Discrete distribution [83 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.12

Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled −3, −1, 0, 1, 2 and 5.

The score for the game, *X*, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for *X*.

Score x	-3	-1	0	1	2	5
$\mathrm{P}(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Markscheme

$\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Markscheme

$-3\times\frac{1}{18}+(-1)\times\frac{4}{18}+0\times\frac{3}{18}+\ldots+5\times\frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$=1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) Jae Hee plays the game twice and adds the two scores together.

Find the probability Jae Hee has a **total** score of −3. [3]

Markscheme

$2\times\frac{1}{18}\times\frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18}\times\frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$=\frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

2. [Maximum mark: 7] EXN.1.SL.TZ0.12

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.

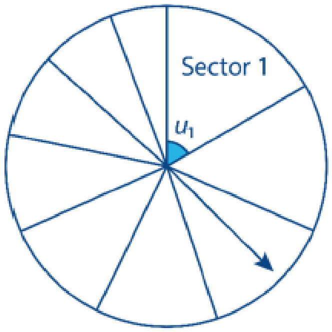


Diagram not to scale

(a) Write down the value of $\sum_{i=1}^{9} u_i$. [1]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

360° **A1**

[1 mark]

It is given that $u_9 = \frac{1}{3}u_1$.

(b) Find the value of u_1. [4]

Markscheme

EITHER

$360 = \frac{9}{2}(u_1 + u_9)$ **M1**

$360 = \frac{9}{2}\left(u_1 + \frac{1}{3}u_1\right) = 6u_1$ **M1A1**

OR

$360=\frac{9}{2}(2u_1+8d)$ **M1**

$u_9=\frac{1}{3}u_1=u_1+8d \Rightarrow u_1=-12d$ **M1**

Substitute this value $360=\frac{9}{2}\left(2u_1-8\times\frac{u_1}{12}\right)$ $\left(=\frac{9}{2}\times\frac{4}{3}u_1=6u_1\right)$ **A1**

THEN

$u_1=60^\circ$ **A1**

[4 marks]

(c) A game is played in which the arrow attached to the centre of the disc is spun and the sector in which the arrow stops is noted. If the arrow stops in sector 1 the player wins 10 points, otherwise they lose 2 points.

Let X be the number of points won

Find $\mathrm{E}(X)$. [2]

Markscheme

$\mathrm{E}(X)=10\times\frac{60}{360}-2\times\frac{300}{360}=0$ **M1A1**

[2 marks]

3. [Maximum mark: 5] 23M.1.SL.TZ2.12

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathbf{P}(X = x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

Markscheme

$0.15 + 0.2 + k + 0.16 + 2k + 0.25 = 1$ *(M1)*

$k = 0.08$ *A1*

[2 marks]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in $\$$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain ($\$$)	-4	-3	-1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

Markscheme

$(-4 \times 0.15) + (-3 \times 0.2) + (-1 \times 0.08) + (0 \times 0.16) + (1 \times 0.16) + (4 \times 0.25)$ *(M1)*

$= -0.12$ *A1*

$\mathrm{E}(X) \neq 0$ therefore the game is not fair *R1*

Note: Do not award ***A0R1*** without an explicit value for $\mathrm{E}(X)$ seen. The ***R1*** can be awarded for comparing **their** $\mathrm{E}(X)$ to zero provided working is shown.

[3 marks]

4. [Maximum mark: 7] 22N.1.SL.TZ0.9

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

Markscheme

$0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

Markscheme

$0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

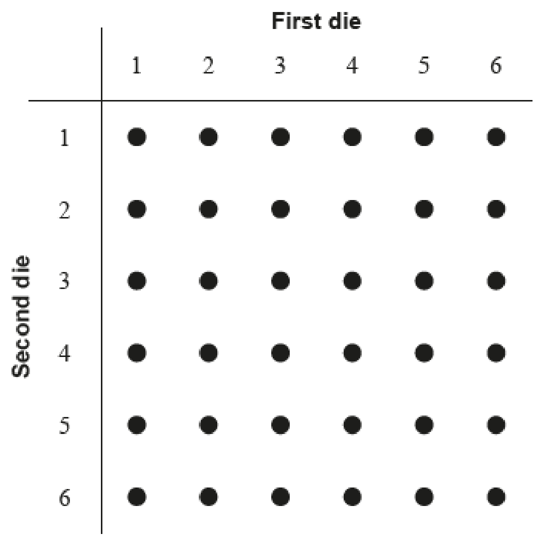
Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k =)\ 24$ (points) ***A1***

