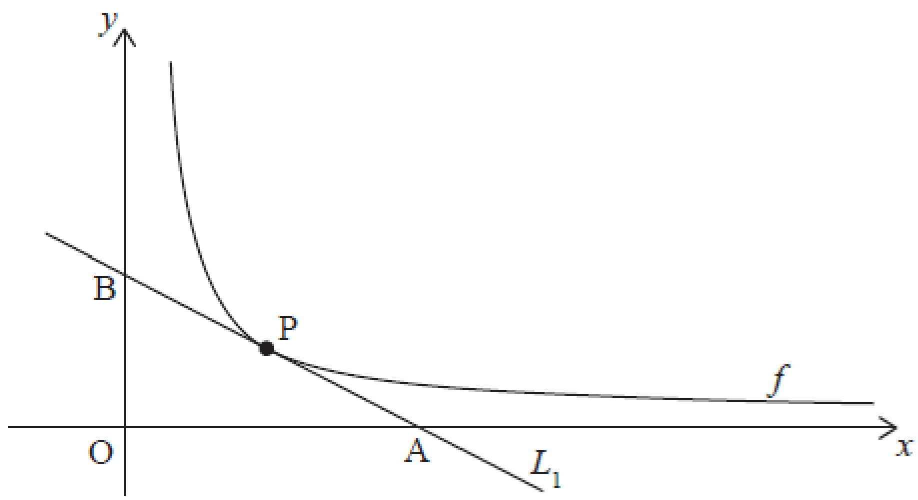
The point A is a local maximum and the point B is a local minimum.

Find the value of p, of q and of r. [8]

14. [Maximum mark: 15] 20N.1.SL.TZ0.S_10

The following diagram shows part of the graph of $f(x) = \frac{k}{x}$, for $x > 0,\ k > 0$.

Let $\mathrm{P}\left(p,\ \frac{k}{p}\right)$ be any point on the graph of f. Line L_1 is the tangent to the graph of f at P.



(a.i) Find $f\prime(p)$ in terms of k and p. [2]

(a.ii) Show that the equation of L_1 is $kx + p^2y - 2pk = 0$. [2]

Line L_1 intersects the x-axis at point $\mathrm{A}(2p,\ 0)$ and the y-axis at point B.

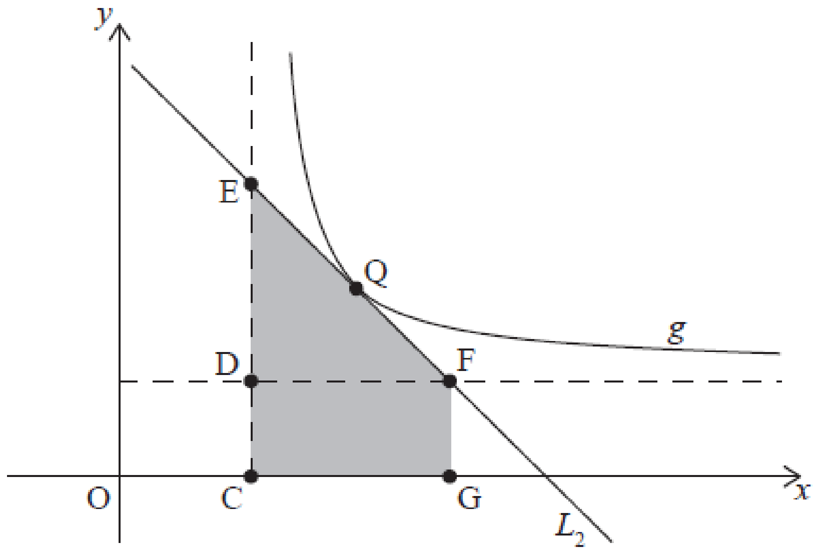
(b) Find the area of triangle AOB in terms of k. [5]

(c) The graph of f is translated by $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ to give the graph of g.

In the following diagram:

- point Q lies on the graph of g
- points C, D and E lie on the vertical asymptote of g
- points D and F lie on the horizontal asymptote of g
- point G lies on the x-axis such that FG is parallel to DC.

Line L_2 is the tangent to the graph of g at Q, and passes through E and F.



Given that triangle EDF and rectangle CDFG have equal areas, find the gradient of L_2 in terms of p. [6]

15. [Maximum mark: 6] 20N.1.SL.TZ0.T_13

Consider the graph of the function $f(x) = x^2 - \frac{k}{x}$.