[3 marks]

5. [Maximum mark: 7] 21M.1.SL.TZ1.10

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$P(T = t)$						

Markscheme

t	1	2	3	4	5	6
$P(T = t)$	$\frac{1}{36}$ (0.027777…)	$\frac{3}{36}$ (0.083333…)	$\frac{5}{36}$ (0.138888…)	$\frac{7}{36}$ (0.194444…)	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ (0.305555…)

A2

Note: Award ***A1*** if three to five probabilities are correct.

[2 marks]

Find the probability that

(b.i) a player scores at least 3 in a game. [1]

Markscheme

$\frac{32}{36}$ $\left(\frac{8}{9},\ 0.888888\ldots,\ 88.9\%\right)$ ***(A1)***

[1 mark]

(b.ii) a player scores 6, given that they scored at least 3. [2]

Markscheme

use of conditional probability ***(M1)***

e.g. denominator of 32 **OR** denominator of $0.888888\ldots$, etc.

$\frac{11}{32}$ $(0.34375,\ 34.4\%)$ ***A1***

[2 marks]

(c) Find the expected score of a game. [2]

Markscheme

$\frac{1\times1+3\times2+5\times3+\ldots+11\times6}{36}$ ***(M1)***

$=\frac{161}{36}$ $\left(4\frac{17}{36},\ 4.47,\ 4.47222\ldots\right)$ ***A1***

[2 marks]

6. [Maximum mark: 6] 20N.2.SL.TZ0.S_3

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\mathrm{P}(X=x)$	q	$4p^2$	p	$0.7-4p^2$

(a) Find an expression for q in terms of p. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of summing probabilities to 1 ***(M1)***

eg $q+4p^2+p+0.7-4p^2=1,\ \ 1-4p^2-p-0.7+4p^2$

$q=0.3-p$ ***A1 N2***

[2 marks]

(b.i) Find the value of p which gives the largest value of $\mathrm{E}(X)$. [3]

Markscheme

correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0\times(0.3-p)+1\times4p^2+2\times p+3\times\left(0.7-4p^2\right)$

valid approach to find when $\mathrm{E}(X)$ is a maximum ***(M1)***

eg max on sketch of $\mathrm{E}(X)$, $8p+2+3\times(-8p)=0$, $\frac{-b}{2a}=\frac{-2}{2\times(-8)}$

$p=\frac{1}{8}\ (=0.125)$ (exact) (accept $x=\frac{1}{8}$) ***A1 N3***

[3 marks]

(b.ii) Hence, find the largest value of $\mathrm{E}(X)$. [1]

Markscheme

2.225

$\frac{89}{40}$ (exact), 2.23 ***A1 N1***

[1 mark]

7. [Maximum mark: 6] 18M.2.SL.TZ1.S_2

A biased four-sided die is rolled. The following table gives the probability of each score.

Score	1	2	3	4
Probability	0.28	k	0.15	0.3

(a) Find the value of k. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of summing to 1 ***(M1)***

eg $0.28 + k + 1.5 + 0.3 = 1,\ 0.73 + k = 1$

$k = 0.27$ ***A1 N2***

[2 marks]

(b) Calculate the expected value of the score. [2]

Markscheme

correct substitution into formula for $E(X)$ ***(A1)***
eg $1 \times 0.28 + 2 \times k + 3 \times 0.15 + 4 \times 0.3$

$E(X) = 2.47$ (exact) ***A1 N2***

[2 marks]

(c) The die is rolled 80 times. On how many rolls would you expect to obtain a three? [2]

Markscheme

valid approach ***(M1)***

eg np, 80×0.15

12 ***A1 N2***

[2 marks]

8. [Maximum mark: 8] 17N.2.SL.TZ0.S_4

A discrete random variable X has the following probability distribution.