(a) Write down $f'(x)$. [3]

The equation of the tangent to the graph of $y = f(x)$ at $x = -2$ is $2y = 4 - 5x$.

(b) Write down the gradient of this tangent. [1]

(c) Find the value of k. [2]

16. [Maximum mark: 15] 19M.1.SL.TZ2.S_9

Let θ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

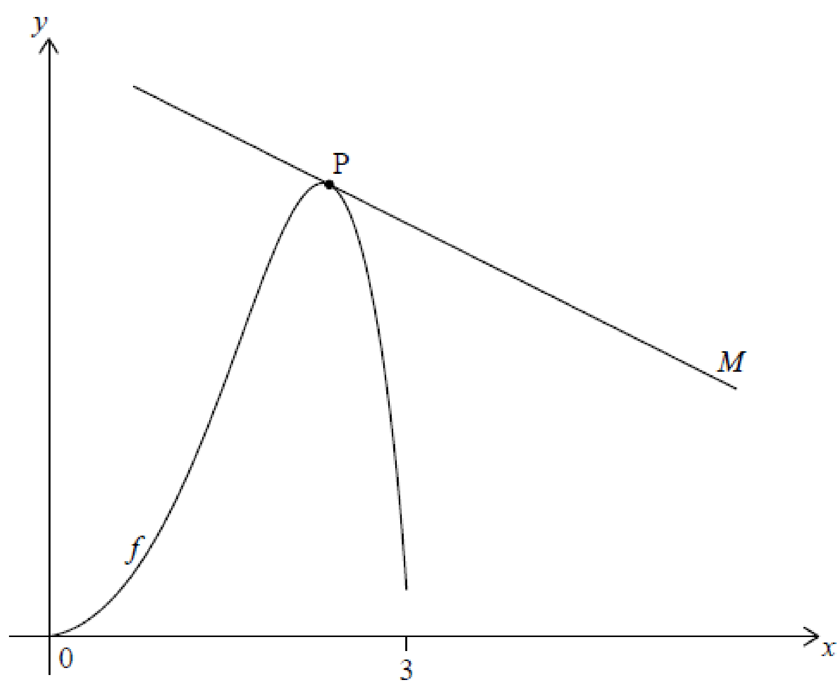
(a) Find the value of $\tan\theta$. [4]

(b) Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L. [2]

Let $f\left(x\right) = \mathrm{e}^{x}\sin x - \frac{3x}{4}$.

(c) Find the derivative of f. [5]

(d) The following diagram shows the graph of f for $0 \le x \le 3$. Line M is a tangent to the graph of f at point P.



Given that M is parallel to L, find the x-coordinate of P. [4]

17. [Maximum mark: 16] 19M.2.SL.TZ1.S_9

Let $f(x) = \frac{16}{x}$. The line L is tangent to the graph of f at $x = 8$.

(a) Find the gradient of L. [2]

L can be expressed in the form $\boldsymbol{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} + t\boldsymbol{u}$.

(b) Find $\boldsymbol{u}$. [2]

The direction vector of $y = x$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(c) Find the acute angle between $y = x$ and L. [5]

(d.i) Find $(f \circ f)(x)$. [3]

(d.ii) Hence, write down $f^{-1}(x)$. [1]

(d.iii) Hence or otherwise, find the obtuse angle formed by the tangent line to f at $x = 8$ and the tangent line to f at $x = 2$. [3]

18. [Maximum mark: 7] 19M.2.SL.TZ2.T_5

Consider the function $f(x) = \frac{1}{3}x^3 + \frac{3}{4}x^2 - x - 1$.

(d) Find $f'(x)$. [3]

(e) Find the gradient of the graph of $y = f(x)$ at $x = 2$. [2]

(f) Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 2$. Give the equation in the form $ax + by + d = 0$ where, a, b, and $d \in \mathbb{Z}$. [2]

19. [Maximum mark: 4] 19M.2.AHL.TZ1.H_1

Let l be the tangent to the curve $y = x\mathrm{e}^{2x}$ at the point $(1, \mathrm{e}^2)$.

Find the coordinates of the point where l meets the x-axis. [4]

20. [Maximum mark: 6] 18N.1.SL.TZ0.T_11

Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

The curve has a tangent at the point P(−1, −2).

(b) Find the gradient of this tangent at point P. [2]

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$. [2]

21. [Maximum mark: 6] 18M.1.SL.TZ2.T_14

Consider the function $f(x) = \frac{x^4}{4}$.

(a) Find $f'(x)$ [1]

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$. [2]

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$. [3]