X	0	1	2	3
$\mathrm{P}(X = x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

(a) Find the value of k. [4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

*eg*total probability = 1

correct equation ***(A1)***

eg$0.475 + 2k^2 + \frac{k}{10} + 6k^2 = 1,\ 8k^2 + 0.1k - 0.525 = 0$

$k = 0.25$ ***A2 N3***

[4 marks]

(b) Write down $\mathrm{P}(X = 2)$. [1]

Markscheme

$\mathrm{P}(X = 2) = 0.025$ ***A1 N1***

[1 mark]

(c) Find $\mathrm{P}(X = 2|X > 0)$. [3]

Markscheme

valid approach for finding $\mathrm{P}(X > 0)$ ***(M1)***

eg$1 - 0.475,\ 2(0.25^2) + 0.025 + 6(0.25^2),\ 1 - \mathrm{P}(X = 0),\ 2k^2 + \frac{k}{10} + 6k^2$

correct substitution into formula for conditional probability ***(A1)***

eg$\frac{0.025}{1-0.475},\ \frac{0.025}{0.525}$

0.0476190

$\mathrm{P}(X = 2|X > 0) = \frac{1}{21}$ (exact), 0.0476 ***A1 N2***

[3 marks]

9. [Maximum mark: 16] 19M.2.SL.TZ1.S_10

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a.i) Find the probability of rolling exactly one red face. [2]

Markscheme

valid approach to find P(one red) (M1)

eg ${}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n, p)$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\binom{3}{1}$

listing all possible cases for exactly one red (may be indicated on tree diagram)

P(1 red) = 0.444 $\left(= \frac{4}{9}\right)$ [0.444, 0.445] ***A1 N2***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

(a.ii) Find the probability of rolling two or more red faces. [3]

Markscheme

valid approach ***(M1)***

eg P($X = 2$) + P($X = 3$), 1 − P($X \le 1$), binomcdf$\left(3, \frac{1}{3}, 2, 3\right)$

correct working ***(A1)***

eg $\frac{2}{9} + \frac{1}{27}$, 0.222 + 0.037 , $1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9}$

0.259259

P(at least two red) = 0.259 $\left(= \frac{7}{27}\right)$ ***A1 N3***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins $10 for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly $10 to his winnings is $\frac{1}{3}$. [5]

Markscheme

recognition that winning \$10 means rolling exactly one green ***(M1)***

recognition that winning \$10 also means rolling at most 1 red ***(M1)***

eg "cannot have 2 or more reds"

correct approach ***A1***

eg P(1G ∩ 0R) + P(1G ∩ 1R), P(1G) − P(1G ∩ 2R),

"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this ***A1*** for purely numerical expressions.

one correct probability for their approach ***(A1)***

eg $3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2$, $\frac{6}{27}$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\frac{1}{9}$, $\frac{2}{9}$

correct working leading to $\frac{1}{3}$ ***A1***

eg $\frac{3}{27}+\frac{6}{27}$, $\frac{12}{27}-\frac{3}{27}$, $\frac{1}{9}+\frac{2}{9}$

probability = $\frac{1}{3}$ ***AG N0***

[5 marks]

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where \$$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c.i) Write down the value of x. [1]

Markscheme

$x=\frac{7}{27}$, 0.259 (check ***FT*** from (a)(ii)) ***A1 N1***

[1 mark]

(c.ii) Hence, find the value of y. [2]

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $\sum = 1,\ x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1,\ 1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$

0.148147 (0.148407 if working with **their** x value to 3 sf)

$y = \frac{4}{27}$ (exact), 0.148 ***A1 N2***

[2 marks]

(d) Ted will always have another turn if he expects an increase to his winnings.

Find the least value of w for which Ted should end the game instead of having another turn. [3]

Markscheme

correct substitution into the formula for expected value ***(A1)***

eg $-w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27}$

correct critical value (accept inequality) ***A1***

eg $w = 34.2857 \left(= \frac{240}{7}\right)$, $w > 34.2857$

$40 ***A1 N2***

[3 marks]

10. [Maximum mark: 15] 18N.1.SL.TZ0.S_9

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

(a.i) Find the probability, in terms of n, that the game will end on her first draw. [1]

Markscheme

$\frac{2}{n}$ ***A1 N1***

[1 mark]

(a.ii) Find the probability, in terms of n, that the game will end on her second draw. [3]

Markscheme

correct probability for one of the draws ***A1***

eg P(not blue first) = $\frac{n-2}{n}$, blue second = $\frac{2}{n-1}$

valid approach ***(M1)***

eg recognizing loss on first in order to win on second, P(B' then B), P(B') × P(B | B'), tree diagram

correct expression in terms of n ***A1 N3***

eg $\frac{n-2}{n} \times \frac{2}{n-1}$, $\frac{2n-4}{n^2-n}$, $\frac{2(n-2)}{n(n-1)}$

[3 marks]

Let $n = 5$. Find the probability that the game will end on her

(b.i) third draw. [2]

Markscheme

correct working ***(A1)***

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$

$\frac{12}{60}$ $\left(= \frac{1}{5}\right)$ ***A1 N2***

[2 marks]

(b.ii) fourth draw. [2]

Markscheme

correct working ***(A1)***

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$

$\frac{6}{60} \quad \left(= \frac{1}{10}\right)$ ***A1 N2***

[2 marks]

(c) Hayley plays the game when $n = 5$. She pays $20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let *M* be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4
Money earned back ($\$M$)	0	20	$8k$	$12k$

Find the value of k so that this is a fair game. [7]

Markscheme

correct probabilities (seen anywhere) ***(A1)(A1)***

eg $\mathrm{P}\left(1\right) = \frac{2}{5}$, $\mathrm{P}\left(2\right) = \frac{6}{20}$ (may be seen on tree diagram)

valid approach to find E (*M*) or expected winnings using **their** probabilities ***(M1)***

eg $\mathrm{P}\left(1\right) \times \left(0\right) + \mathrm{P}\left(2\right) \times \left(20\right) + \mathrm{P}\left(3\right) \times \left(8k\right) + \mathrm{P}\left(4\right) \times \left(12k\right)$,

$\mathrm{P}\left(1\right) \times \left(-20\right) + \mathrm{P}\left(2\right) \times \left(0\right) + \mathrm{P}\left(3\right) \times \left(8k - 20\right) + \mathrm{P}\left(4\right) \times \left(12k - 20\right)$

correct working to find E (*M*) or expected winnings ***(A1)***

eg $\frac{2}{5}\left(0\right) + \frac{3}{10}\left(20\right) + \frac{1}{5}\left(8k\right) + \frac{1}{10}\left(12k\right)$,

$\frac{2}{5}\left(-20\right) + \frac{3}{10}\left(0\right) + \frac{1}{5}\left(8k - 20\right) + \frac{1}{10}\left(12k - 20\right)$

correct equation for fair game ***A1***

eg $\frac{3}{10}\left(20\right) + \frac{1}{5}\left(8k\right) + \frac{1}{10}\left(12k\right) = 20$, $\frac{2}{5}\left(-20\right) + \frac{1}{5}\left(8k - 20\right) + \frac{1}{10}\left(12k - 20\right) = 0$

correct working to combine terms in k ***(A1)*** *eg* $-8+\frac{14}{5}k-4-2=0$, $6+\frac{14}{5}k=20$, $\frac{14}{5}k=14$

$k=5$ ***A1 N0***

Note: Do not award the final ***A1*** if the candidate's ***FT*** probabilities do not sum to 1.

[7 marks]